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A Case Study on Investigating the Effects of Shear and Torsion on the Requirement of Longitudinal Rebar in Reinforced Concrete Beam

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ABSTRACT

Longitudinal rebar is an important component of reinforced concrete (RC) beam because it has to be designed to resist shearing and torsional forces. In Thailand, there are two equations to determine the area of the longitudinal rebar which were provided by EIT Standard. The results obtained from those equations, however, have been doubted when considered in a relationship directly to the applied shear and torsion magnitudes. Concerning to the validation of the matter, this paper presents the investigation of the doubtful outcome from such two equations and get insight into the effects of shear and torsion on the requirement of the longitudinal rebar area. A sensitivity analysis, sets of computational data, including nonlinear finite element analysis, were conducted in the study. Results showed that the requirement ofthe longitudinal rebar area calculated using equations given by EIT standard has turned out some of the unreasonable solutions when considered depending on the magnitude of the applied shear and torsion. The result acquired by finite element analysis was more reasonable in comparison to the calculation following to EIT Standard. When the high shear force and torsion were applied, the beam required more the area of longitudinal rebar. For as the presence of stirrup in term of one leg area of a closed stirrup to center-to-center spacing ratio has negligibly affected to the determination of the requirement of the longitudinal rebar area. This is due to the main function of stirrup is to resist shear and torsion.

Keywords: Longitudinal Rebar; RC Beam; RC Design; Shear and Torsion

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Introduction

Commonly, reinforced concrete beams (RC beams) are designed to subjected to flexural moments, transversed shears and torsional forces, or the combination of them. In prior years, torsion was considered as a secondary effect to the reinforced concrete beam and did not give much explicit attention in the design. While torsional forces were present, they were included in the design by using a conservative overall safety factor. Currently, the methods of analysis and design have converged the result to be more precise and lead to less conservativism. Torsion can be distinguished into two categories: primary torsion, also called equilibrium torsion or statically determinate torsion; and secondary torsion, known as compatibility torsion or statically indeterminate torsion [1]. In design, the reinforced concrete beams are mostly satisfied to the applied shear and torsion with the reinforcement of stirrup, web reinforcement, which function as to prolong the 45 degrees spiral patterned crack and to strengthen the beams. In addition to the stirrup, the longitudinal rebar was specified to be distributed around the beam section, especially at the corners, when the beams have to subjected to shear and torsion [2]. In Thailand, EIT Standard [3] has provided equations to determine the requirement of the overall cross-sectional longitudinal rebar area to be arranged in the beam, but the correlation of them and the applied shear and torsional forces has been found in questionable notice. Hence, the effects of shear and torsion on longitudinal rebar were investigated in detail in this study.

Literature Review

Around the world, codes and standards have provided for the arrangement of the longitudinal torsion reinforcement by many countries. It was given priorly in 1985 by British Standard BS 8110-1 and 2 [4, 5] that the longitudinal rebar should be distributed evenly round the inside perimeter of the torsion member. Aside from the transversed reinforcement, Eurocode Standard [6] recommended that the area of longitudinal rebar required a control of the beam cracking. The area of the longitudinal reinforcement was also recommended by CEB-FIB Code [7], as well as ACI 318-19 Standard [8], to prevent excessive cracking and give strengthening when the members subjected to shearing and torsional forces. In Thailand, EIT Standard [3] has provided two equations, Eq.(1) and Eq.(2), to calculate the minimum area and required area of the longitudinal rebar, A_ℓ. The equations are as follows:

$$A_\ell = 2A_t\left(\frac{x_1 + y_1}{s}\right) \tag{1}$$

and

$$A_\ell = \left[\frac{28xs}{f_y}\left(\frac{T_u}{T_u + \frac{V_u}{3C_t}}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right) \quad (2)$$

Where as V_u is the applied shearing force, T_u is the applied torsional force, A_t is the ratio of one leg area of a closed stirrup to center-to-center spacing, S is the distance of a closed stirrup, f_y is yield strength of the rebar, x_1 and y_1 are the distances from centroidal axis between each side of a stirrup in width and depth directions, respectively.

In research, an experiment and finite element analysis were conducted on six cantilever reinforced concrete beams to investigate the effect of longitudinal rebar under pure torsion [9]. Without stirrup, the agreement of the test and finite element analysis shown that the effect of the longitudinal rebar on the post-cracking stage was high, and should be incorporated into the code and standard. In 2015, a set of full-scale reinforced concrete beams with carbon fiber reinforced polymer (CFRP) rebar and stirrups in a variation of reinforcement types and stirrup spacing was tested under pure torsion loading to study on the strength and behavior [10]. The test results, however, showed that the CFRP beams exhibited little differences in strength, cracking behavior, post-peak torsional stiffness compared to the counterpart of steel RC beam. Although the result of the beams with closer spacing of stirrups revealed the higher torsional resistance and improved the post-peak stiffness, the further study should be carried out to examine the minimum-maximum longitudinal rebar ratio, and the effects of a combination of shear, torsion and bending. Based on a 2019 study of steel fiber-reinforced concrete (SFRC) in torsion, a minimum torsional reinforcement ratio was recommended [11]. The results indicated that the minimum ratio specified by some of the design codes may not satisfy the desired strength, and not prevent the brittle failure of the members after cracking. The suggested equation to determine the minimum torsional reinforcement ratio was proposed by including the area of the longitudinal rebar and stirrup, and related to the compressive strength of the concrete. Consequently, in 2020 the maximum torsional reinforcement ratio was suggested by the theoretical study, parametric study, to reflect on significant variables such as the tensile resistance of concrete, the cross-sectional area, the concrete softening factor and the averaged stress factor [12]. Researchers proposed a basic equation to determine the maximum torsional reinforcement ratio for reinforced concrete beams. This equation can predict key behaviors, including rebar yielding and potential failure modes, when the beam is subjected to torsion. In 2023, An experimental study reported that the longitudinal rebar was not only affected to the failure modes, ultimate torque, torsional stiffness and energy dissipation for the reactive powder concrete beams, but also the spiral reinforcement configuration, the spiral reinforcement ratio and the steel fiber area when subjected to torsion [13]. However, the arrangement of the longitudinal steel rebar is not only effect to improve the ductility of the reinforced concrete structures, but also to the planning of building constructions [14].

Selected Case Study

A reinforced concrete beam section of 30x60 cm^2 was chosen as an example in this study. The typical reinforcement parameters and reinforcing details are shown in Figure 1(a). With 2.5 cm covering, a traditional 9 mm diameter and 10 cm spacing of stirrup with the distance x_1 and y_1 were designed as 24.1 and 54.1 cm (see Figure 1(b)). The diameter of the longitudinal rebar was given as 25 mm arranged in the four corners. The properties of material were given for concrete strength as f_c' = 280 kg/cm^2, yielding strength of longitudinal rebar f_y = 4000 kg/cm^2, and stirrup f_v = 4000 kg/cm^2. The schematic of structure for 3000 mm in length cantilever beam subjected to torsion, shear and moment was depicted in Figure 2 for investigating the effects of shear and torsion on the requirement of the longitudinal rebar area, A_ℓ.

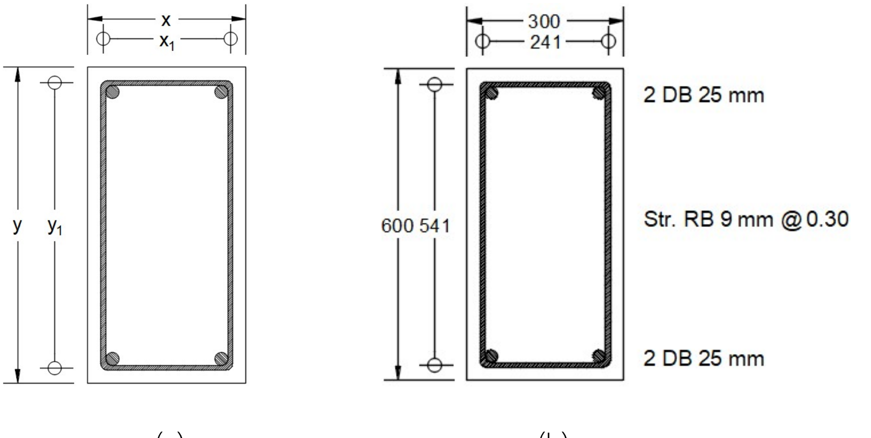


Figure 1 Reinforcement detail for stirrup and longitudinal rebar: (a) typical section and (b) selected example.

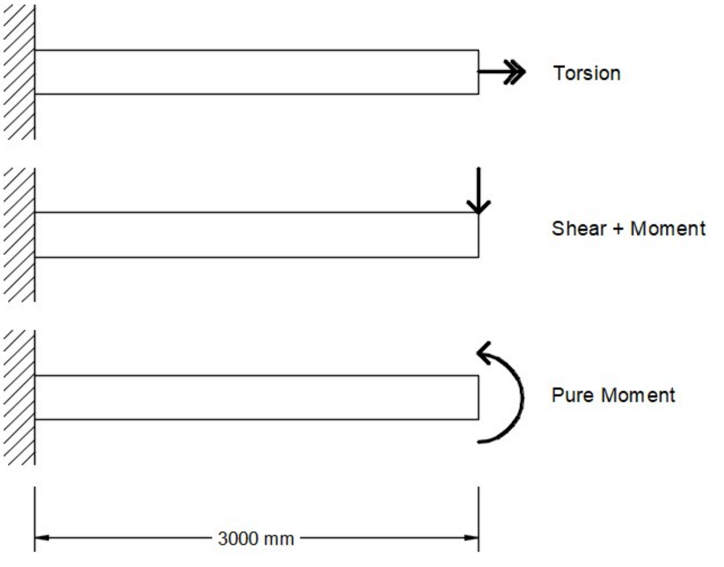


Figure 2 Schematic of cantilever beam subjected to torsion, shear and moment, and pure bending moment.

Finite Element Simulation

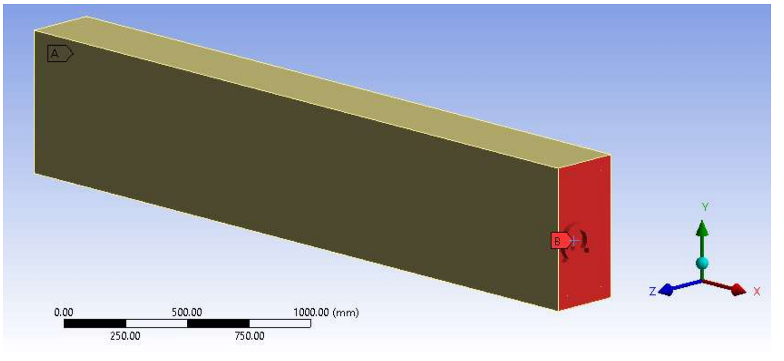
The nonlinear finite element analysis was carried out by using ANSYS 2022 R1 software [15] to investigate on the effects of shear and torsion on the requirement of the longitudinal rebar area. To limit in the tensile strength of concrete, only 10 percent of the compressive strength was defined, as shown in Table 1, including the necessary nonlinear material properties. The nonlinear properties of the steel stirrup and rebar were shown in Table 2. The simulation of the cantilever reinforced concrete beam was illustrated in Figure 3 for overall beam and reinforcing steel. The boundary conditions were fixed end support at the left of the beam, while at the right end was to be subjected to shear, torsion and flexural moment for the analysis procedure. The element size throughout the entire model was mapped mesh at the default, 150 mm approximately. The element type of concrete was SOLID186, which is a higher order 3-D 20-node solid element, having three degrees of freedom per node, that exhibits quadratic displacement behavior which is suitable for plasticity and large deflection. For reinforcing steel, REINF264 element type was adopted in which it has standard 3-D link with only uniaxial stiffness to provide reinforcing to the based solid elements. The nodal locations, degrees of freedom, and connectivity of the REINF264 element are identical to those of the base element.

Table 1 Concrete properties.

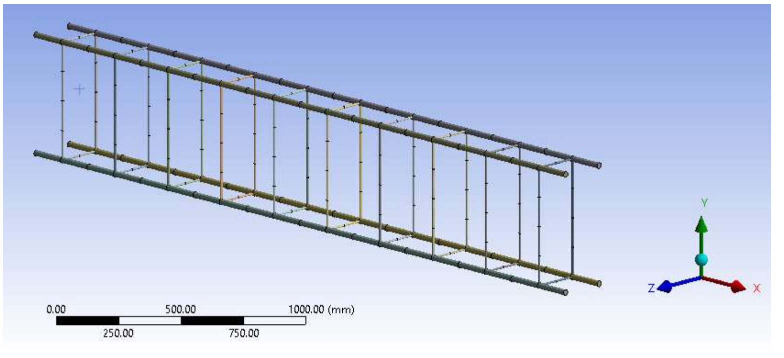
Properties	Define
Young's modulus	252671.32 kg/cm^2
Poisson's ratio	0.3
Shear modulus	9530.2 kg/cm^2
Uniaxial compressive strength	280 kg/cm^2
Uniaxial tensile strength	28 kg/cm^2
Biaxial compressive strength	328.24 kg/cm^2
Dilatancy angle	30 Degrees
Plastic strain at uniaxial compressive strength	0.001
Ultimate effective plastic strain in compression	0.01
Relative stress at start of nonlinear hardening	0.4
Residual compressive relative stress	0.2
Plastic strain limit in tension	0.01
Residual tensile relative stress	0.2

Table 2 Steel stirrup and rebar properties

Properties	Define
Young's modulus	2040000 kg/cm^2
Poisson's ratio	0.3
Shear modulus	784610.44 kg/cm^2
Yield strength	4000 kg/cm^2
Tangent modulus	14785.89 kg/cm^2



(a)



(b)

Figure 3 Finite element model for cantilever reinforced concrete beam: (a) RC beam and (b) stirrup and rebar.

Result and Discussion

1.Sensitivity Analysis

In order to examine all the parameters probably affected to the requirement of the longitudinal rebar area, A_ℓ, and also to put forward those were more relevant to the outcome of A_ℓ, a sensitivity analysis was carried out for the investigation. The parameters involved in the consideration were the applied shear force, torsion, one leg area of a closed stirrup A_t, stirrup spacing s, concrete covering, yield strength of the rebar f_y, beam width x, and beam depth y, all of which was set for the default at 15000 kg, 4500 kg-m, 0.64 cm^2, 30 cm, 2.5 cm, 4000 kg/cm^2, 30 cm and 60 cm, respectively. Based on EIT Standard [3], the result of the sensitivity analysis in the range of 20 to 180 percent parameter change is presented in Figure 4 with the controlled requirement of the longitudinal rebar area A_ℓ = 4.46 cm^2. The beam width, following with beam depth and yield strength of the rebar shown most sensitive effects on the requirement of the longitudinal rebar area. The less affected parameters were stirrup spacing, one leg area of stirrup and concrete covering. As it is remarkable, the two medium affected parameters, applied shear force and torsion, have shown the prominent outcomes of the requirement of the longitudinal rebar area. When the percent of the applied shear force and torsion were changed to higher, the requirement of the longitudinal rebar area were contrarily less. Furthermore, when the percent change of the applied shear force was toward to the less, the requirement of the longitudinal rebar area was not in the characteristic of mostly linear trends, but in a dubious reversed curve.

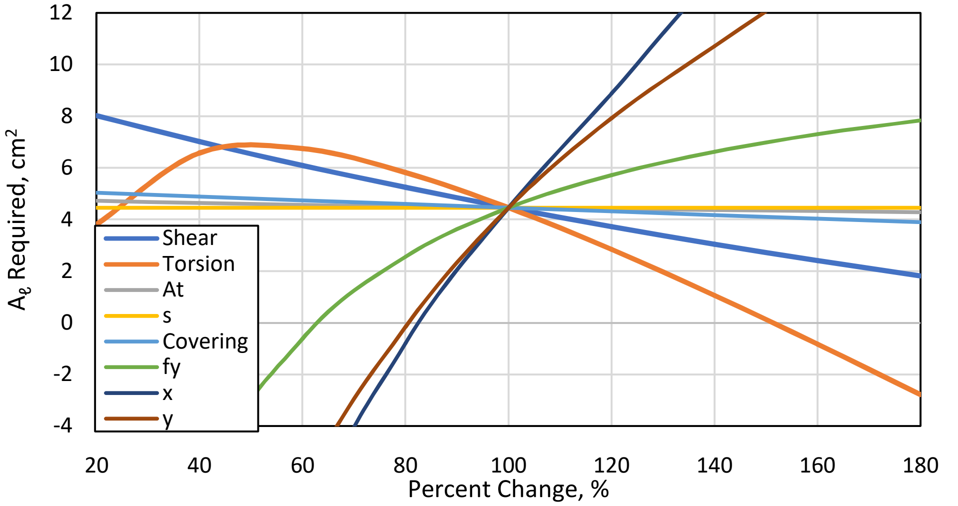


Figure 4 Sensitivity analysis of parameters affecting requirement oflongitudinal rebar area A_ℓ.

2.Shear and Torsion

Figure 5 shows the area requirement of the longitudinal rebar area, A_ℓ, following EIT Standard [3] for the represent reinforced concrete beam caused by shearing force. Regardless of the applicable design limitation, the graph was plotted on the range of torsion applied between 1185 kg-m to 7500 kg-m, which was included the minimum requirement of the longitudinal rebar. As to the case of 9 mm dimeter and 10 cm spacing of the given stirrup, the results show that the higher applied shearing force requires gradually less longitudinal rebar in all value of torsion. In addition, the longitudinal rebar is remarkably required even less when the applied torsion varies higher. In the graph, the minimum longitudinal rebar requirement, Eq.(1), was also plotted, and it is worth noting with the equation that A_ℓ will still be required further relating to the ratio of torsional rebar area to spacing, $\frac{A_t}{s}$. In the case of large torsion, it requires a large $\frac{A_t}{s}$ and, accordingly, more

A_ℓ to sustain the load. But when the torsion was small and the stirrup was overdesigned in a high ratio of torsional rebar area to spacing, this will affect to the more requirement of A_ℓ, unnecessarily. Moreover, the ratio $\frac{A_t}{s}$ used for the computation of the requirement of the longitudinal rebar should be chosen referred to the actual design arrangement, rather than the theoretical design calculation.

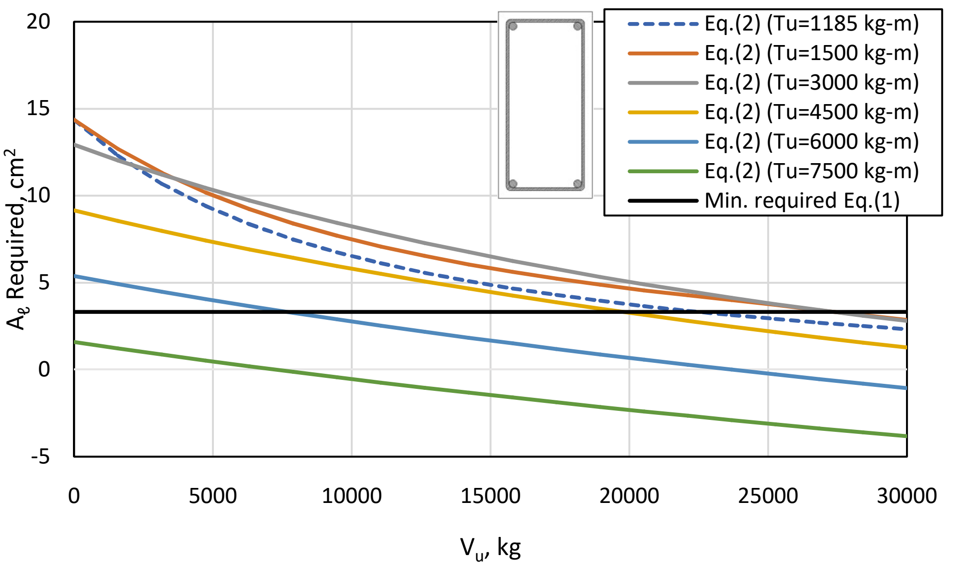


Figure 5 Requirement of calculated longitudinal rebar area caused by shearing force.

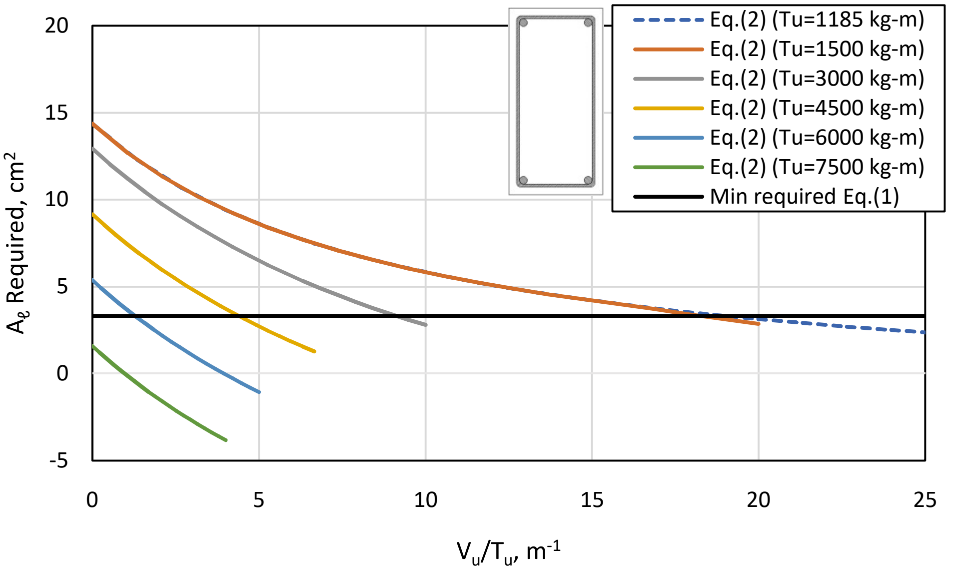


Figure 6 Requirement of calculated longitudinal rebar area against shear to torsion ratio.

In other aspect of combined shear and torsion, Figure 6 shows the requirement of the longitudinal rebar, A_ℓ, respected to shear to torsion ratio, and it has been found that the trends of the rebar needed in more upward curves. From the graph, it can be seen that the quantity of the required longitudinal rebar when high shearing forces are applied was gradually low and, thus, it was stipulated with the minimum longitudinal rebar requirement, Eq.(1). However, there still are the ranges of small to medium shearing forces that the requirement of the longitudinal rebar will be determined by Eq.(2) in which it was involved with the effect of applied shearing forces.

3.Finite Element Analysis

The results of nonlinear finite element analysis investigating on the cantilever reinforced concrete beam, which aim on the requirement of the longitudinal rebar, A_ℓ, caused by shear and torsion are illustrated in Figure 7 for the model subjected to pure torsion, and Figure 8 for that subjected to shear including flexural moment. The results show the behavior in the views of total deformation, equivalent plastic strain, equivalent (Von-Misses) stress and axial normal stresses that occurred at the concrete fiber and the reinforcing steel. To acquire the axial stresses at the longitudinal rebar that were caused solely by shear force, the axial normal stresses occurred were subtracted with those caused by pure bending moment (Figure 9). Then the maximum axial stress without the involvement of flexural moment occurred at one of the four corner rebar was measured and converted to be the required longitudinal rebar area, A_ℓ. To be compared the result with the calculation following the standard Eq.(2), the converted requirement of the longitudinal rebar area, A_ℓ, obtained individually from nonlinear finite element analysis within elastic material range was multiplied by 4 for four cornered rebar. The finite element analysis result can be plotted as shown in Figure 10 and it turned out that the requirement of the longitudinal rebar area was reasonably depended on shear and torsion, contrasting to those obtained by the standard equations. When the beam subjected to either higher shear and torsion, the longitudinal rebar area A_ℓ was required more in the manner of a little bit upward curve. It is also remarkable that most of the requirement of the longitudinal rebar area were exceeded the minimum requirement given by EIT Standard.

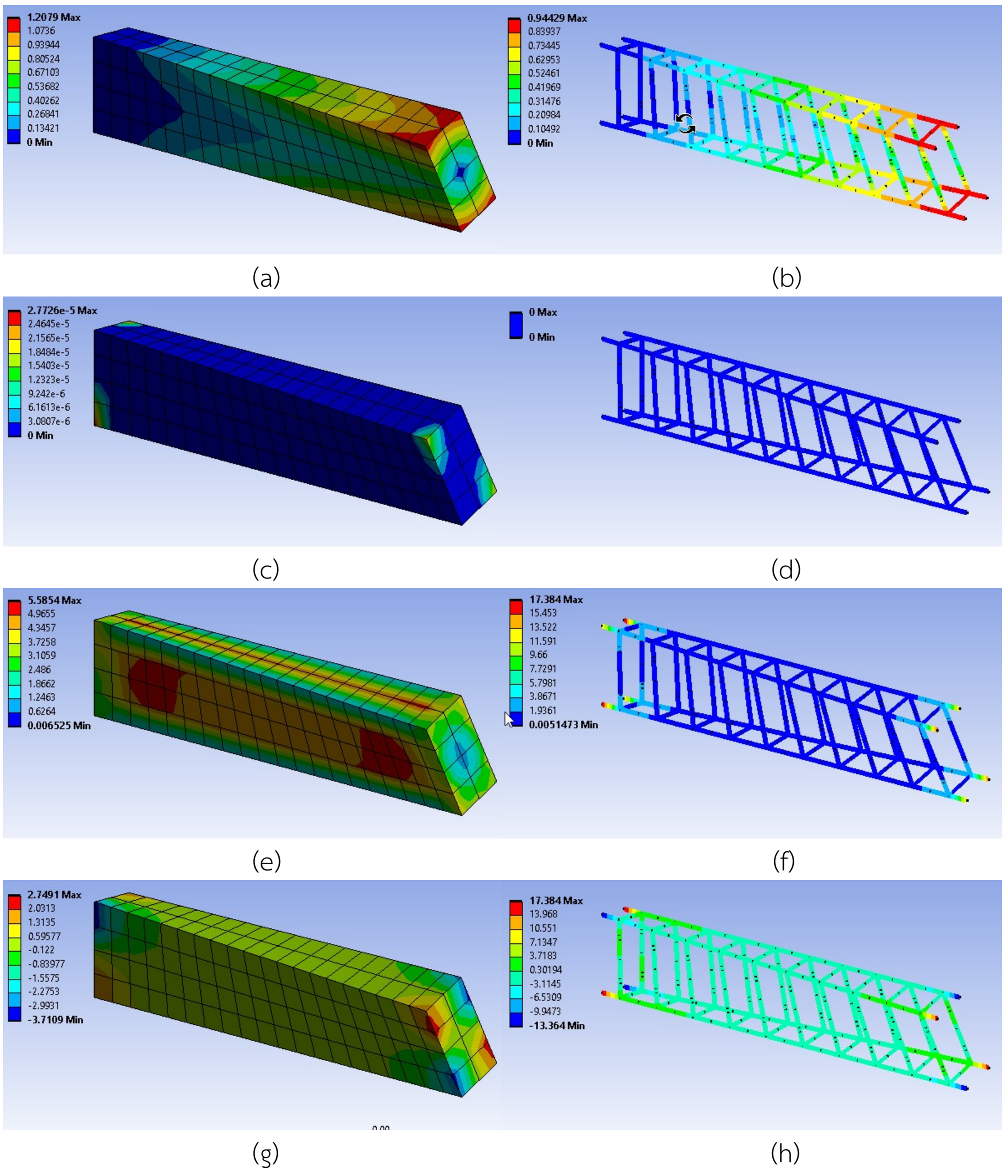


Figure 7 Finite element analysis result for pure torsion (T_u=4500 kg-m): (a) total deformation of concrete, (b) total deformation of reinforcing steel, (c) equivalent plastic strain of concrete, (d) equivalent plastic strain of reinforcing steel, (e) equivalent (Von-Misses) stress of concrete, (f) equivalent (Von-Misses) stress of reinforcing steel, (g) normal stress of concrete and (h) normal stress of reinforcing steel.

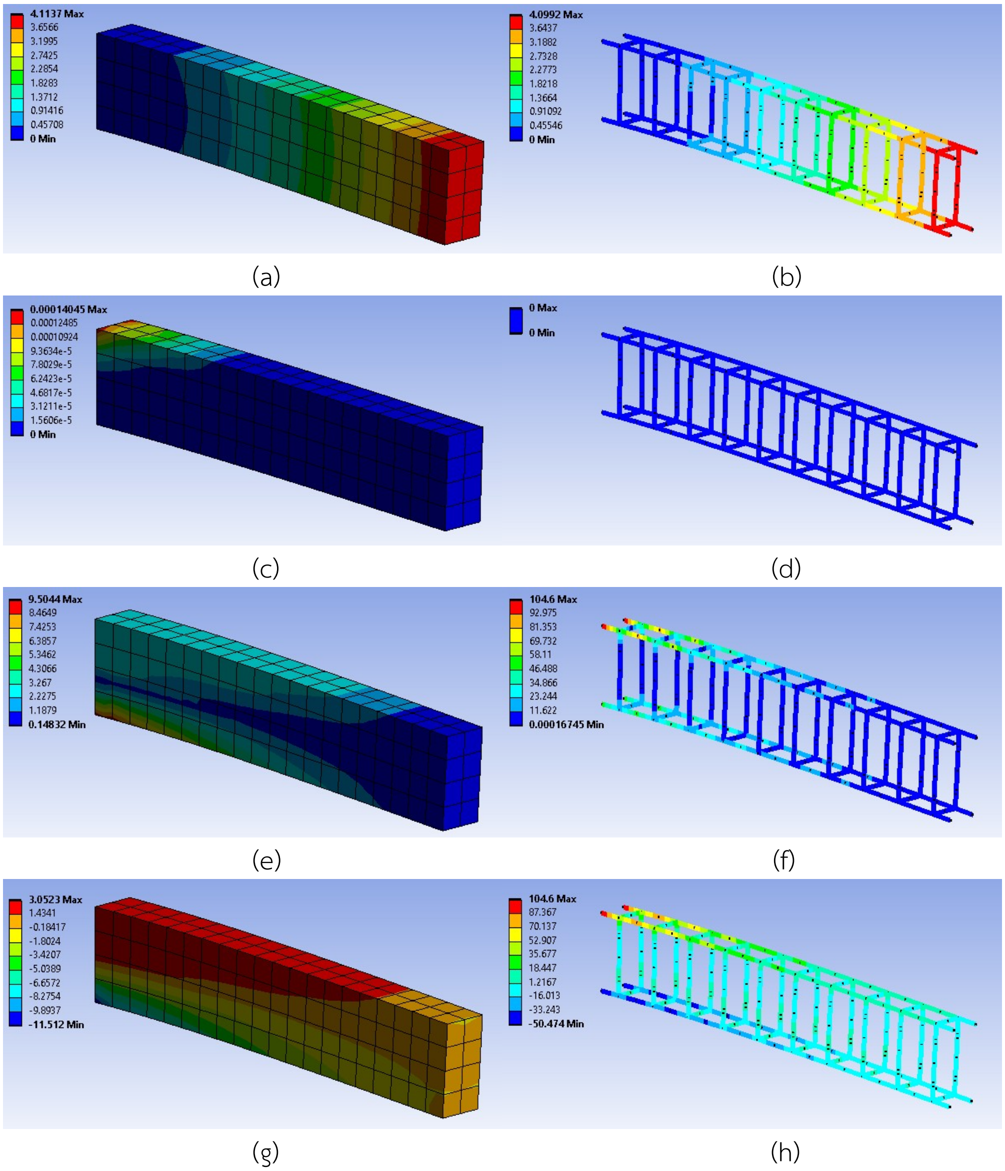


Figure 8 Finite element analysis result for shear and flexural moment (P_u=5000 kg): (a) total deformation of concrete, (b) total deformation of reinforcing steel, (c) equivalent plastic strain of concrete, (d) equivalent plastic strain of reinforcing steel, (e) equivalent (Von-Misses) stress of concrete, (f) equivalent (Von-Misses) stress of reinforcing steel, (g) normal stress of concrete and (h) normal stress of reinforcing steel.

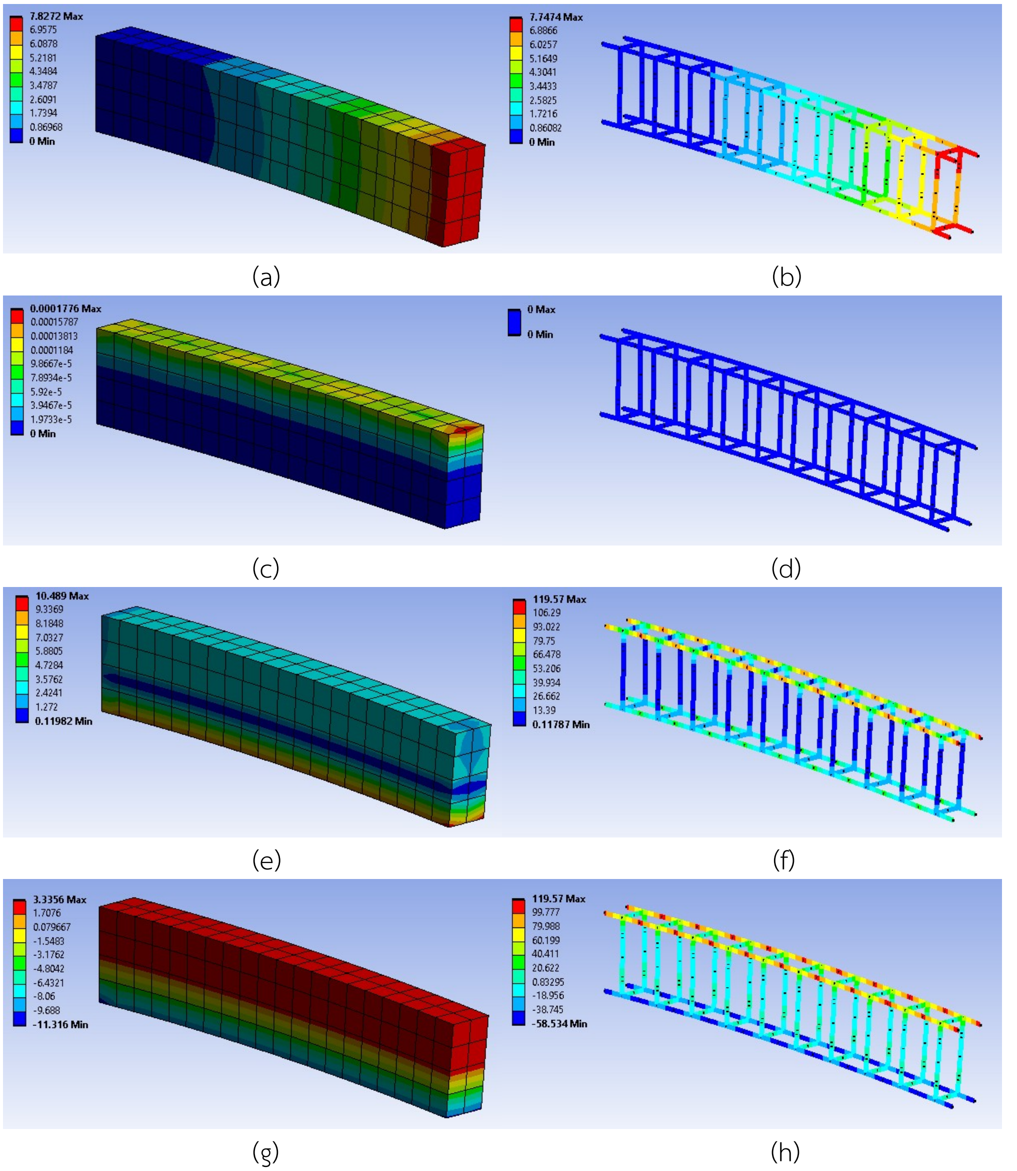


Figure 9 Finite element analysis result for pure bending moment (M_u=15000 kg-m): (a) total deformation of concrete, (b) total deformation of reinforcing steel, (c) equivalent plastic strain of concrete, (d) equivalent plastic strain of reinforcing steel, (e) equivalent (Von -Misses) stress of concrete, (f) equivalent (Von-Misses) stress of reinforcing steel, (g) normal stress of concrete and (h) normal stress of reinforcing steel.

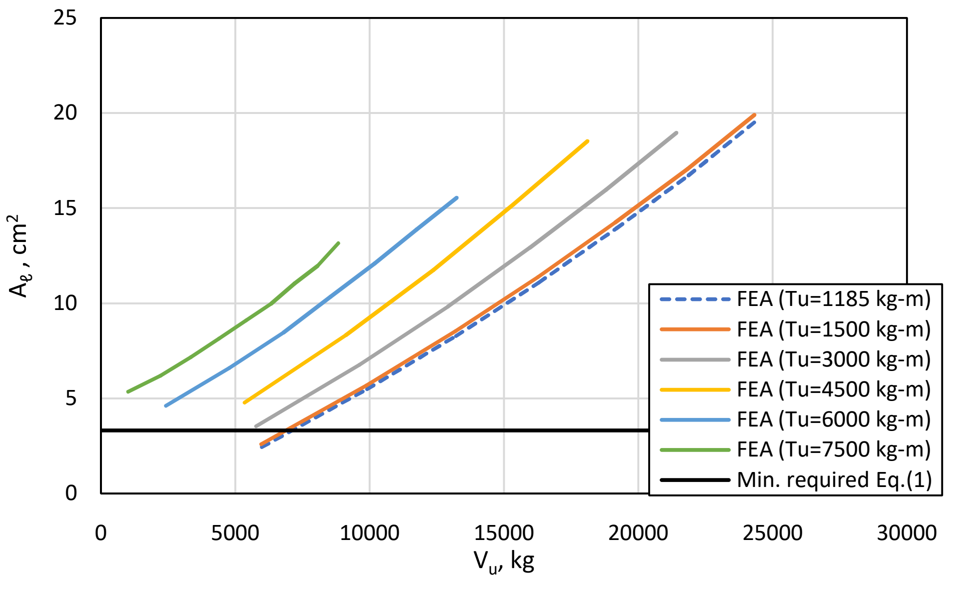


Figure 10 Result of finite element analysis for requirement of longitudinal rebar area caused by shear and torsion.

4.Effect of Stirrup

Stirrup or web reinforcement is a transverse component functioned as a reinforcement to prevent the failure modes in reinforced concrete beam subjected to shear and torsion. It also involved in the calculation process according to Eq.(1) and Eq.(2) provided by EIT Standard to calculate the longitudinal rebar area, A_ℓ. The desired quantity of stirrup was conventionally arranged in the term of one leg area of a closed stirrup to center-to-center spacing ratio, $\frac{A_t}{s}$. Following EIT Standard, the requirement of the longitudinal rebar area in a generally applicable range of $\frac{A_t}{s}$ for the case of zero shear force was plotted in Figure 11 to investigate the effect of stirrup involvement. From the graph, although it turned out that in a certain torsion applied, $\frac{A_t}{s}$ ratio has a slight difference on the longitudinal rebar area which was determined by Eq.(2), but for Eq.(1) it affected in almost linear proportion as refer to the minimum requirement, $A_{\ell\,min}$. When the applied torsion is less and $\frac{A_t}{s}$ ratio greater, the requirement of the longitudinal rebar area will be gradually governed by $A_{\ell\,min}$ proportionally.

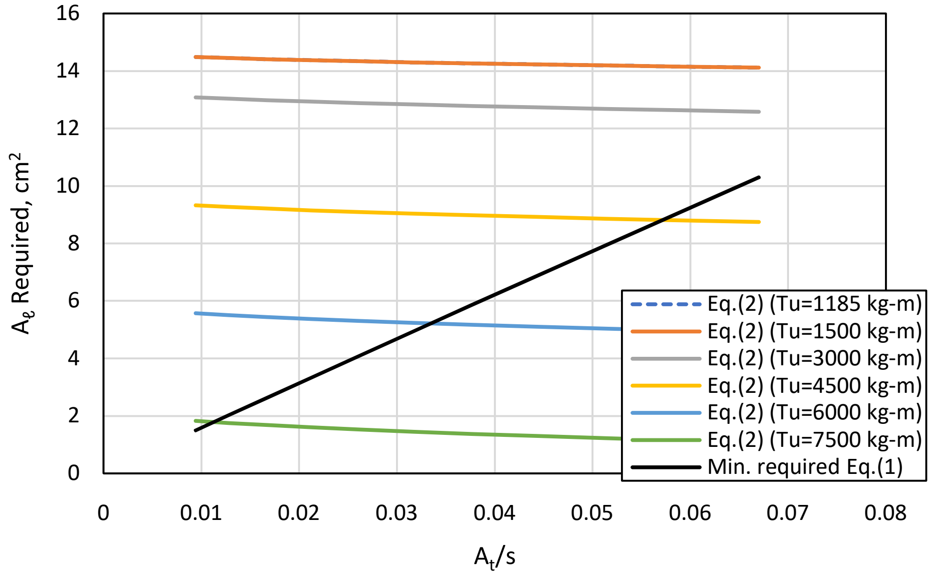


Figure 11 Requirement of calculated longitudinal rebar area depended on $\frac{A_t}{s}$ ratio at zero shear force.

5.Ultimate Behavior

To extend the study beyond the results within the elastic range of materials that was studied in the previous sections, the load-deformation behavior of the cantilever reinforced concrete beam subjected to shear and torsion has been observed over the nonlinear finite element analysis. The load-deformation curve in Figure 12 shows for the case under 4500 kg-m applied torsion that the reinforced concrete beam has been found the first crack around the support region with the load of 4436.69 kg. Afterward, the beam can sustain the load further to pass the yielding found on the longitudinal rebar at the load of 8381.85 kg, and then reach the ultimate state at 9571.25 kg with 1.86 cm in total deformation at the end of the beam. For flexural behavior, the beam can undergo the applied load with the given torsion to the first crack, rebar yielding and ultimate moment reaction at 14051.98 kg-m, 25552.49 kg-m and 29033.63 kg-m, respectively.

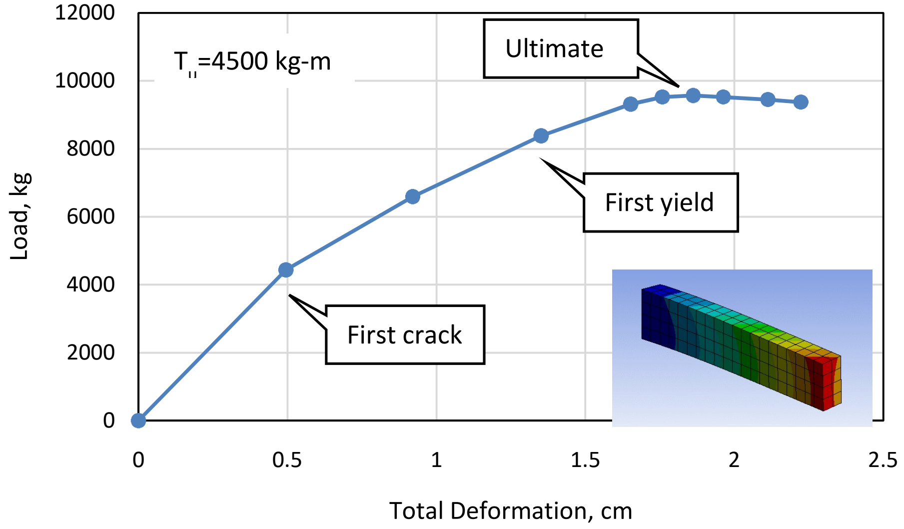


Figure 12 Result of load-deformation curve of a cantilever reinforce concrete beam subjected to shear and torsion.

Conclusions

In this study, the conclusion of investigating on the area requirement of longitudinal rebar subjected to shear and torsion on the represent cases of reinforced concrete beam can be drawn as follows:

The requirement of the longitudinal rebar area, A_ℓ, calculated using equations given by EIT standard has represented the unreasonable mannerwhen considered depending on the magnitude of shear and torsion applied. In the cases of high shear force and torsion, the reinforced concrete beam contrastingly required less the total area of longitudinal rebar.

The requirement of the longitudinal rebar area, A_ℓ, acquired by finite element analysis is more reasonable as compared to the calculation obtained from EIT Standard. When the high shear force and torsion were applied, the reinforced concrete beam required more the area of the longitudinal rebar, proportionally, in the manner of a little bit upward curve. Moreover, the A_ℓ requirement for the most part of shear and torsion exceeded the minimum requirement, $A_{\ell\,min}$.

The designed stirrup in the term of $\frac{A_t}{s}$ has slightly affected to the determination of the longitudinal rebar area, A_ℓ, but it proportionally affected to the minimum requirement, $A_{\ell\,min}$ given by EIT Standard.

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