Transformed Lindley Distribution and Application for Acceptance Sampling Plan

Kanittha Yimnak[1]* and Panittha Meechobtham[2]

[1*]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, Thailand, 12110.

[2]Faculty of Business Administration, Panyapiwat Institute of Management, Nonthaburi, Thailand, 11120.

E-mail: kanittha_y@rmutt.ac.th

Abstract

A two-parameter Lindley distribution transformed using Log-expo transformation (LET-TLD) and involved one parameter for more flexible with lifetime distribution is proposed in term the probability density function, cumulative distribution function , survival function, hazard function, some moments and applying for real data, respectively. Then, the transformed distribution is applied for acceptance sampling plan (ASP) based on a truncated life test under LET-TLD. The developed ASP is presented along with the minimum sample sizes desired to testify that the specified mean life is attained, the values of operating characteristic function and the producer's risk of ASP under LET-TLD. The results show that the developed ASP is gainful in producing lot.

Keywords: Two-parameter Lindley distribution; Log-expo transformation; Acceptance sampling plan

1. Introduction

Lifetime distribution is wildly knowledge in work about medical, engineering, insurance, finance and other. Lifetime distribution is always explained by exponential distribution, Weibull distribution and Lindley distribution [1,2]. Lindley distribution (TLD), introduced by Lindley [3], is one of the popular distributions for describing lifetime distributions. It is developed continuously by adding some parameters or transformation methods to fit lifetime distribution, for example, Shanker and Mishra [4] proposed the probability density function (PDF) and cumulative distribution function (CDF) of two-parameter Lindley distribution (TLD) as equation (1) and (2) [4,5].

$$f(x;\alpha,\theta)=\frac{\theta^2}{\theta+\alpha}(1+\alpha x)e^{-\theta x} \quad ; \quad x>0,\theta>0 \text{ and } \alpha>-\theta, \tag{1}$$

$$F(x;\alpha,\theta)=1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x} \; ; \; x>0,\theta>0 \text{ and } \alpha>-\theta, \tag{2}$$

where $X \sim TLD(\alpha,\theta)$ is a random variable, α and θ be the shape and scale parameters, respectively. Transformation of lifetime distributions is one of the famous methods to increase the performance of the lifetime distributions. There are many transformation methods, for example, Maurya et al.[6] presented

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***Corresponding author**

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logarithm transformed method by taking some distributions and having parsimonious in parameter along with increase elasticity of the parent distribution, Aslam et al. [7] proposed the log-expo transformation by adding one shape parameter (λ) for Fre′chet, Exponential and Lomax distributions. They found that the new generators are more flexible lifetime distribution and correspond the real data. The PDF (g(x)) and CDF (G(x)) of the log-expo transformation method shows in equation (3) and (4), respectively,

$$g(x;\lambda,\Theta)=\frac{\lambda f(x;\Theta)e^{-\lambda F(x;\Theta)}}{log\left\{2-e^{-\lambda}\right\}\left\{2-e^{-\lambda F(x;\Theta)}\right\}}, \tag{3}$$

$$G(x;\lambda,\Theta)=\frac{log\left\{2-e^{-\lambda F(x;\Theta)}\right\}}{log\left\{2-e^{-\lambda}\right\}}, \tag{4}$$

where $g(x,\Theta)$ and $G(x,\Theta)$ are the PDF and the CDF of baseline distributions with the parameter Θ. To increase the consistency between the lifetime distribution and the real data. Lindley distribution is transformed using log-expo transformation method along with studying some statistical properties. Then, the transformed distribution is applied for acceptance sampling plan (ASP). The ASP is the important process for statistical process control because it has been widely used to see if the lot is acceptance or not by investigating samples. In modern times, there are many researchers developing the ASP to correspond the product lifetime data, for example, Lio et al. [8] presented acceptance sampling plans from truncated life tests based on the urr type XII percentiles, Al-Nasser et al. [9] proposed acceptance sampling plan based on truncated life tests for exponentiated Fre′chet distribution, Gui and Aslam [10] presented acceptance sampling plans based on truncated life tests for weighted exponential distribution and Al-Omari et al. [11] proposed ASP based on truncated life test for Akash distribution with application to electric carts data. To show the application of the transformed Lindley distribution, the transformed distribution is applied for ASP to increase the efficiency for ASP. Moreover, there are studying the efficiency of the developed ASP. This research presents as follows: section 2 offers Lindley distribution transformed using log-expo method along with statistical properties and applying real data. Section 3 presents the ASP under the transformed Lindley distribution and finally, the conclusions are presented in section 4.

2. A two-parameter Lindley distribution transformed using Log-expo Transformation (LET-TLD)

Let $X \sim LET\text{-}TLD\ (\lambda,\alpha,\theta)$, the PDF and CDF of the two-parameter Lindley distribution transformed using a log-expo that based on Aslam et al.[7]'s research is revealed in equation (5) and (6),

$$g_1(x;\lambda,\alpha,\theta)=\frac{\lambda\theta^2(1+\alpha x)e^{-\theta x}e^{-\lambda\left(1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x}\right)}}{(\theta+\alpha)log\left\{2-e^{-\lambda}\right\}\left\{2-e^{-\lambda\left(1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x}\right)}\right\}}, \tag{5}$$

$$G_1(x;\lambda,\Theta)=\frac{\log\left\{2-e^{-\lambda\left(1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x}\right)}\right\}}{\log\left\{2-e^{-\lambda}\right\}}. \tag{6}$$

The PDF and CDF plots of LET-TLD are shown as figure 1.

2.1 Survival and hazard functions

The survival function($S(x)$) is a property that a subject survives longer than x and the hazard ($H(x)$) function is the ratio of the PDF to ($S(x)$). The survival and hazard function of LET-TLD can be written as equations (7) and (8),

$$S_1(x;\lambda,\alpha,\theta)=1-\ G_1(x;\lambda,\alpha,\theta)=1-\frac{\log\left\{2-e^{-\lambda\left(1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x}\right)}\right\}}{\log\left\{2-e^{-\lambda}\right\}}, \tag{7}$$

$$H_1(x;\lambda,\alpha,\theta)=\frac{g_1(x;\lambda,\alpha,\theta)}{S_1(x;\lambda,\alpha,\theta)}=\frac{\lambda\theta^2(1+\alpha x)e^{-\theta x}M}{(\theta+\alpha)(2-M)\left(\log\left(2-e^{-\lambda}\right)-\log(2-M)\right)}, \tag{8}$$

where $M=e^{-\lambda\left(1-\frac{\theta+\alpha+\alpha\theta x}{\theta+\alpha}e^{-\theta x}\right)}$. The $S(x)$ and $H(x)$ plots of LET-TLD are shown as figure 2.

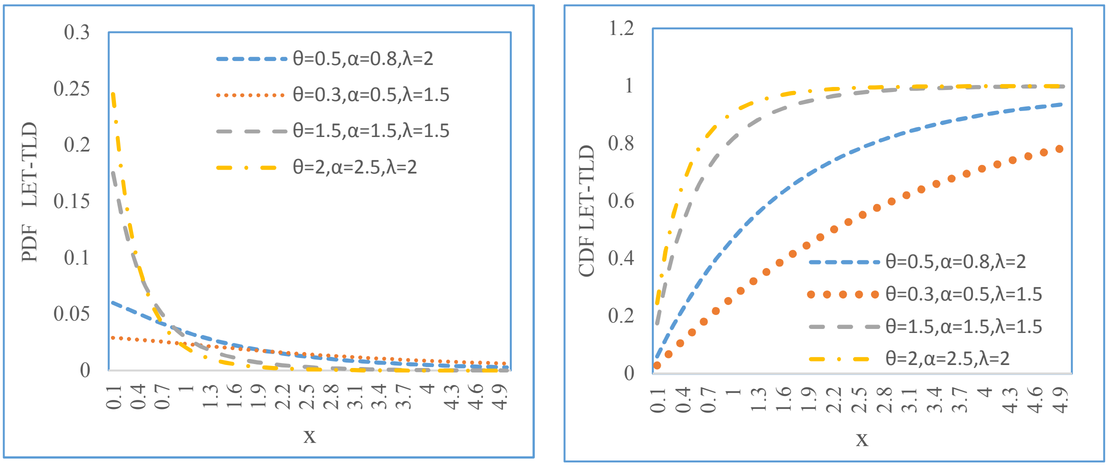

Figure 1. PDF and CDF plots of LET-TLD.

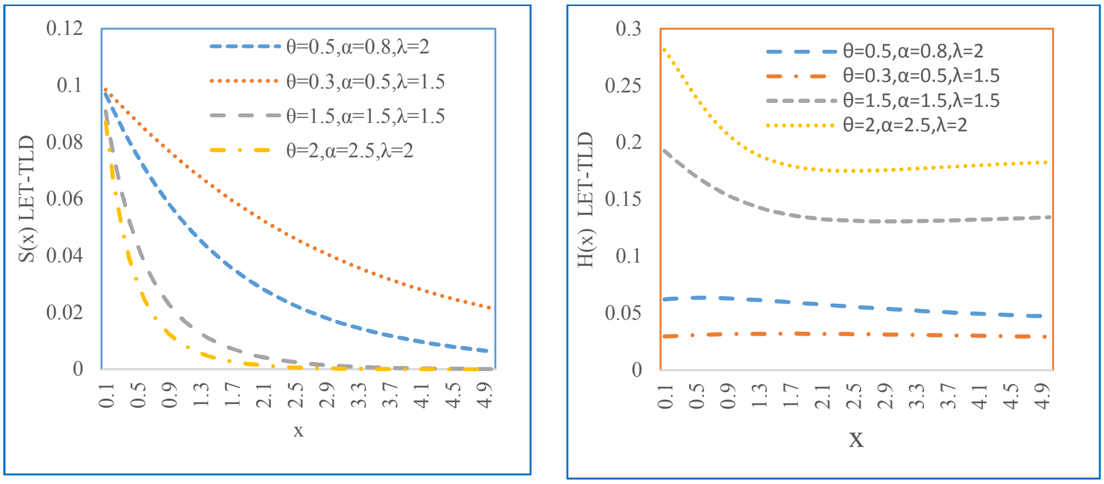

Figure 2. *S(x)* and *H(x)* plots of LET-TLD

Figure 1 reveals the PDF and CDF plots of LET-TLD. The PDF graphs are quite steep and rapidly decreasing as the parameter values increase. Figure 2 presents *S(x)* and *H(x)* plots of LET-TLD. The *S(x)* graphs are exponential decreasing graph while the *H(x)* graphs tend to decrease and after that they stay steady.

2.2 Moments of LET-TLD

Because the moments of LET-TLD are not closed form, the 1st, 2nd, 3rd and 4th moments ($\mu_x^{(n)}, n = 1,2,3,4$), standard deviation (SD), skewness coefficient (SC) and kurtosis coefficient (KC) for the different parameters are calculated using a package program as table 1. In table 1, they are found that mostly LET-TLD models are highly skewed on the positive side (SC >1[12]) and are relatively high kurtosis coefficient (KC>3, KC<3).

Table 1 Moments of LET-TLD for different combinations of parameters.

Moment	λ=3 α=0.5 θ=0.8	λ=3 α=1.5 θ=2.5	λ=0.5 α=0.7 θ=0.5	λ=0.5 α=3 θ=6
$M_X^{(1)}$	0.7255	0.1898	1.6124	0.1886
$M_X^{(2)}$	1.1656	0.1038	4.5496	0.0776
$M_X^{(3)}$	2.8246	0.1088	15.3088	0.0497
$M_X^{(4)}$	19.896	0.2052	870.60	0.0292
SD	0.7995	0.2603	1.3963	0.2050
SC	2.0571	3.5915	0.6191	2.2295
KC	35.6033	30.7258	216.3717	2.5315

2.3 Application for a real data

The proposed distributions are applied for two real lifetime data. The descriptive statistics of the data sets are represented in table 2 and the details of the datasets used for this numerical experiment are as:

Dataset 1 : the dataset represents the failure times of the air conditioning system of an airplane [13].

23	261	87	7	120	14	62	47	225	71	246	21
42	20	5	12	120	11	3	14	71	11	14	11
16	90	1	16	52	95						

Dataset2: this data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane [14].

74 57 48 29 502 12 70 21 29 386 59 27 153 26 326

Descriptive statistics of the two datasets show in table2. Table 2 reveals that the two datasets are highly skewed on the positive side.

Table 2 Mean, standard deviation (SD), skewness coefficients (SC) and kurtosis coefficients (KC) of the two datasets.

Data set	**Mean**	**SD**	**SC**	**KC**
1	59.6000	71.88477	1.784	2.569
2	121.2667	154.26854	1.698	1.723

The performance of parameter estimation for LET-TLD is examined by –LL, Akaike's information criterion (AIC), Bayesian information criterion (BIC) and Kolmogorov-Smirnov test (KS) as equation (9), (10) and (11),

$$AIC = -2\,log\left(LL\right) + 2p \quad , \tag{9}$$

$$BIC = -2\,log\left(LL\right) + p\,log(n)\,, \tag{10}$$

$$KS = \sup_{x}\left|F_n\left(x\right) - F_0\left(x\right)\right| \;, \tag{11}$$

where n is a number of samples, p is a number of parameters. An appropriate distribution is distribution that gives the lowest AIC, BIC and the KS with a p-values > 0.05.

Table 3 The parameter estimate, -LL, AIC, BIC and K-S statistics of the dataset 1

Distribution	**Paramete r**	**Paramet er estimate**	**-LL**	**AIC**	**BIC**	**KS Statistics (p-values)**
TLD	α	-0.00058	152.6397	309.2794	312.0818	0.2100 (0.1205)
	θ	0.01634				
LET-TLD	λ	1.35694	151.4899	308.9798	313.1834	0.1349 (0.5932)
	α	-1.01×10^{-6}				
	θ	0.01021				

Table 3 and table 4 represent the parameter estimations, –LL, AIC, BIC and KS statistics. They are found that the LET-TLD provides the parameter estimators regarding the best fit for real lifetime data set because this gives the lower of - LL, AIC and BIC than the TLD. In addition, LET-TLD provides KS tests with a p-values >0.05 being more than TLD.

Table 4 The parameter estimate, -LL, AIC, BIC and KS statistics of dataset 2

Distribution	Parameter	Parameter estimate	-LL	AIC	BIC	KS Statistics (p-values)
TLD	α	-0.00153	87.12668	178.2534	179.6695	0.3096 (0.0881)
	θ	0.00503				
LET-TLD	λ	1.58735	86.1840	178.3680	180.4921	0.1930 (0.5612)
	α	0.00051				
	θ	0.00513				

3. Acceptance Sampling Plan Under LET-TLD

Suppose that the product lifetime is LET-TLD pattern as shown in the equation (5) for which the variables and symbols are defined as follows [10,11]:

t means the longest lifetime of the product being measured

t_0 means specified lifetime of the product being measured

c means the number of products being failure within the test time (t)

μ means average actual lifetime of product

μ_0 means specified average lifetime of product

Given the product lifetime being test for failure at the time t_0 when the product lifetime is valued in the range of [0, t], the product lots to be tested will be accepted if the number of products being failure does not exceed the specified amount (c) on the hypothesis that in case of large product lots, the ASP is determined to be used under the binomial distribution, in the condition that $1-P^*$ is risk that customers or buyers will receive defective product lots under the hypothesis $\mu < \mu_0$. In other words, actual average lifetime (μ) has the value not exceeding the specified lifetime (μ_0), that is, such probability will be less than $1-P^*$. Such hypothesis testing can be efficient only in case where a computation for the smallest and most appropriate sample size (n) is carried out. By this, the appropriate sample size computation is shown in the inequality (12),

$$\sum_{i=0}^{c}\binom{n}{i}p^i\left(1-p\right)^{n-i} \le 1-P^*, \tag{12}$$

where c is the number of products being failure within the time t , $P^* \in (0,1)$ and $p = F(t;\mu_0)$ are the probability that a product is found to be defective within the time t which will correlate to the ratio t/μ_0. The p value is shown in the equation (13),

$$p = \frac{\log\left\{2-e^{-\lambda\left(1-\frac{\theta+\alpha+\alpha\theta U}{\theta+\alpha}e^{-\theta U}\right)}\right\}}{\log\left\{2-e^{-\lambda}\right\}}, \tag{13}$$

when given $U = m_0\left(\dfrac{d}{\mu / \mu_0}\right)$ where $d = t_0 / \mu_0$ and m_0 is mean which is not in a closed-form expression, if the number of products being failure within the time t do not exceed than c or in other words $F(t;\mu) \le F(t;\mu_0)$,which hints $\mu_0 \le \mu$, a computation for the smallest sample size for ASP under LET-TLD is carried out by giving t / μ_0=0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 and P^* = 0.75, 0.90 and 0.95 respectively. The values of t / μ_0 and P^* are determined according to [10,11]. For the operating characteristic function of ASP $(n,c,t/\mu_0)$ or the probability of acceptance of product lots, in selecting the smallest sample size n or the number of acceptable degraded product prior to the specified time, the acceptance sampling plan is defined in the equation (14),

$$\begin{aligned} OC(p) &= P(\text{Acceptaning a lot} \mid \mu < \mu_0) \\ &= \sum_{i=0}^{c}\binom{n}{i} p^i (1-p)^{n-i}, \end{aligned} \tag{14}$$

where $p = F(t_0;\mu)$ and producer's risk or the risk that a product will be rejected by customers despite the fact that they should have accepted the product $\mu > \mu_0$ is shown in the equation (15),

$$\begin{aligned} PR &= P(\text{Rejecting a lot}) \\ &= \sum_{i=c+1}^{n}\binom{n}{i} p^i (1-p)^{n-i}, \end{aligned} \tag{15}$$

As $p = F\left(\dfrac{t}{\mu_0}\dfrac{\mu_0}{\mu}\right)$ is the function of μ / μ_0 where μ / μ_0 is the smallest integer to cause p to have a correlation according to the inequality (16),

$$\sum_{i=c+1}^{n}\binom{n}{i} p^i (1-p)^{n-i} \le \Re, \tag{16}$$

where $\Re$ is a value of risk of producer and p is shown in the equation (17),

$$p = \frac{\log\left\{2 - e^{-\lambda\left(1 - \frac{\theta+\alpha+\alpha\theta U}{\theta+\alpha} e^{-\theta U}\right)}\right\}}{\log\left\{2 - e^{-\lambda}\right\}}. \tag{17}$$

If given r is a value at risk of producer under the developed ASP, the values of μ / μ_0 are the smallest ratio to confirm that a value at risk of producer is not higher than r when r is the least positive value that affects the value $p = F\left(\dfrac{t}{\mu_0}\dfrac{\mu_0}{\mu}\right)$ to correlate to the inequality (18),

$$\sum_{i=c+1}^{n}\binom{n}{i} p^i (1-p)^{n-i} \le r. \tag{18}$$

The numerical experiments and discussions are shown as the next topic.

Numerical experiments and Discussions

The LET-TLD is applied for ASP based on truncated lifetime with setting $\lambda = 3, \alpha = 0.5, \theta = 0.8$, $mean = M_x^{(1)} = 0.7255$ and $\lambda = 0.5, \alpha = 0.7, \theta = 0.5$, $mean = M_x^{(1)} = 1.6124$, respectively (refer to table 1). Table 5 represents the minimum sample size (*n*) and the least of μ / μ_0 for $r = 0.05$. The ASP under LET-TLD provides the minimum sample size being rather small. However, the values of μ / μ_0 increase when the values of t / μ_0 increase. The values of OCF is closed to 1 when μ / μ_0 increase and they decrease when *n* decrease (see in table 6). The results correspond [10], [11] and [12]. For interpretation of the research results, assume that the lifetime of the products follows the LET-TLD with parameters $\lambda = 3, \alpha = 0.5, \theta = 0.8$, $\mu_0 = 1000$ hours $\mu \geq \mu_0$, $P^* = 0.95$ and the life test will be finished at $t_0 = 942$ hours. The ASP under LET-TLD provides the minimum sample size *n* = 7 units when $P^* = 0.95, t_0 / \mu_0 = 0.942$ and $c = 2$.

4. Conclusions

Lindley distribution transformed using log-expo transformation (LET-TLD) with one additional parameter is proposed for lifetime distribution, including statistical properties and parameter estimation. The proposed distribution is explained by using the two real lifetime datasets, a high skewed distribution on the positive side. The LET-TLD is flexible and sustainable with lifetime distribution. Then, the proposed distribution is applied for acceptance sampling plan based on truncated life tests. Since the increasing of the parameters λ, α and θ affects to the slope and the decrease of the PDF graphs, the parameter values set in this research refer to table1 for SC>1 $(\lambda = 3, \alpha = 0.5, \theta = 0.8)$ and SC<1 $(\lambda = 0.5, \alpha = 0.7, \theta = 0.5)$, respectively. The essential tables are represented for the minimum sample size (*n*) required to guarantee an absolute mean life of the test units. The values of operating characteristic function and the producer's risks are also offered. The results show that the ASP under LET-TLD is profitable in producing lot.

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Table 5 Minimum sample sizes (*n*) and minimum ratio of μ / μ_0 for $r = 0.05$ to be tested for a time *t* to assert with probability P^* and acceptance number *c* that $\mu \geq \mu_0$ in LET-TLD.

P^*	c	$\lambda = 3, \alpha = 0.5, \theta = 0.8$								$\lambda = 0.5, \alpha = 0.7, \theta = 0.5$							
		t / μ_0								t / μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	2	2	1	1	1	1	1	1	4	3	2	2	1	1	1	1
	μ / μ_0	39.046	58.569	38.845	48.548	72.807	97.066	121.355	145.614	24.942	28.115	25.108	31.380	23.788	31.714	39.650	47.576
	1	4	3	3	3	2	2	2	2	9	6	5	4	3	3	2	2
	μ / μ_0	9.578	10.050	13.411	16.760	14.106	18.806	23.512	28.212	7.734	7.557	8.277	8.076	8.674	11.564	8.551	10.261
	2	6	5	4	4	3	3	3	3	13	9	7	6	4	4	3	3
	μ / μ_0	5.834	6.868	6.618	8.271	7.443	9.922	12.405	14.885	11.320	11.601	11.885	12.602	12.112	16.147	14.458	17.348
0.90	0	3	2	2	2	1	1	1	1	7	5	4	3	2	2	1	1
	μ / μ_0	58.686	58.569	78.154	97.677	72.807	97.066	121.355	145.614	43.537	46.711	49.924	46.889	47.06	62.74	39.65	47.576
	1	6	4	4	3	3	2	2	2	12	8	6	5	4	3	3	2
	μ / μ_0	15.287	14.368	19.172	16.760	25.135	18.806	23.512	28.212	10.424	10.255	10.084	10.345	12.112	11.564	14.458	10.261
	2	8	6	5	5	4	4	3	3	17	11	9	7	5	4	4	4
	μ / μ_0	8.324	8.751	9.165	11.454	12.404	16.536	12.405	14.885	6.3538	6.0138	6.4529	6.0884	6.1233	6.1015	7.6283	9.1532
0.95	0	4	3	2	2	2	1	1	1	9	6	5	4	3	2	2	1
	μ / μ_0	78.326	88.029	78.154	97.677	146.484	97.066	121.355	145.614	55.932	56.008	62.331	62.396	70.319	62.740	78.440	47.576
	1	7	5	4	4	3	3	3	2	14	10	7	6	4	4	3	3
	μ / μ_0	18.133	18.656	19.172	23.961	25.135	33.510	41.896	28.212	12.216	12.947	11.885	12.602	12.112	16.147	14.458	17.348
	2	9	7	6	5	4	4	4	4	19	13	10	8	6	5	4	4
	μ / μ_0	9.564	10.622	11.677	11.454	12.404	16.536	20.675	24.807	7.134	7.188	7.240	7.079	7.636	8.164	7.628	9.153

Table 6 OCF for the sampling plan (*n*,*c*=2, t / μ_0) with a given probability $P^* = 0.95$ in LET-TLD.

t / μ_0	$\lambda = 3, \alpha = 0.5, \theta = 0.8$							$\lambda = 0.5, \alpha = 0.7, \theta = 0.5$						
	n	μ / μ_0						*n*	μ / μ_0					
		2	4	6	8	10	12		2	4	6	8	10	12
0.628	9	0.3005	0.7052	0.8604	0.9246	0.9550	0.9711	19	0.4138	0.8140	0.9241	0.9624	0.9788	0.9869
0.942	7	0.2573	0.6583	0.8302	0.9055	0.9425	0.9626	13	0.3996	0.8092	0.9225	0.9618	0.9785	0.9868
1.257	6	0.2213	0.6136	0.7993	0.8852	0.9289	0.9531	10	0.3882	0.8047	0.9209	0.9611	0.9782	0.9866
1.571	5	0.2490	0.6328	0.8093	0.8907	0.9321	0.9551	8	0.3996	0.8122	0.9249	0.9634	0.9796	0.9875
2.356	4	0.2425	0.6076	0.7869	0.8740	0.9199	0.9461	6	0.3421	0.7785	0.9096	0.9556	0.9752	0.9848
3.141	4	0.1301	0.4528	0.6659	0.7869	0.8574	0.9004	5	0.2974	0.7461	0.8939	0.9474	0.9705	0.9819
3.927	4	0.0714	0.3321	0.5522	0.6958	0.7869	0.8460	4	0.3500	0.7780	0.9094	0.9557	0.9754	0.9850
4.712	4	0.0403	0.2425	0.4527	0.6076	0.7140	0.7869	4	0.2332	0.6862	0.8619	0.9299	0.9602	0.9754

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