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The Forecasting Efficiency of Fuzzy Time Series Model Based on Fuzzy Inverse for Forecasting Thailand Fruit Price

Rungsarit Intaramo[1] and Kanittha Yimnak[2,*]

[1]Department of Mathematics and Statistics, Faculty of Science, Thaksin University, Thailand

[2]Department of Applied Statistics, Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi, Thailand

Email: yimnak@gmail.com

Abstract

Fuzzy time series model based on fuzzy inverse and SARIMA model were applied for forecasting Tubtim Chandra Rose Apple and Shogun Orange prices in Thailand in different seasons, and the data gained from the study is uncertain. As a result of data uncertainty, the accuracy of two methods was compared by the mean absolute percentage error (MAPE). The results show that the forecast values by the modified fuzzy time series forecasting method is more accurate than the SARIMA model.

Keywords: Fuzzy Time Series, SARIMA Model, Uncertain Data.

1. Introduction

The efficiency of forecasting model is an interesting topic for many researchers because of an uncertainty of future effects on decision maker needed in forecast. There are many traditional time series forecasting methods such as Holt winter, smoothing exponential and Box-Jenkins are well-known and famous for time series data because there are many convenient program for solving.

However, those methods are limited to statistical assumptions. If the analyzed time series data does not comply with the assumptions or they are uncertain data, it affects the accuracy of forecasting. Fuzzy time series forecasting method is a good alternative for uncertain data. The fuzzy time series forecasting model was first proposed by Song and Chissom [1]. They used the model to predict functional new enrollment prediction. After that these models was continuously improved to predict temperature [2], stock index [3], exchange rate [4] and data mining [5]. Nowadays, Fuzzy time series forecasting method is continuously improved by many researchers to increase the accuracy; for example, Preetika Saxena et al. [6] used fuzzy time series with higher forecast accuracy rate forecasting enrollments. Then Kun Zhang et al. [7] developed Preetika Saxena et al. [6] method by designing fuzzy time series prediction model based on fuzzy inverse, and they found that the mean square error and prediction error of ameliorated model are smaller and the precision is higher.

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***Corresponding author**

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In this paper, Fuzzy time series model based on fuzzy inverse was applied for forecasting the prices per kilogram of Shogun Orange and Tubtim Chandra Rose Apple in Thailand. The prices per kilogram of Shogun Orange (middle size) and Tubtim Chandra Rose Apple (middle size) are the retail price in Bangkok collected from Bureau of Trade and Economic indices [8]. These fruit prices tend to unexpectedly fluctuate due to seasonal influences. The accuracy of the presented method and the traditional method was compared by the mean absolute percentage error (MAPE).

2. SARIMA model

SARIMA(p,d,q)(P,D,Q)$_S$ is a seasonal ARIMA model with S observation per period. The equation of SARIMA model are as follows ([9]; [10]; [11]):

$$\phi_P(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D Z_t = \theta_q(B)\Theta_Q(B^S)a_t$$

Where:

$$\phi_p(B) = \left(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p\right)$$
$$\Phi_P(B^s) = (1 - \Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_P B^{Ps})$$
$$\theta_q(B) = \left(1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q\right)$$
$$\Theta_Q(B^s) = \left(1 - \Theta_1 B^s - \Theta_2 B^{2s} - \ldots - \Theta_Q B^{Qs}\right)$$

Where:

B = The backward shift operator

d and D = The non-seasonal and seasonal order of differences, respectively and usually abbreviated as SARIMA(p,d,q)(P,D,Q)$_S$.

The SARIMA model reduces to pure ARIMA(p,d,q) if there is no seasonal effect. Tubtim Chandra Rose Apple price, Figure1 shows ACF and PACF with D=12 for Tubtim Chandra Rose Apple price.

The SARIMA models corresponding Tubtim Chandra Rose Apple price were compared by the mean absolute percentage error (MAPE)

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - f_i}{y_i}\right| \times 100 \ ,$$

The models that correspond Tubtim Chandra Rose Apple price are as shown in Table1. The best model that there is a smallest error for forecasting Tubtim Chandra Rose Apple price in Thailand is SARIMA$(1,0,0)(1,1,1)_{12}$.

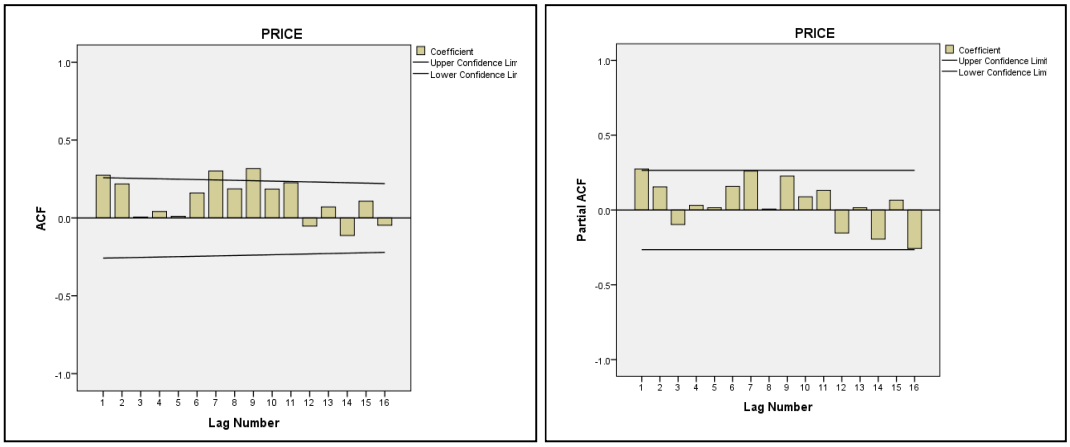

Figure 1 ACF and PACF of Tubtim Chandra Rose Apple price with D=12

Table 1 The MAPE of SARIMA model for forecasting Tubtim Chandra Rose Apple price in Thailand

Model	MAPE
$SARIMA(1,1,0)_{12}$	5.028
$SARIMA(0,1,1)_{12}$	5.010
$SARIMA(1,1,1)_{12}$	5.025
$SARIMA(1,0,0)(1,1,0)_{12}$	4.993
$SARIMA(1,0,0)(0,1,1)_{12}$	4.881
$SARIMA(1,0,0)(1,1,1)_{12}$	4.731
$SARIMA(0,0,1)(1,1,0)_{12}$	5.016
$SARIMA(0,0,1)(0,1,1)_{12}$	4.831
$SARIMA(0,0,1)(1,1,1)_{12}$	4.721

3. Basic Concepts

This section describes the basic concepts used by Preetika Saxena et al. [6] for fuzzy time series.

Definition 3.1 the definition of the percentage changes in real data is the following:

$$\frac{y_i - y_{i-1}}{y_{i-1}} \times 100\%,$$

y_i, y_{i-1} are the real data of history at time $i, i-1$, respectively.

Definition 3.2 let $P = \{P_1, P_2, \dots, P_n\}$ be nonempty discrete set, and X be named fuzzy subset in P. A map $\mu_X: P \to [0,1]$ is called membership function of fuzzy subset X. $\mu_X(P_i), (P_i \in P, i = 1,2, \dots, n)$ is called membership degree of fuzzy set for the element P_i to fuzzy set X. Fuzzy set X can be written as

$$X = \frac{\mu_X(P_1)}{P_1} + \frac{\mu_X(P_2)}{P_2} + \cdots + \frac{\mu_X(P_n)}{P_n}.$$

4. Fuzzy Time Series Prediction Model Based on Fuzzy Inverse

Kun Zhang et al. [7] developed fuzzy time series prediction model from Preetika Saxena et al. [6] who proposed a fuzzy time series forecasting model of inverse based on fuzzy number. The basic steps are as follows:

Step 1: Collecting the real historical data

Step 2: Calculating the percent month to month changes of real historical data for Tubtim Chandra Rose Apple price,

$$P_{Feb-12} = \frac{Y_{Feb-12} - Y_{Jan-12}}{Y_{Jan-12}} \times 100 = \frac{36.71 - 46.39}{46.39} \times 100 = -20.867\%$$

Step 3: Constructing the discrete domain

The percentage changes of the Tubtim Chandra Rose Apple price from Jan-2012 to Sep-2017 in the minimum and maximum values are $P_{Feb-12} = -20.867$ and $P_{Mar-12} = 31.463$. The discrete domain is established by the percentage change of the Tubtim Chandra Rose Apple price:

$$P = \left\{P^1_{Feb-12} = -20.867, P^2_{Jan-15} = -18.847, \dots, P^{67}_{Jul-13} = 16.518, P^{68}_{Mar-12} = 31.463\right\}.$$

Step 4: Establishing fuzzy inverse formula and the prediction formula

Establishing fuzzy subset on discourse domain *P*:

$$x_1 = \frac{1}{P^1_{Feb-12}} + \frac{0.05}{P^2_{Jan-15}} + \frac{0}{P^3_{May-12}} + \cdots + \frac{0}{P^{68}_{Mar-12}} \;\; ; \;\; i = 1$$

$$x_i = \frac{0}{P^1_{Feb-12}} + \cdots + \frac{0.05}{P^{i-1}_x} + \frac{1}{P^i_y} + \frac{0.05}{P^{i+1}_z} \ldots + \frac{0}{P^{68}_{Mar-12}} \;\; ; \;\; 2 \le i \le 67$$

$$x_{68} = \frac{0}{P^1_{Feb-12}} + \ldots + \frac{0}{P^{67}_{Mar-15}} + \frac{0.05}{,P^{67}_{Jul-13}} + \frac{1}{P^{68}_{Mar-12}} \;\; ; \;\; i = 68$$

The fuzzy numbers above can be written by:

$$J_1 = \frac{1}{P^1_{Feb-12}} + \frac{0.05}{P^2_{Jan-15}}; \;\; i = 1$$

$$J_i = \frac{0.05}{P^{i-1}_x} + \frac{1}{P^i_y} + \frac{0.05}{P^{i+1}_z} \;\; ; \;\; 2 \le i \le 67$$

$$J_{68} = \frac{0.05}{P^{67}_{Jul-13}} + \frac{1}{P^{68}_{Mar-12}} \;\; ; \;\; i = 68$$

The operations of fuzzy numbers, fuzzy number is actually a real number. For example:

$$\frac{0}{P^i_x} \; (\text{i=1,2,3,...,68} \; ; \; x = \; Jan-12, Feb-12, \ldots, Sep-17)$$

For example:

$$J_1 = \frac{1}{-20.8665660} + \frac{0.05}{-18.8468782} \;\; = \;\; -0.0505765$$

$$J_2 = \frac{0.05}{-20.8665660} + \frac{1}{-18.8468782} + \frac{0.05}{-14.0241365} \;\; = \;\; -0.0590206$$

The inverse fuzzy number formula on discourse domain P is the following:

$$I_m = \begin{cases} \dfrac{1+0.05}{\dfrac{1}{P^1_{Feb-12}}+\dfrac{0.05}{P^2_{Jan-15}}} & ; i = 1 \\ \dfrac{0.05+1+0.05}{\dfrac{0.05}{P^{i-1}_x}+\dfrac{1}{P^i_y}+\dfrac{0.05}{P^{i+1}_z}} & ; \ 2 \le i \le 67 \\ \dfrac{0.05+1}{\dfrac{0.05}{P^{67}_{Jul-13}}+\dfrac{1}{P^{68}_{Mar-12}}} & ; \ i = 68 \end{cases}$$

Thus $I_i^m ; i = Feb-12, Mar-12, \dots, Seb-17$ means the inverse fuzzy number is a real number, and y is the month which membership grade is 1.

Besides, $\alpha \div 0$ is undefined. The percentage of Tubtim Chandra Rose Apple price is 0 so the fraction $\alpha \div 0$ is 0. In the inverse fuzzy number, the inverse fuzzy number is actually a real number.

For example:

$$I^1_{Feb-12} = \frac{1.05}{-0.0505765} = -20.761$$

$$I^2_{Jul-14} = \frac{1.10}{-0.0590206} = -18.638$$

Step 5: Establishing the prediction formula

The forecasted values is $f_i = Y_{i-1} \times (1 + I_i^m\%)$.

f_i is the predicted data of i, Y_{i-1} is the real history data of $i-1$.

Step 6: Calculating the predicted data of historical data

$$f_{Feb-12} = 46.39 \times (1 + (-20.761\%)) = 36.759.$$

The percentage change, operations of fuzzy numbers and forecast values of Tubtim Chandra Rose Apple price are shown in Table 2.

Table 3 reveals the percentage change, the operations of fuzzy numbers and the forecast values of Shogun Orange price which consist of seasonal influences. Figure 2 and 3 represent the lines of real prices of Tubtim Chandra Rose Apple and Shogun Orange, the lines of forecasting values by SARIMA model and fuzzy time series prediction model based on fuzzy inverse respectively.

Figure 2 and 3 reveal the actual data, the forecasted values by SARIMA model and the proposed method are coincided. In addition, the line of forecasted valued by SARIMA model and the proposed method are alike. However, the MAPE using proposed method is less than Box-Jenkins method (see in Table 4).

5. Conclusions

Both of the forecasting methods correspond to Thailand fruit price influenced by season in the sense of accuracy. Those are often uncertain data. By comparison of the forecasting accuracy, the errors by the SARIMA model are 135.2 and 281.611 times higher than fuzzy model for Tubtim Chandra Rose Apple price data and Shogun Orange price data respectively. Fuzzy time series model based on fuzzy inverse is quite easy for calculating and it is a good alternative for high-precision forecasting. However, the proposed method is suitable for forecasting in a short time. In addition, the SARIMA model is convenient method for using conventional package programs such as SPSS, SAS and MINITAB.

Table 2 The percentage change, the operations of fuzzy numbers and the forecast values of Tubtim Chandra Rose Apple price

Date	Price	P	J	I	F
Jan-12	46.39				
Feb-12	36.71	-20.867	-0.051	-20.761	36.759
Mar-12	48.26	31.463	0.035	30.163	47.783
Apr-12	48.06	-0.414	-2.623	-0.419	48.058
May-12	41.32	-14.024	-0.078	-14.126	41.271
Jun-12	45.9	11.084	0.100	11.042	45.883
Jul-12	48.89	6.514	0.169	6.499	48.883
Aug-12	51.81	5.973	0.184	5.993	51.820
Sep-12	52.22	0.791	1.378	0.798	52.224
Oct-12	57.5	10.111	0.110	10.042	57.464
Nov-12	57.13	-0.643	-1.698	-0.648	57.127
Dec-12	54.97	-3.781	-0.290	-3.794	54.963
Jan-13	53.25	-3.129	-0.353	-3.113	53.259
Feb-13	50.13	-5.859	-0.186	-5.902	50.107
Mar-13	49.2	-1.855	-0.587	-1.874	49.191
Apr-13	47.05	-4.370	-0.252	-4.360	47.055
May-13	47.94	1.892	0.585	1.881	47.935
Jun-13	47.1	-1.752	-0.637	-1.727	47.112
Jul-13	54.88	16.518	0.065	16.845	55.034
Aug-13	58.13	5.922	0.187	5.885	58.110
Sep-13	58.5	0.637	1.765	0.623	58.492
Oct-13	58.63	0.222	4.392	0.250	58.647
Nov-13	62.75	7.027	0.156	7.070	62.775
Dec-13	62.38	-0.590	-1.894	-0.581	62.386
Jan-14	62.8	0.673	1.628	0.676	62.802
Feb-14	54.77	-12.787	-0.086	-12.827	54.745
Mar-14	53.81	-1.753	-0.626	-1.757	53.808
Apr-14	50.05	-6.988	-0.159	-6.935	50.078
May-14	53.44	6.773	0.163	6.766	53.436
Jun-14	48.54	-9.169	-0.121	-9.057	48.600
Jul-14	56.19	15.760	0.070	15.630	56.127
Aug-14	62.5	11.230	0.097	11.287	62.532
Sep-14	65.11	4.176	0.265	4.154	65.096
Oct-14	68.41	5.068	0.218	5.051	68.399
Nov-14	65.97	-3.567	-0.308	-3.572	65.966
Dec-14	69.03	4.638	0.237	4.651	69.038
Jan-15	56.02	-18.847	-0.059	-18.638	56.165
Feb-15	55.91	-0.196	-4.993	-0.220	55.897
Mar-15	53.18	-4.883	-0.225	-4.894	53.174
Apr-15	52.73	-0.846	-1.297	-0.848	52.729
May-15	55.11	4.514	0.244	4.517	55.112
Jun-15	54.89	-0.399	-2.880	-0.382	54.900
Jul-15	61.93	12.826	0.086	12.851	61.944
Aug-15	63.41	2.390	0.459	2.397	63.415
Sep-15	67.84	6.986	0.158	6.983	67.838
Oct-15	68.52	1.002	1.093	1.006	68.523
Nov-15	67.61	-1.328	-0.841	-1.309	67.623
Dec-15	67.84	0.340	3.309	0.332	67.835
Jan-16	60.51	-10.805	-0.102	-10.787	60.522
Feb-16	54.94	-9.205	-0.119	-9.266	54.903
Mar-16	57.39	4.459	0.247	4.461	57.391
Apr-16	54.03	-5.855	-0.190	-5.802	54.060
May-16	56.82	5.164	0.212	5.190	56.834
Jun-16	61.42	8.096	0.137	8.043	61.390
Jul-16	62.59	1.905	0.572	1.922	62.601
Aug-16	63.64	1.678	0.655	1.680	63.641
Sep-16	63.86	0.346	3.159	0.348	63.862
Oct-16	66.7	4.447	0.248	4.435	66.692
Nov-16	67.22	0.780	1.420	0.775	67.217
Dec-16	69.6	3.541	0.315	3.488	69.565
Jan-17	64.6	-7.184	-0.152	-7.246	64.557
Feb-17	56.5	-12.539	-0.088	-12.459	56.552
Mar-17	55.05	-2.566	-0.433	-2.543	55.063
Apr-17	55.9	1.544	0.727	1.512	55.883
May-17	53.95	-3.488	-0.317	-3.474	53.958
Jun-17	58.35	8.156	0.134	8.225	58.388
Jul-17	62.45	7.027	0.157	7.025	62.449
Aug-17	66.75	6.886	0.160	6.885	66.750
Sep-17	67.03	0.419	2.607	0.422	67.032

Table 3 The percentage change, the operations of fuzzy numbers and the forecast values of Shogun Orange price

DATE	Price	P	J	I	F
Jan-10	38.670				
Feb-10	47.170	21.981	0.050	21.886	47.133
Mar-10	52.330	10.939	0.101	10.938	52.330
Apr-10	47.170	-9.861	-0.111	-9.885	47.157
May-10	67.670	43.460	0.025	43.804	67.832
Jun-10	67.670	0.000	0.000	0.000	67.670
Jul-10	67.670	0.000	0.000	0.000	67.670
Aug-10	74.330	9.842	0.112	9.865	74.345
Sep-10	77.250	3.928	0.277	3.967	77.279
Oct-10	64.250	-16.828	-0.063	-16.632	64.402
Nov-10	61.000	-5.058	-0.218	-5.035	61.015
Dec-10	55.080	-9.705	-0.114	-9.625	55.129
⋮	⋮	⋮	⋮	⋮	⋮
Jan-14	92.000	-13.477	-0.081	-13.533	91.940
Feb-14	86.500	-5.978	-0.184	-5.979	86.500
Mar-14	89.000	2.890	0.383	2.871	88.984
Apr-14	89.000	0.000	0.000	0.000	89.000
May-14	89.000	0.000	0.000	0.000	89.000
Jun-14	89.000	0.000	0.000	0.000	89.000
Jul-14	78.170	-12.169	-0.091	-12.141	78.194
Aug-14	86.670	10.874	0.102	10.825	86.632
Sep-14	85.000	-1.927	-0.579	-1.901	85.023
Oct-14	87.080	2.447	0.449	2.451	87.084
Nov-14	87.380	0.345	3.291	0.334	87.371
Dec-14	84.500	-3.296	-0.332	-3.313	84.485
⋮	⋮	⋮	⋮	⋮	⋮
Jun-17	88.090	-6.595	-0.167	-6.591	88.094
Jul-17	93.660	6.323	0.174	6.320	93.657
Aug-17	91.730	-2.061	-0.535	-2.058	91.733
Sep-17	113.550	23.787	0.045	24.195	113.924

Table 4 Comparison of the MAPE of the two methods

Price	Model	MAPE
Tubtim Chandra Rose Apple	$SARIMA(1,0,0)(1,1,1)_{12}$	4.732
	FI	0.035
Shogun Orange	$SARIMA(1,1,1)(1,1,1)_{12}$	5.069
	FI	0.018

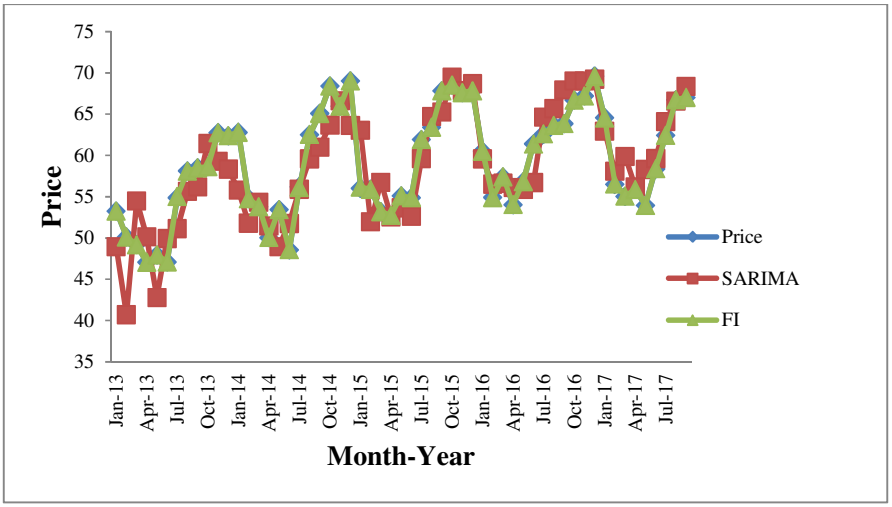

Figure 2 Lines of real Tubtim Chandra Rose Apple prices, forecasting values by SARIMA model and fuzzy time series prediction model based on fuzzy inverse

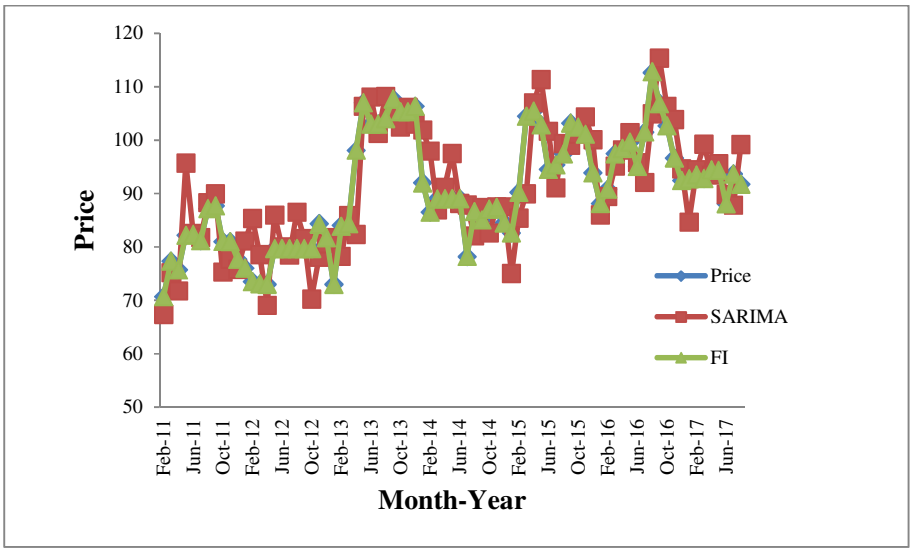

Figure 3 Lines of real of Shogun Orange prices, forecasting values by SARIMA model and fuzzy time series prediction model based on fuzzy inverse

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