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DIVISIBILITY CONDITIONS BASED ON DIGITS OF POSITIVE INTEGERS

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Abstract

In this research, we study the necessary and sufficient conditions for any positive integer N to be divisible by a given positive integer m whenever $N \geq m$. The conditions are related to digits in each place value of the positive integer N . Applications of these conditions are also provided via examples and corollaries. Some of the examples and corollaries show that our main results are generalizations of previous results.

Keywords: digit, place value, divisibility

Introduction

There are several studies concerning divisibility of integers. In 2007, Konick and Mercier (Konick & Mercier, 2007) proposed an interesting divisibility-test problem in the item 323: Let N be a positive integer whose decimal representation is

$$N = a_n 10^n + \cdots + a_2 10^2 + a_1 10 + a_0,$$

where $0 < a_n \leq 9$ and for $k = 0, \dots, n-1, 0 \leq a_k \leq 9$.

Show that

- $3|N \leftrightarrow 3|[a_n + a_{n-1} + \cdots + a_1 + a_0]$.
- $4|N \leftrightarrow 4|[10a_1 + a_0]$.
- $6|N \leftrightarrow 6|[4(a_n + \cdots + a_2 + a_1) + a_0]$.
- $7|N \leftrightarrow 7|[(100a_2 + 10a_1 + a_0) - (100a_5 + 10a_4 + a_3) + (100a_8 + 10a_7 + a_6) - \cdots]$.
- $8|N \leftrightarrow 8|[100a_2 + 10a_1 + a_0]$.
- $9|N \leftrightarrow 9|[a_n + a_{n-1} + \cdots + a_1 + a_0]$.
- $11|N \leftrightarrow 11|[a_n - a_{n-1} + \cdots + (-1)^n a_0]$.

Recently in 2021, Jampawai (Jampawai, 2021) studied conditions on digits in the places of positive integers divisible by 7. He finds that $7|N$ if and only if 7 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, 3, 2, -1, -3, -2, 1, 3, 2, -1, -3, -2, ..., respectively, of the positive integer N . His idea is motivated by Nahir's research [3-4]. In comparison to problem (d) presented by Konick and Mercier, Jampawai's statement offers greater ease of application.

Indeed, a group of students from Chaiyabhum Bhakdeechumphon School, Chaiyabhum, Thailand, applied Jampawai's idea to study conditions on digits in the places of positive integers divisible by 13. They also presented their results in 2023 Mathematical mini-Project contest of Nakhon Ratchasima Rajabhat University Science Week, Nakhon Ratchasima, Thailand. However, their results have not yet been published.

Alongside these ongoing explorations, further formal studies on the divisibility of integers can be seen in [5-6], for example.

In this work, we study the necessary and sufficient conditions for any positive integer N to be divisible by a given positive integer m whenever $N \geq m$. We also solve some parts of the Konick and Mercier's problems by our main theorem and show that Jampawai's result is a special case of our main theorem.

Main Results

Throughout this section, let α be a positive integer such that

$$10^\alpha \equiv \beta \pmod{m},$$

where m is any positive integer, let

$$N = \sum_{i=0}^n 10^i a_i$$

be an $(n+1)$ -digit positive integer (i.e. $a_n a_{n-1} \cdots a_1 a_0$) with $N \geq m$, and let

$$n = \alpha k + l$$

for some integers k, l with $0 \leq l < \alpha$. The following theorem shows the necessary and sufficient conditions for N to be divisible by m .

Theorem 1. $m|N$ if and only if

$$m \mid \left[\sum_{j=0}^{k-1} \beta^j \left(\sum_{i=0}^{\alpha-1} b_i a_{\alpha j+i} \right) + \beta^k \sum_{i=0}^l b_i a_{\alpha k+i} \right],$$

where $b_i \equiv 10^i \pmod{m}$ for any $i \in \{0, 1, 2, \dots, \alpha-1\}$.

Proof. For any $j \in \{0, 1, 2, \dots, k-1\}$, observe that

$$\sum_{i=0}^{\alpha-1} 10^{\alpha j+i} a_{\alpha j+i} \equiv \sum_{i=0}^{\alpha-1} \beta^j 10^i a_{\alpha j+i} \pmod{m}.$$

Consider

$$\begin{aligned} N &= \sum_{i=0}^n 10^i a_i \\ &= \sum_{i=0}^{\alpha k-1} 10^i a_i + \sum_{i=\alpha k}^{\alpha k+l} 10^i a_i \end{aligned}$$

$$\begin{aligned}
&= \sum_{j=0}^{k-1} (10^{\alpha j + (\alpha-1)} a_{\alpha j + (\alpha-1)} + \dots + 10^{\alpha j + 1} a_{\alpha j + 1} + 10^{\alpha j} a_{\alpha j}) \\
&\quad + \sum_{i=0}^l 10^{\alpha k + i} a_{\alpha k + i} \\
&= \sum_{j=0}^{k-1} \sum_{i=0}^{\alpha-1} 10^{\alpha j + i} a_{\alpha j + i} + \sum_{i=0}^l 10^{\alpha k + i} a_{\alpha k + i} \\
&\equiv \sum_{j=0}^{k-1} \beta^j \left(\sum_{i=0}^{\alpha-1} 10^i a_{\alpha j + i} \right) + \beta^k \sum_{i=0}^l 10^i a_{\alpha k + i} \pmod{m}
\end{aligned}$$

This completes the proof by the definition of $b_i \equiv 10^i \pmod{m}$ for any $i \in \{0, 1, 2, \dots, \alpha - 1\}$. $\square$

Corollary 2. Put $\beta = 1$. Then $m|N$ if and only if

$$m \left| \sum_{j=0}^{k-1} \left(\sum_{i=0}^{\alpha-1} b_i a_{\alpha j + i} \right) + \sum_{i=0}^l b_i a_{\alpha k + i} \right|,$$

where $b_i \equiv 10^i \pmod{m}$ for any $i \in \{0, 1, 2, \dots, \alpha - 1\}$.

We next show the application of Corollary 2 when $m = 3$, $m = 9$, and $m = 101$ as follows:

Example 3. Put $m = 3$ or $m = 9$. Since

$$10^1 \equiv 1 \pmod{m},$$

we have that $\alpha = 1$. Then $n = k + l$ for some integers k, l with $0 \leq l < 1$ and so $n = k, l = 0$. It follows from Corollary 2 that $m|N$ if and only if

$$m \left| \sum_{j=0}^{n-1} \left(\sum_{i=0}^0 b_i a_{j+i} \right) + \sum_{i=0}^0 b_i a_{n+i} \right|,$$

where $b_0 \equiv 10^0 \pmod{m}$. Observe that we can possibly put $b_0 = 1 \equiv 10^0 \pmod{m}$.

Consequently, we obtain that $m|N$ if and only if

$$m \left| \sum_{i=0}^n a_i \right|.$$

This implies that $m|N$ if and only if m divides the sum on each digit in each place of the positive integer N . The following two sub-examples are applications of this statement: Recall that $m \in \{3, 9\}$.

1. Put $N = 12345678$ which is divisible by m . Observe that $a_0 = 8, a_1 = 7, a_2 = 6, a_3 = 5, a_4 = 4, a_5 = 3, a_6 = 2, a_7 = 1$ and so $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$ which is also divisible by m .
2. Put $N = 1234567$ which is not divisible by m . Observe that $a_0 = 7, a_1 = 6, a_2 = 5, a_3 = 4, a_4 = 3, a_5 = 2, a_6 = 1$ and so $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ which is not divisible by m too.

Example 4. Put $m = 101$. Since

$$10^4 \equiv 1 \pmod{101},$$

we have that $\alpha = 4$. Then $n = 4k + l$ for some integers k, l with $0 \leq l < 4$. It follows from Corollary 2 that $101|N$ if and only if

$$101 \left| \sum_{j=0}^{k-1} \left(\sum_{i=0}^3 b_i a_{4j+i} \right) + \sum_{i=0}^l b_i a_{4k+i} \right|,$$

where $b_i \equiv 10^i \pmod{101}$ for any $i \in \{0, 1, 2, 3\}$. Observe that we can possibly put

$$\begin{aligned} b_0 &= 1 \equiv 10^0 \pmod{101}, \\ b_1 &= 10 \equiv 10^1 \pmod{101}, \\ b_2 &= -1 \equiv 10^2 \pmod{101}, \\ b_3 &= -10 \equiv 10^3 \pmod{101}. \end{aligned}$$

Consequently, we obtain that $101|N$ if and only if

$$101 \left| \sum_{j=0}^{k-1} (a_{4j} + 10a_{4j+1} - a_{4j+2} - 10a_{4j+3}) + \sum_{i=0}^l b_i a_{4k+i} \right|.$$

This implies that $101|N$ if and only if 101 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and $1, 10, -1, -10, 1, 10, -1, -10, 1, 10, -1, -10, \dots$, respectively, of the positive integer N . The following two sub-examples are applications of this statement:

1. Put $N = 1212303$ which is divisible by 101 . Observe that $a_0 = 3, a_1 = 0, a_2 = 3, a_3 = 2, a_4 = 1, a_5 = 2, a_6 = 1$ and so $3 + 10(0) - 3 - 10(2) + 1 + 10(2) - 1 = 0$ which is also divisible by 101 .
2. Put $N = 1212312$ which is not divisible by 101 . Observe that $a_0 = 2, a_1 = 1, a_2 = 3, a_3 = 2, a_4 = 1, a_5 = 2, a_6 = 1$ and so $2 + 10(1) - 3 - 10(2) + 1 + 10(2) - 1 = 9$ which is not divisible by 101 too.

We conclude from Example 3 and Example 4 as in the following two corollaries:

Corollary 5. For $m \in \{3, 9\}$, $m|N$ if and only if m divides the sum of all digits in each place of the positive integer $N \geq m$.

Corollary 6. $101|N$ if and only if 101 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and $1, 10, -1, -10, 1, 10, -1, -10, 1, 10, -1, -10, \dots$, respectively, of the positive integer $N \geq 101$.

We can deduce a simple practical fact from Corollary 6 as follows:

Corollary 7. If $\sum_{i=0} a_{4i} = \sum_{i=0} a_{4i+2}$ and $\sum_{i=0} a_{4i+1} = \sum_{i=0} a_{4i+3}$, then $101|N$ for an $(n+1)$ -digit positive integer $N \geq 101$, i.e., $a_n a_{n-1} \dots a_1 a_0$.

Example 8. By Corollary 7, the following integers are divisible by 101:

1. 1212 (because $2 = 2$ and $1 = 1$)
2. 23331 (because $1 + 2 = 3$ and $3 = 3$)
3. 679831 (because $1 + 7 = 8$ and $3 + 6 = 9$)
4. 9238167 (because $7 + 3 = 1 + 9$ and $6 + 2 = 8$)
5. 168456789 (because $9 + 5 + 1 = 7 + 8$ and $8 + 4 = 6 + 6$)

In another direction, putting $\beta = -1$ in Theorem 1 yields the simpler statement:

Corollary 9. Put $\beta = -1$. Then $m|N$ if and only if

$$m \mid \left[\sum_{j=0}^{k-1} (-1)^j \left(\sum_{i=0}^{\alpha-1} b_i a_{\alpha j+i} \right) + (-1)^k \sum_{i=0}^l b_i a_{\alpha k+i} \right],$$

where $b_i \equiv 10^i \pmod{m}$ for any $i \in \{0, 1, 2, \dots, \alpha-1\}$.

We next show the applications of Corollary 9 when $m = 11$ and $m = 17$ as follows:

Example 10. Put $m = 11$. Since

$$10^1 \equiv -1 \pmod{11},$$

we have that $\alpha = 1$. Then $n = k + l$ for some integers k, l with $0 \leq l < 1$ and so $n = k, l = 0$. It follows from Corollary 9 that $11|N$ if and only if

$$11 \mid \left[\sum_{j=0}^{n-1} (-1)^j \left(\sum_{i=0}^0 b_i a_{j+i} \right) + (-1)^n \sum_{i=0}^0 b_i a_{n+i} \right],$$

where $b_0 \equiv 10^0 \pmod{11}$. Observe that we can possibly put $b_0 = 1 \equiv 10^0 \pmod{11}$.

Consequently, we obtain that $11|N$ if and only if

$$11 \mid \sum_{i=0}^n (-1)^i a_i.$$

This implies that $11|N$ if and only if 11 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and $1, -1, 1, -1, 1, -1, 1, -1, 1, -1, \dots$, respectively, of the positive integer N . The following two sub-examples are applications of this statement:

1. Put $N = 780197$ which is divisible by 11. Observe that $a_0 = 7, a_1 = 9, a_2 = 1, a_3 = 0, a_4 = 8, a_5 = 7$ and so $7 - 9 + 1 - 0 + 8 - 7 = 0$ which is also divisible by 11.
2. Put $N = 1780198$ which is not divisible by 11. Observe that $a_0 = 8, a_1 = 9, a_2 = 1, a_3 = 0, a_4 = 8, a_5 = 7, a_6 = 1$ and so $8 - 9 + 1 - 0 + 8 - 7 + 1 = 2$ which is not divisible by 11 too.

Example 11. Put $m = 17$. Since

$$10^8 \equiv -1 \pmod{17},$$

we have that $\alpha = 8$. Then $n = 8k + l$ for some integers k, l with $0 \leq l < 8$. It follows from Corollary 9 that $17|N$ if and only if

$$17 \left| \sum_{j=0}^{k-1} (-1)^j \left(\sum_{i=0}^7 b_i a_{8j+i} \right) + (-1)^k \sum_{i=0}^l b_i a_{8k+i} \right|,$$

where $b_i \equiv 10^i \pmod{17}$ for any $i \in \{0, 1, 2, 3, 4, 5, 6, 7\}$. Observe that we can possibly put

$$\begin{aligned} b_0 &= 1 \equiv 10^0 \pmod{17}, \\ b_1 &= -7 \equiv 10^1 \pmod{17}, \\ b_2 &= -2 \equiv 10^2 \pmod{17}, \\ b_3 &= -3 \equiv 10^3 \pmod{17}, \\ b_4 &= 4 \equiv 10^4 \pmod{17}, \\ b_5 &= 6 \equiv 10^5 \pmod{17}, \\ b_6 &= -8 \equiv 10^6 \pmod{17}, \\ b_7 &= 5 \equiv 10^7 \pmod{17}. \end{aligned}$$

Consequently, we obtain that $17|N$ if and only if

$$17 \left| \sum_{j=0}^{k-1} (-1)^j (a_{8j} - 7a_{8j+1} - 2a_{8j+2} - 3a_{8j+3} + 4a_{8j+4} + 6a_{8j+5} - 8a_{8j+6} + 5a_{8j+7}) \right. \\ \left. + (-1)^k \sum_{i=0}^l b_i a_{8k+i} \right|.$$

This implies that $17|N$ if and only if 17 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, -7, -2, -3, 4, 6, -8, 5, -1, 7, 2, 3, -4, -6, 8, -5, 1, -7, -2, -3, 4, 6, -8, 5, ..., respectively, of the positive integer N . The following two sub-examples are applications of this statement:

1. Put $N = 167901234572091$ which is divisible by 17. Observe that $a_0 = 1, a_1 = 9, a_2 = 0, a_3 = 2, a_4 = 7, a_5 = 5, a_6 = 4, a_7 = 3, a_8 = 2, a_9 = 1, a_{10} = 0, a_{11} = 9, a_{12} = 7, a_{13} = 6, a_{14} = 1$ and so $1 - 7(9) - 2(0) - 3(2) + 4(7) + 6(5) - 8(4) + 5(3) - 1(2) + 7(1) + 2(0) + 3(9) - 4(7) - 6(6) + 8(1) = -51$ which is also divisible by 17.
2. Put $N = 852467913$ which is not divisible by 17. Observe that $a_0 = 3, a_1 = 1, a_2 = 9, a_3 = 7, a_4 = 6, a_5 = 4, a_6 = 2, a_7 = 5, a_8 = 8$ and so $3 - 7(1) - 2(9) - 3(7) + 4(6) + 6(4) - 8(2) + 5(5) - 8 = 6$ which is not divisible by 17 too.

From Example 10 and Example 11, we have the following two corollaries:

Corollary 12. $11|N$ if and only if 11 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, ..., respectively, of the positive integer $N \geq 11$.

Corollary 13. $17|N$ if and only if 17 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions

place, ten-millions place, hundred-millions place, billions place, ... and 1, -7, -2, -3, 4, 6, -8, 5, -1, 7, 2, 3, -4, -6, 8, -5, 1, -7, -2, -3, 4, 6, -8, 5, ..., respectively, of the positive integer $N \geq 17$.

Moreover, Corollary 12 leads to a simpler statement as follows:

Corollary 14. If $\sum_{i=0}^n a_{2i} = \sum_{i=0}^n a_{2i+1}$, then $11|N$ for an $(n+1)$ -digit positive integer $N \geq 11$, i.e., $a_n a_{n-1} \cdots a_1 a_0$.

Example 15. By Corollary 14, the following integers are divisible by 11:

1. 1243 (because $3 + 2 = 4 + 1$)
2. 34353 (because $3 + 3 + 3 = 5 + 4$)
3. 895763 (because $3 + 7 + 9 = 6 + 5 + 8$)
4. 7148680 (because $0 + 6 + 4 + 7 = 8 + 8 + 1$)
5. 78987898 (because $8 + 8 + 8 + 8 = 9 + 7 + 9 + 7$)

Finally, we consider $m = 7$ and get the same statement in the main result of (Jampawai, 2021).

Example 16. Put $m = 7$. Since

$$10^3 \equiv -1 \pmod{7},$$

we have that $\alpha = 3$. Then $n = 3k + l$ for some integers k, l with $0 \leq l < 3$. It follows from Corollary 9 that $7|N$ if and only if

$$7 \left| \sum_{j=0}^{k-1} (-1)^j \left(\sum_{i=0}^2 b_i a_{3j+i} \right) + (-1)^k \sum_{i=0}^l b_i a_{3k+i} \right|,$$

where $b_i \equiv 10^i \pmod{7}$ for any $i \in \{0, 1, 2\}$. Observe that we can possibly put

$$b_0 = 1 \equiv 10^0 \pmod{7},$$

$$b_1 = 3 \equiv 10^1 \pmod{7},$$

$$b_2 = 2 \equiv 10^2 \pmod{7}.$$

Consequently, we obtain that $7|N$ if and only if

$$7 \left| \sum_{j=0}^{k-1} (-1)^j (a_{3j} + 3a_{3j+1} + 2a_{3j+2}) + (-1)^k \sum_{i=0}^l b_i a_{3k+i} \right|.$$

This implies that $7|N$ if and only if 7 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, 3, 2, -1, -3, -2, 1, 3, 2, -1, -3, -2, ..., respectively, of the positive integer N .

Corollary 17. $7|N$ if and only if 7 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, 3, 2, -1, -3, -2, 1, 3, 2, -1, -3, -2, ..., respectively, of the positive integer $N \geq 7$.

The application of Corollary 17 can be seen in (Jampawai, 2021). In conclusion, we can say that our main results are generalizations of (Jampawai, 2021).

Conclusions

Let α be a positive integer such that

$$10^\alpha \equiv \beta \pmod{m},$$

where m is any positive integer, let

$$N = \sum_{i=0}^n 10^i a_i$$

be an $(n+1)$ -digit positive integer (i.e. $a_n a_{n-1} \cdots a_1 a_0$) with $N \geq m$, let

$$n = \alpha k + l$$

for some integers k, l with $0 \leq l < \alpha$, and let

$$b_i \equiv 10^i \pmod{m}$$

for any $i \in \{0, 1, 2, \dots, \alpha-1\}$. We have proved that $m|N$ if and only if

$$m \left| \sum_{j=0}^{k-1} \beta^j \left(\sum_{i=0}^{\alpha-1} b_i a_{\alpha j + i} \right) + \beta^k \sum_{i=0}^l b_i a_{\alpha k + i} \right|.$$

It follows from putting $\beta = 1$ that $m|N$ if and only if

$$m \left| \sum_{j=0}^{k-1} \left(\sum_{i=0}^{\alpha-1} b_i a_{\alpha j + i} \right) + \sum_{i=0}^l b_i a_{\alpha k + i} \right|.$$

On the other hand, putting $\beta = -1$ yields the fact that $m|N$ if and only if

$$m \left| \sum_{j=0}^{k-1} (-1)^j \left(\sum_{i=0}^{\alpha-1} b_i a_{\alpha j + i} \right) + (-1)^k \sum_{i=0}^l b_i a_{\alpha k + i} \right|.$$

Although these facts may not be immediately practical, we yield some simpler conditions for specific values of m as follows:

1. For $m \in \{3, 9\}$, $m|N$ if and only if m divides the sum of all digits in each place of the positive integer $N \geq m$.
2. $101|N$ if and only if 101 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and $1, 10, -1, -10, 1, 10, -1, -10, 1, 10, -1, -10, \dots$, respectively, of the positive integer $N \geq 101$.
3. If $\sum_{i=0} a_{4i} = \sum_{i=0} a_{4i+2}$ and $\sum_{i=0} a_{4i+1} = \sum_{i=0} a_{4i+3}$, then $101|N$ for an $(n+1)$ -digit positive integer $N \geq 101$, i.e., $a_n a_{n-1} \cdots a_1 a_0$.
4. $11|N$ if and only if 11 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions

place, ten-millions place, hundred-millions place, billions place, ... and 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, ..., respectively, of the positive integer $N \geq 11$.

5. If $\sum_{i=0} a_{2i} = \sum_{i=0} a_{2i+1}$, then $11|N$ for an $(n+1)$ -digit positive integer $N \geq 11$, i.e., $a_n a_{n-1} \cdots a_1 a_0$.
6. $17|N$ if and only if 17 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, -7, -2, -3, 4, 6, -8, 5, -1, 7, 2, 3, -4, -6, 8, -5, 1, -7, -2, -3, 4, 6, -8, 5, ..., respectively, of the positive integer $N \geq 17$.
7. $7|N$ if and only if 7 divides the sum of products between each digit in the ones place, tens place, hundreds place, thousands place, ten-thousands place, hundred-thousands place, millions place, ten-millions place, hundred-millions place, billions place, ... and 1, 3, 2, -1, -3, -2, 1, 3, 2, -1, -3, -2, ..., respectively, of the positive integer $N \geq 7$.

Other practical divisibility conditions for specific positive integers can also be deduced from our main results.

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