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Mechanism of Spur Gear Hobbing

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Abstract

For the involute gear tooth generation, the hobbing method is used generally since the accurate gear can be obtained efficiently if there are no form restrictions on the gear blank. Adding to a good hobbing machine and a precise hob, the correct setting of the hob and gear blank are needed to generate precise gear teeth. A relatively simple method is developed to find the approximate relation between the tooth profile error and the hob or the usage of it by simplifying the action of the generation of the gear toot.

Keywords: Spur gear hobbing, Tooth profile error

1. Introduction

The cutting teeth of hob generate teeth surface of the hobbing is as same as it of a crossed helical gear set, it is necessary to think about the complicated movement in space to estimate the tooth profile errors [1]. It makes analysis of gear hobbing mechanism difficult. For generating the gear tooth form of a true involute curve, the hob needs to have cutting edge of an involute curve. The form of the cutting edge slightly convex consequently. But the standard hob has straight cutting edges because of the slight difference of both forms and easiness of manufacturing of the teeth. This report is limited to simulation of the hobbing of spur gear by standard hob with gashes parallel with hob axis and radial cutting face [2]. Cihan Ozel [3] developed a spur gears manufacturing of simulation for improvement of cutting errors of the tooth profile curves. Tamura and Lui [4] developed a measuring algorithm for helical gear using CMM to inspect the manufacturing error. Ming-Haung et al. [5] used the conjugate shape generation method to construct the mathematical model of tooth profile of spur gears that were ground in a threaded wheel gear grinder with operational error. A single flank rolling test simulator is developed to investigate the relationships between the operational errors and the kinematic transmission errors. Cuneyt Fetvaci [6] developed gear cutter to simulate of involute spur gears machined and computer graphs for generating cutting to inspect before gears manufacturing. The hob reference profile is the normal section of the tooth of the basic rack tooth profile [7].

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2. Kinematics of gear hobbing

The mechanics of a hobbing is the same as an engagement of the gear to be cut and helical gear of a number of teeth equal to the number of start of the hob thread. Figure 1 shows the state of having finished a spur gear by a hob. The hob arbor is inclined to the work gear surface at an angle of lead angle of the hob. In this report, we call a line perpendicular to both the hob arbor axis and the work gear axis a "generating center line". Every cutting tooth of the hob is numbered as shown in Figure 1. A cutting tooth on the generating center is numbered zero and attached sign of plus to the cutting teeth of leading side.

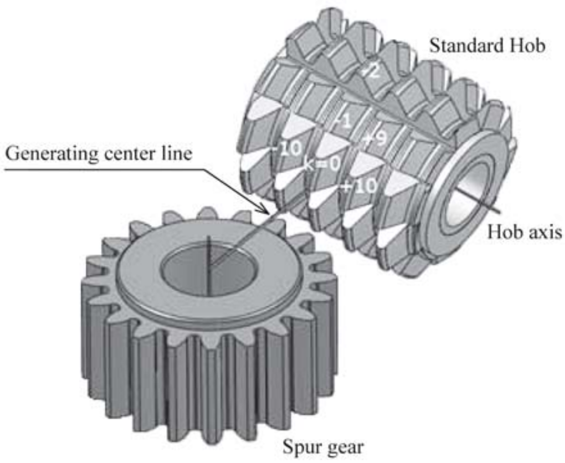

Figure 1 Cutting tooth number of 10 gashed hob and generating center line.

Figure 2 is projected pictures showing a movement of a point on a cutting edge of the hob. The picture (A) is the horizontal view (top view), the picture (B) is the vertical view (frontal view) from the work table side, the picture (C) is axial view (primary auxiliary view), and the axial true form of the cutting tooth can be shown on picture (D) which is secondary auxiliary view. The point P_k is the intersection of a cutting a cutting edge and the line of action. The suffix k shows the number of cutting teeth. The point generates involute tooth profile finally and is called it "generating point" below. The relative motion between the cutting teeth of the hob and the gear blank seems to be complicated. But as both the parts are simply rotated around each axis, the deriving equations to estimate the tooth profile error are provided as follows [8-9].

2.1 Movement of hob

A projected picture of the movement of a generating point, P_k is shown schematically in Figure 2. The axial projection drawing of the cutting edge of the hob with axial gashes and radial cutting face is shown at a position (D). When the hob rotated at an angle θ_i, the generating point P_k moves to the point P_{ki} and when the hob carried out one revolution, the trajectory of the point is shown by the thick line in each projected picture under a condition of no feed of the hob. The projection of the trajectory of the point is a circle in a side-view (C) and a thick line perpendicular to hob axis as shown in the vertical plane (B) and is an ellipse in the top view (A). In usual hobbing process, the hob is fed along to work gear axis keeping a relation of tooth generation without axial movement. The feed is not taking into consideration in the following equations because the involute curve is formed during a few rotation of each cutting edge. Here, two dimensional orthogonal coordinate x-y is used in the top view plane perpendicular to gear axis to express both the positions of generating point and ideal involute tooth profile. The origin of the coordinate is at an intersection of hob axis and generating center line. The x coordinate is on the horizontal plane and rectangular to the generating center and the generating center line is used as y coordinate. When the hob of which axis is inclined at angle Γ is rotated at angle of θ_i, the coordinate of the generating point is obtained by equations (1) and (2).

$$x_{ki} = R_k \cdot \sin\theta_i \cdot \sin\Gamma - Q_k \cdot \cos^2\alpha_x \cdot \cos\Gamma \quad (1)$$

$$y_{ki} = R_k \cdot \cos\theta_i \quad (2)$$

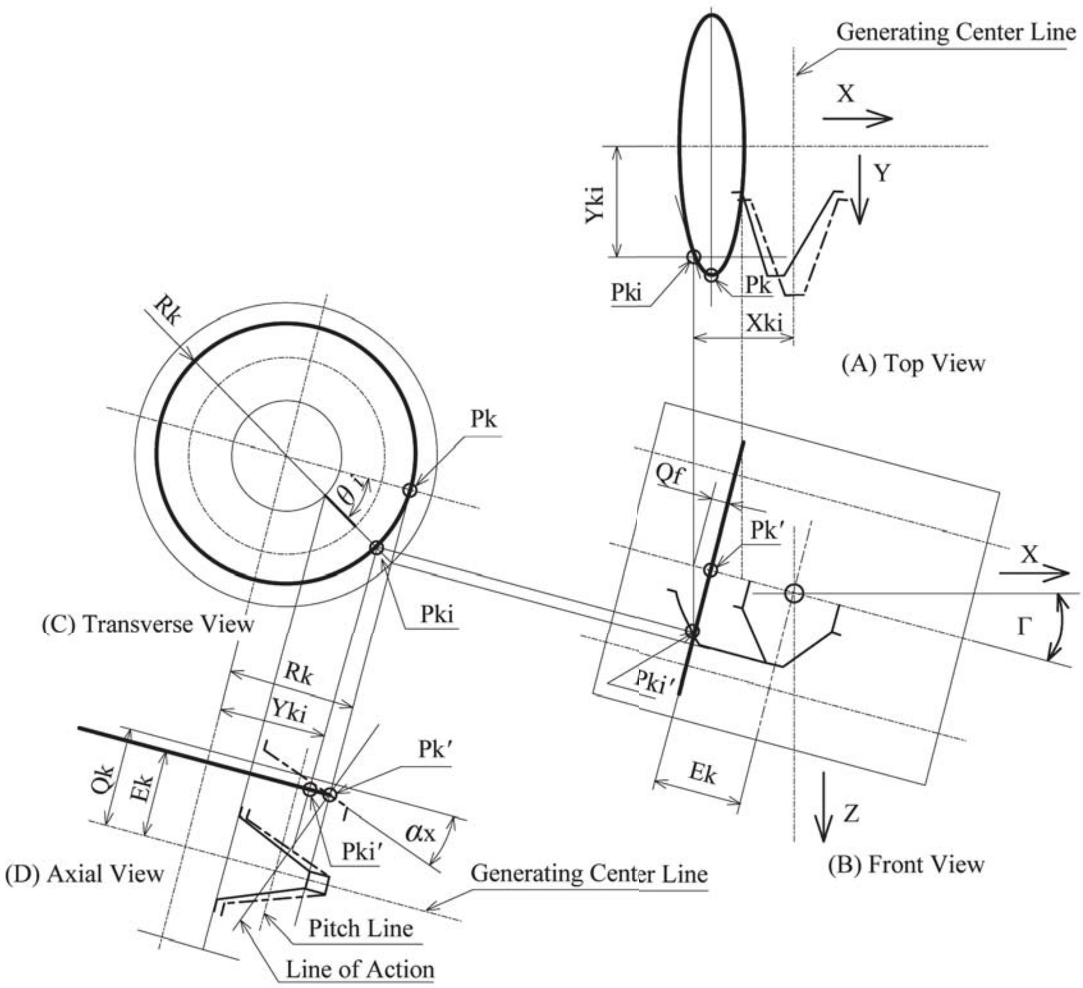

Figure 2 Schematic projected pictures of movement of a point on a cutting edge.

Here, R_k is distance from the hob axis to the point P_k and Q_k a distance from the generating center to the pitch point of a No. *k* cutting edge are denoted by equations (3) and (4)

$$R_k = \frac{d_0}{2} + Q_k \cdot \sin\alpha_x \cdot \cos\alpha_x \tag{3}$$

$$Q_k = k \cdot \frac{L_x}{G} + \frac{S_x}{2} \tag{4}$$

Here, *G* is number of gashed hob of cutting tooth , γ_0 is a lead angle of the hob thread, L_k is an axial lead of hob thread, S_x is tooth thickness of the hob and α_x is axial pressure angle of cutting tooth. The following equations (5), (6) and (7) commonly used to set them in case that symbol m is normal module and n is number of start of the hob and α_0 is pressure angle of hob.

$$L_x = \frac{\pi \cdot m \cdot n}{\cos\gamma_0} \tag{5}$$

$$\sin\gamma_0 = \frac{n \cdot m}{d_0} \tag{6}$$

$$\tan\alpha_x = \frac{\tan\alpha_0}{\cos\gamma_0} \tag{7}$$

2.2 Movement of work gear

The gear blank rotates on a work table of the hobbing machine synchronizing with rotation of hob. As the gear blank rotates by one lead of hob during one rotation of the hob, it is difficult to know how the cutting edge acts to the gear blank. The action of a generating point to gear tooth flank can be known by the operation which stops the motion of gear blank. A method to know the situation approximately is shown below.

The coordinates (x_j, y_j) of an involute curve (a) which sets a base circle radius to R_b in Figure 3 are denoted by equation (8) and (9).

$$x_j = R_b \cdot \varsigma \cdot \cos\varsigma - R_b \cdot \sin\varsigma \tag{8}$$

$$y_j = R_b \cdot \cos\varsigma + R_b \cdot \varsigma \cdot \sin\varsigma \tag{9}$$

Here, ζ is involute angle. The involute curve should be rotated to a position (b) when the No. *k* cutting edge rotates at angle θ_i. The coordinate is converted by the following equation (10) and (11).

$$x'_j = x_j \cdot \cos\phi_{ki} + y_j \cdot \sin\phi_{ki} \qquad (10)$$

$$y'_j = y_j \cdot \cos\phi_{ki} - x_j \cdot \sin\phi_{ki} \qquad (11)$$

Here, ϕ_{ki} is a rotational angle of the reference point from the y axis when a No. k cutting tooth rotates at an angle θ_i by the following equations (12), (13), (14) and (15).

$$\phi_{ki} = \phi_b + \phi_k + \phi_i \qquad (12)$$

$$\phi_b = \frac{\frac{m \cdot z}{2} \cdot \sin\alpha + \frac{n \cdot m}{4} \cdot \cos\alpha}{R_b} - \alpha \qquad (13)$$

$$\phi_k = \frac{2 \cdot \pi \cdot n}{z \cdot G} \cdot k \qquad (14)$$

$$\phi_i = \frac{n \cdot \theta_i}{z} \qquad (15)$$

Here, ϕ_b is a rotational angle for basic involute curve which contacts with a center cutting tooth, k=0 cutting tooth, α is pressure angle of rack and ϕ_k is an angle for changing an ideal position of the involute profile correspond to No. k cutting tooth. More compensated with an angle of ϕ_i, the coordinate of true involute curve is fixed to correspond to every rotation of a cutting tooth.

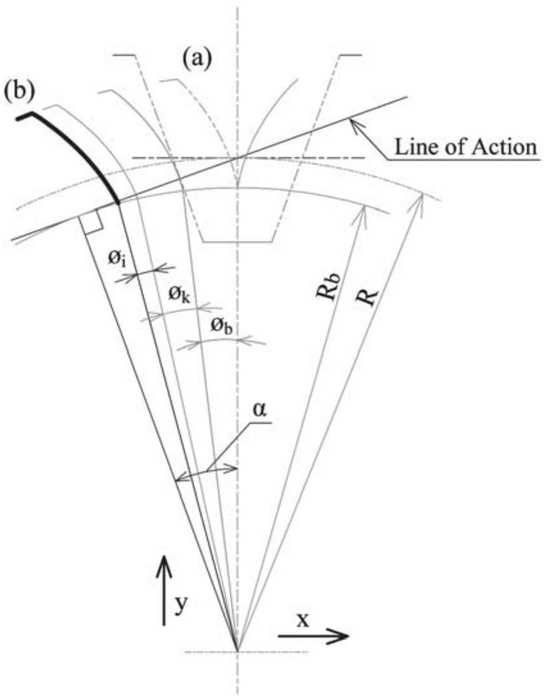

Figure 3 Rotation of gear tooth accompanied with hob rotation.

2.3 Tooth profile error

The minimum distance between the generating point calculated by equations (1) and (2) and the true involute curve presented by equations (10) and (11) becomes profile error, Δx with shown Figure 4. The approximate error is estimated by the following method.

At first, the coordinate of a generating point is set and the coordinate $I_j(x_j', y_j')$ of the involute curve are calculated adjusting a division angle ζ with the number of gear teeth and memorized. And next, the involute curve coordinate where the y_j'' is equal to generating point coordinate y_{ki} is set and the difference of two x coordinate, x_{ki} and x_j'' is checked for each cutting tooth related to tooth generation. The tooth error is found by getting the minimum difference while the hob is rotated.

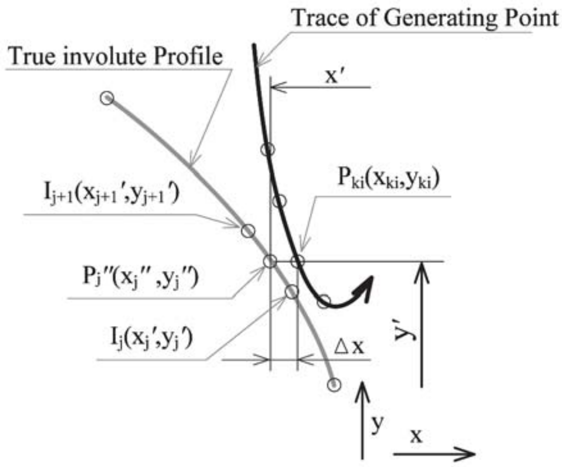

Figure 4 Distance between a generating point and true involute profile.

3. Simulation

3.1 Tooth profile error caused by standard hob

Four standard hobs with straight cutting edge use for simulation are shown in Table 1.

Table 1 Dimension of standard hob

Module	Outside diameter (mm)	Pitch diameter (mm)	Lead angle (degree)
3	70	62.5	2.75
5	95	82.5	3.47
10	130	105	5.47
16	190	150	6.12

Simulation of tooth profile error of right flank of the gear with 20 teeth is shown in Figure 5. The hob used in this simulation is made in right geometry. There is no error at pitch point shown by k=2 and the tooth is cut extra mount nearby tip and root of the gear. The situation is sometimes effective in order to avoid interference on gearing.

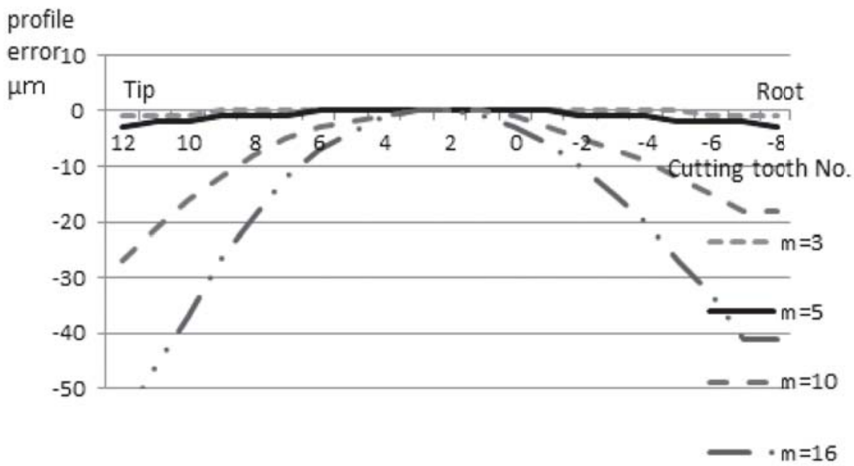

Figure 5 Simulated profile error of gears caused by standard hob showed in Table 1.

3.2 Relationship between profile error and gear teeth

Figure 6 shows simulated profile errors when the gear with number of teeth, z = 20, 50, 100, and 500 will be cut by module 10 standard hob with 20 degrees of pressure angles, pitch diameter, d_0 = 105 mm. The error values are not effected by number of gear teeth and almost same as the difference between the tooth profile of standard hob and of involute hob.

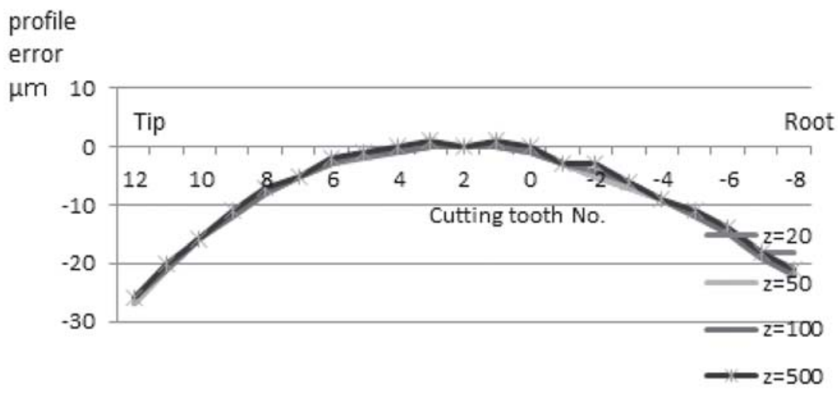

Figure 6 Simulated profile error of gears with typical number of teeth (m=10).

3.3 Relationship between profile error and hob inclination

Figure 7 shows simulated profile error of module 5 gears with 20 teeth which are cut under different set angle, regular angle γ_0, $\gamma_0 + 0.5^\circ$ and $\gamma_0 + 1^\circ$. The tooth profiles are same but tooth thickness is changed to smaller. The tooth profile of hobbed gears with same condition as simulation is shown in Figure 8.

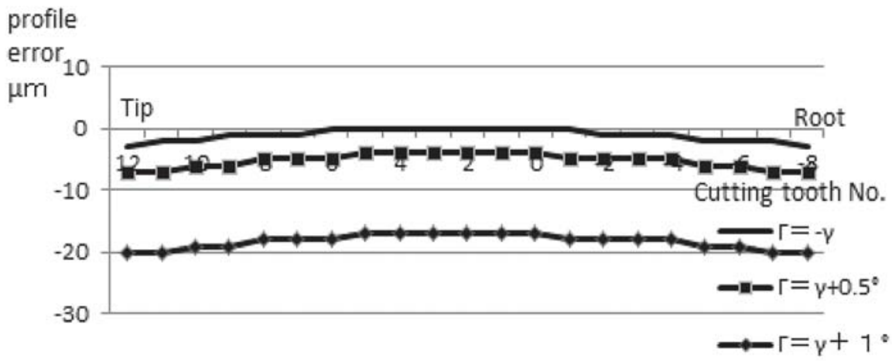

Figure 7 Simulated profile error of gears of different setting angle (m=5).

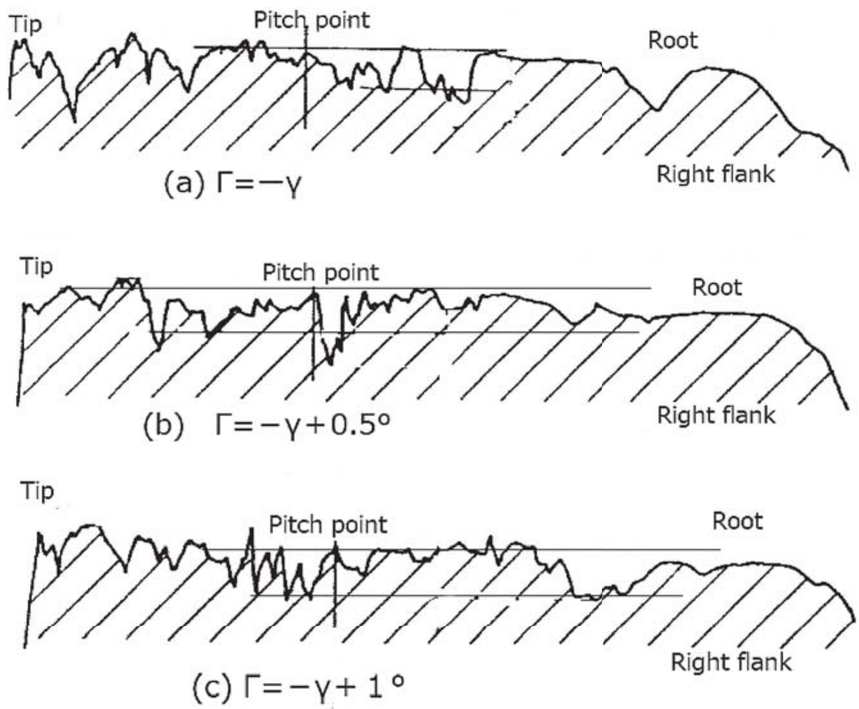

Figure 8 Profile error of S45C gears hobbed by different setting angle (m=5).

3.4 Relationship between profile error and hob run-out

Figure 9 shows simulated profile errors occurred by run-out of the hob. Theoretical profile error is acquired by multiplying the run-out value by sine pressure angle. In case of run-out is 0.04 mm, the error becomes about 0.014 mm. The profile error at the tip is larger a little than theoretical one because the error caused by straight cutting edge is included. The tooth profiles of hobbed gear are shown in Figure 10 [10].

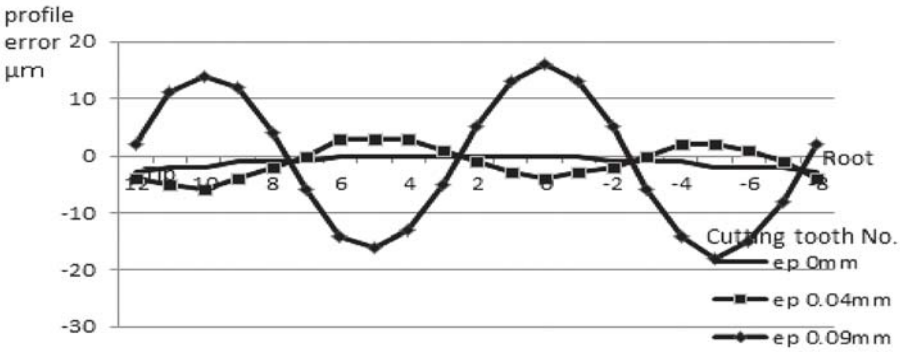

Figure 9 Simulated profile error of gears of different setting angle (m=5).

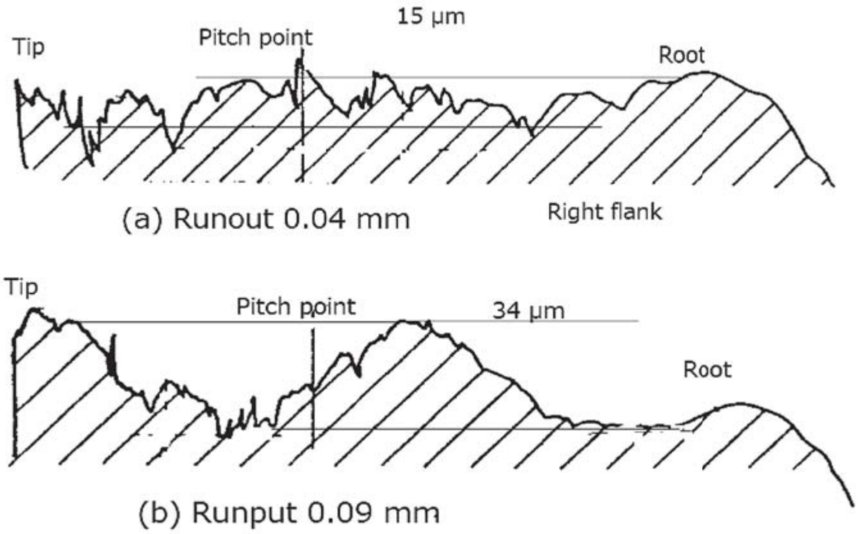

Figure 10 Profile error of gears caused by run-out (m=5).

3.5 Relationship between profile error and number of start of hob thread

The simulated profile errors of gear with module 5 in case of using the same size of hob are shown in Figure 11. As the double start hob makes significant error and it cannot be neglected in case of triple started hob, the profiles of cutting tooth should be modified to near an involute hob.

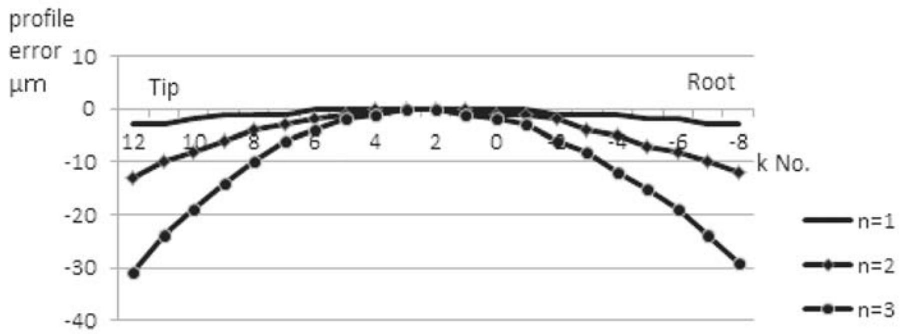

Figure 11 Simulated profile error by multi thread hob (m=5).

4. Generation of rack profile by hobbing

Since there is little difference in tooth profile error of gears with different number of teeth as mentioned in section 3.2, the process for finding tooth error becomes relatively simple by generating a gear with infinitive number of teeth, i.e., a rack. As the motion of the gear is changed from rotational to liner, the coordinate transformation is not needed and the distance between a generating point and rack surface is obtained simply by following process.

When a No. *k* cutting tooth rotates at angle of θ_i , a pitch point of rack is in the location of the distance x_j' calculated by equations (16), (17), (18) and (19).

$$x'_j = L_b + L_k + L_i \tag{16}$$

$$L_b = \frac{\pi \cdot m}{4} \tag{17}$$

$$L_k = \frac{\pi \cdot m \cdot n}{G} \cdot k \tag{18}$$

$$L_i = m \cdot n \cdot \frac{\theta_i}{2} \tag{19}$$

Here, L_b shown in Figure 12 is a distance of pitch point of rack should be generated by $k = 0$ cutting tooth. L_k is amount of shift of rack should be finished by number *k* cutting tooth and L_i is additional amount of sift when the cutting tooth rotates at angle of θ_i.

A tooth profile error Δx shown in Figure 12 is calculated by equation (20).

$$\Delta x = x'_j - \left(y_{ki} - \frac{d_0}{2} \right) \cdot \sin\alpha - x_{ki} \tag{20}$$

The minimum distance between a generating point and the rack surface is found by changing the hob rotation. The minimum distance means the maximum tooth profile error. Figure 13 shows a profile error simulated by this procedure. It shows almost same error as in Figure 6.

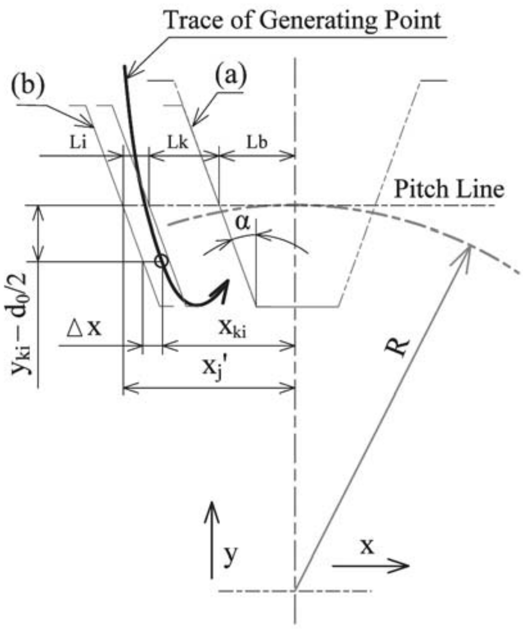

Figure 12 Distance between a generating point and a rack surface.

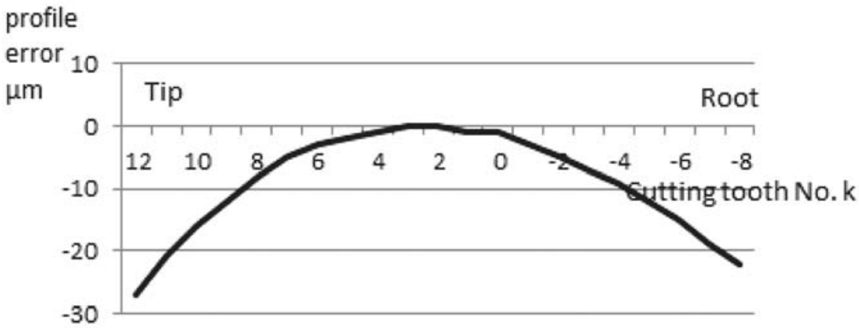

Figure 13 Simulated profile error by a rack tooth generation (m=10).

5. Conclusion

The relatively of gear tooth profile is expressed as an aggregation of an envelope of all cutting edges. A tooth profile error is estimated approximately by checking motions of both a point which is intersection of the cutting edge and the line of action and a gear should be finished. Moreover, as the tooth profile error is not related to number of the hobbed gear teeth, a tooth generation of a gear with infinitive teeth is advantageous to quick calculation.

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