Research Article
Journal of Advanced Development in Engineering and Science
Vol. 14 ● No. 41 • September – December 2024

A Fuzzy Approach to Determine Production Lot Size for Capacitated Single-Stage Production Process with Fuzzy Demand

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Received: 29 October 2023
Revised: 16 July 2024
Accepted: 6 September 2024

ABSTRACT

This paper addresses the production lot size problem for a fuzzy single-stage, multiple -item, capacitated lot-sizing model in the context of unrelated parallel machines, known as the F-CLSPP model. This problem is particularly useful for SMEs or new product production planning, where there is a lack of historical quantitative data, and the available data comes primarily from expert experience. In this paper, the problem is formulated as a fuzzy mixed-integer programming model in the form of a dynamic lot size and scheduling problem. To make the F-CLSPP model mathematically solvable, a chance-constraint programming concept and a possibility approach are proposed to transform it into an equivalent crisp CLSPP model. The fuzzy constraints are converted into equivalent crisp constraints using the extension principle, allowing the model to be solved with basic software. This procedure and model are tested with an illustrative numerical example, and the results demonstrate that this approach can provide valuable production planning information and assist in decision-making based on the confidence level in the data.

Keywords: Fuzzy Demand; Possibility Approach; Single-Stage Production Process

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Introduction

This research focuses on lot sizing production planning of order quantity produced on parallel machines to determine how much to produce of each item and each period in the planning horizon to satisfy the customer demand when there is no historical data of demand trend but there is verbal information from experience of expert. This verbal demand is called fuzzy demand in this paper. Fuzzy theory and stochastic methods are both used to handle uncertainty, but they approach it in different ways. Fuzzy theory deals with uncertainty that arises from vagueness or imprecision. It is particularly useful when the data or information is subjective, qualitative, or lacks clear boundaries. For example, when experts describe demand as "high" or "low" without specific numerical values, fuzzy theory can model this kind of imprecision. Stochastic method handle uncertainty that arises from randomness or inherent variability. Stochastic methods assume that the uncertain variables follow a known probability distribution, such as normal, binomial, or Poisson distributions. For example, if demand varies according to a known probability distribution, stochastic methods are appropriate. In this paper, the fuzzy demand will be focused, which it enables the management of uncertain or ambiguous demand data, which often arises when there is insufficient historical data or when the data is highly variable.

The use of unrelated parallel machines complicates the problem as we not only have to determine the quantity and timing to produce, but we also have to assign production lots to machines. Each item can be produced on any of the machines and several different items can be produced on the same machine in the same time period. A setup time and a setup cost are incurred before starting production and the setup is sequence independent [1]. Fiorotto et al. [2] studied the capacitated lot sizing problem with multiples items, setup time and unrelated parallel machines. The Dantzig-Wolfe decomposition was applied to a strong reformulation of the problem. Kim and Glock [3] studied the case where a manufacturer produces a single type of product on multiple parallel machines. They proposed a deterministic mathematical model for supporting production and distribution planning in this scenario and analyzed the behavior of the proposed model in an extensive numerical experiment using an implementation of the proposed model in a commercial solver. There are several papers dealing with surveying, extending, and applying the capacitated lot-sizing problem in parallel machines [4-8].

The purpose of this study is to propose the procedures of solving the lot sizing production planning problem in the form of mixed integer linear programming when demand is the fuzzy. As a result, this mathematical model is an uncertain programming problem. Generally, solving uncertain programming must to trans-form it into an equivalent crisp program and then to obtain an optimal solution by some crisp deterministic algorithms [9-11]. The chance-constrained programming (CCP), was proposed by Charnes and Cooper [12], is the approach to find the solution for optimization problems under uncertainty by transforming the stochastic programming with probabilistic criteria into equivalents of the original stochastic problems [13]. In this work, the CCP approach will be applied to convert fuzzy constraints to equivalent crisp. Chotayakul and Punyangarm [11] studied the capacitated lot sizing problem with multiple items, setup time, unrelated parallel machines and stochastic demand which was assumed to be a normal distribution. The problem is formulated as a SMIP. The stochastic constraints are transformed into equivalent deterministic programming ones by using the CCP approach and then obtain an optimal solution by deterministic mixed-integer linear programming model. The proposed algorithm is evaluated through a numerical example. Computational experiment demonstrates that the proposed method has goodquality result for the test problem. Ketsarapong et al. [14] also applied the CCP model to convert the uncapacitated fuzzy single item lot sizing problem model (F-USILSP) to a mathematically solvable equivalent crisp USILSP (EC-USILSP).

In this paper, the fuzzy set theory is applied to solve this information uncertainty for production inventory model. We focus the membership function of fuzzy demands as the triangular and trapezoidal fuzzy numbers, which are basically linear membership function of fuzzy parameters [15, 16]. There are many researchers studying and using the fuzzy set theory to apply in the inventory model where there are fuzzy parameters appear in the models [17-20]. The aim of this research presents amethodology to solve the production lot size problem of a fuzzy single-stage multiple items capacitated lot-sizing in the setting of unrelated parallel machines (known as F-CLSPP) model.

The rest of the paper is organized as follows: Section 2 presents the steps of research methodology including (1) data collection process and (2) developing production lot size model based on fuzzy demand. Section 3 presents an illustrative numerical example to test the proposed methodology. Finally, the last Section is the conclusions.

Research Methodology

The problem studied in this paper involves developing production lot size model to deal with multi-products in a single stage production on unrelated parallel machines with limited capacity which the demand of each item is considered as fuzzy. This production planning model is designed in case of there is lack of historical quantitative data that it often occurs in SMEs business or new product production planning. The research methodology is organized as follows; (1) the data collection process (2) a developing production lot size model based on fuzzy demand.

1.Data collection process

The data collection process is the process to obtain inputs data for the production lot size model based on fuzzy demand. The fuzzy demand is a qualitative information that it must be collected from experienced decision makers. This problem model is to determine how many units of each product should be produced in each period over a given planning horizon. The objective of the production lot size model is to minimize total production costs, such as holding cost, setup cost and cost to produce, to meet customer demand of the products on time. In this paper, we first formulate this problem as a fuzzy mixed-integer programming formulation in form of dynamic lot size and scheduling problem.

To present the problem formulation, we define the input parameters used in formulation the model in the following:

Parameters

I = {1,..., n} set of items;
J = {1,..., r} set of machines;
T = {1,..., m} set of periods;
$\tilde{d}_{it}$: fuzzy demand of item i in period t;
s_{ij} : setup cost of item i on machine j;
p_{ij} : unit production cost of item i on machine j;
h_i : unit inventory cost of item i;
f_{ij} : setup time of item i on machine j;
b_{ij} : time to produce one unit of item i on machine j;
C_{jt} : capacity (in unit of time) of machine j in period t;
$\tilde{d}_{ikT}$: sufficiently large number, where $\tilde{d}_{ikT} = \sum_{t=1}^{T} \tilde{d}_{it}, \forall t$.

2.Developing production lot size model based on fuzzy demand

This subsection discusses the steps of developing production lot size model based on fuzzy demand. First, a developing a fuzzy single-stage capacitated lot-sizing in the setting of unrelated parallel machines (F-CLSPP) model of the problem is discussed. Then, transforming F-CLSPP model to equivalent crisp CLSPP (EC-CLSPP) model is presented by using a Possibility Approach. Last subsection shows how to develop EC-CLSPP in a form of mixed integer linear programming.

2.1 Developing F-CLSPP model

The problem formulation is developed in form of mixed-integer program for the single-stage capacitated multi-items lot-sizing problem with fuzzy demand on unrelated parallel machines. The fuzzy production lot size model, where demand is a fuzzy variable, in form of F-CLSPP model can be modeled as follows:

Decision Variables

x_{ijt}: amount of unit produced of item *i* on machine *j* in period *t*;

y_{ijt}: binary variable, indicating the production or not of item *i* on machine *j* in period *t*;

I_{it}: quantity of inventory of item *i* at the end of period *t*;

F-CLSPP model

$$\text{Minimize } \sum_{t=1}^{m}\sum_{j=1}^{r}\sum_{i=1}^{n}(s_{ij}y_{ijt}+p_{ij}x_{ijt})+\sum_{t=1}^{m}\sum_{i=1}^{n}h_iI_{it} \tag{1}$$

subject to

$$\sum_{j=1}^{r}x_{ijt}+I_{i,t-1}-I_{it}\geq \tilde{d}_{it};\quad i = 1, \ldots, n;\ t = 1, \ldots, m, \tag{2}$$

$$\sum_{i=1}^{n}(b_{ij}x_{ijt}+f_{ij}y_{ijt})\leq C_{jt};\quad j = 1, \ldots, r;\ t = 1, \ldots, m, \tag{3}$$

$$x_{ijt}\leq \tilde{d}_{ikT}y_{ijt};\ i = 1, \ldots, n;\quad j = 1, \ldots, r;\ t = 1, \ldots, m, \tag{4}$$

$$y_{ijt}\in\{0,1\}, x_{ijt}\geq 0, I_{it}\geq 0;\quad i = 1, \ldots, n;\ j = 1, \ldots, r;\ t = 1, \ldots, m. \tag{5}$$

The objective function (1) is to minimize the total costs of setup cost, production cost and inventory cost. The constraints in equation (2) guarantee the inventory balance in each period where demand is a fuzzy variable. Note that, in general, the inventory balance constraint is initially declared as an equality constraint for deterministic problem, but the inequality constraint is used here due to a fuzzy condition. Therefore, in this study, these constraints are changed to “>” by relaxing the upper bound of the constraints in order to maintain feasibility and to ensure that supply meets the demand. The capacity constraints in equation (3) limit the total production and setup times to the available capacity in each machine and for each period. Constraints (4) are the machine setup constraints. Finally, constraints (5) define the binary setup variables and non-negative variables for produced quantities and inventory level. These constraints assure that no backlogging occurs.

Since this F-CLSPP model in equations (1) - (5) is in form of fuzzy mixed integer linear programming (FMIP), which it cannot be solved by classical mathematical methods. In this paper, the fuzzy linear program will be transformed to an equivalent deterministic program by using Chance-Constrained Programming (CCP) [12].

2.2 Transforming F-CLSPP model to EC-CLSPP model

The traditional method of CCP is used to convert the stochastic program into the equivalent deterministic program. In this paper, this approach will be applied for converting F-CLSPP model to EC-CLSPP model.

The CCP is one of the well-known approaches to find the best solution for optimization problems under uncertainty, where the objective function or some of the constraints ensure that the probability of one or more events occurring is less than a prescribed threshold. In this paper, the possibility constraints (In Equation (2) and Equation (4), fuzzy demands) are rewritten as equivalent crisp deterministic constraint by using the CCP approach. At the end of each and every time period, the possibility that the demand will support customers' need is set to be at least α_{it}. As a fuzzy constraint, Equation (2) and Equation (4) can be guaranteed greater than or equal to a pre-specified minimum possibility and can be written in the following form,

EC-CLSPP model

$$\text{Minimize} \quad \sum_{t=1}^{m}\sum_{j=1}^{r}\sum_{i=1}^{n}(s_{ij}y_{ijt}+p_{ij}x_{ijt}) + \sum_{t=1}^{m}\sum_{i=1}^{n}h_iI_{it} \tag{6}$$

subject to

$$\pi\left(\sum_{j=1}^{r}x_{ijt}+I_{i,t-1}-I_{it}\geq \tilde{d}_{it}\right)\geq \alpha_{it}; i = 1, \ldots, n;\ t = 1, \ldots, m, \tag{7}$$

$$\sum_{i=1}^{n}(b_{ij}x_{ijt}+f_{ij}y_{ijt})\leq C_{jt}; \quad j = 1, \ldots, r;\ t = 1, \ldots, m, \tag{8}$$

$$\pi\left(x_{ijt}\leq \tilde{d}_{ikT}y_{ijt}\right)\geq \alpha_{it}; \quad i = 1, \ldots, n;\ j = 1, \ldots, r;\ t = 1, \ldots, m, \tag{9}$$

$$y_{ijt}\in\{0,1\}, x_{ijt}\geq 0, I_{it}\geq 0; i = 1, \ldots, n;\ j = 1, \ldots, r;\ t = 1, \ldots, m, \tag{10}$$

where π means possibility and α_{it} means possibility level, which α_{it} = [0,1]. The proposed EC-CLSPP model (6) – (10) is a fuzzy mixed integer linear programming (FMILP) problem. This mathematical form cannot be solved by general approach which in this paper the fuzzy logic method is proposed to solve this problem by developing EC-CLSPP in the form of MILP model and then to obtain an optimal solution by some crisp deterministic algorithms.

2.3 Developing EC-CLSPP in a form of MILP model

In this research, the fuzzy constraints in Equation (7) and (9) can be converted to be equivalent crisp constraints by using "the extension principle", that is one of the basic methods of fuzzy set theory, and Lemma 1, that was shown in Lertworasirikul, et al. [21]. The details of Lemma 1 based on possibility measure are shown as follows:

Lemma 1. Let $\tilde{a}_i$ for i = 1, ..., n be fuzzy variables with normal and convex membership functions and b be a crisp variable. The lower and upper bounds of the α – level set of $\tilde{a}_i$ are denoted by $(\tilde{a}_i)_\alpha^L$ and $(\tilde{a}_i)_\alpha^U$, respectively. Then, for any given possibility levels α_1, α_2 and α_3 with $0\leq\alpha_1,\alpha_2,\alpha_3\leq 1$,

(i) $\pi(\tilde{a}_1+\cdots+\tilde{a}_n\leq b)\geq\alpha_1 \text{ iff } (\tilde{a}_1)_{\alpha_1}^L+\cdots+(\tilde{a}_n)_{\alpha_1}^L\leq b,$ (11)

(ii) $\pi(\tilde{a}_1+\cdots+\tilde{a}_n\geq b)\geq\alpha_2 \text{ iff } (\tilde{a}_1)_{\alpha_2}^U+\cdots+(\tilde{a}_n)_{\alpha_2}^U\geq b,$ (12)

(iii) $\pi(\tilde{a}_1+\cdots+\tilde{a}_n= b)\geq\alpha_3 \text{ iff } (\tilde{a}_1)_{\alpha_3}^L+\cdots+(\tilde{a}_n)_{\alpha_3}^L\leq b, \text{and } (\tilde{a}_1)_{\alpha_3}^U+\cdots+(\tilde{a}_n)_{\alpha_3}^U\geq b.$ (13)

From Lemma 1, the fuzzy constraints in Equations (7) and (9) can be converted to be an equivalent crisp constraint by Equations (11) and (12), respectively. Therefore, the EC-CLSPP can be formulated as the following MILP model.

EC-CLSPP in a form of MILP model

$$\text{Minimize} \quad \sum_{t=1}^{m}\sum_{j=1}^{r}\sum_{i=1}^{n}(s_{ij}y_{ijt}+p_{ij}x_{ijt}) + \sum_{t=1}^{m}\sum_{i=1}^{n}h_iI_{it} \tag{14}$$

subject to

$$\sum_{j=1}^{r}x_{ijt}+I_{i,t-1}-I_{it}\geq \left(\tilde{d}_{it}\right)_\alpha^L; \ i = 1, \ldots, n;\ t = 1, \ldots, m, \tag{15}$$

$$\sum_{i=1}^{n}(b_{ij}x_{ijt}+f_{ij}y_{ijt})\leq C_{jt}; \quad j = 1, \ldots, r;\ t = 1, \ldots, m, \tag{16}$$

$x_{ijt} \leq \left(\tilde{d}_{ikT}\right)_{\alpha}^{U} y_{ijt}$; *i* = 1, ... *n*; *j* = 1, ..., *r*; *t* = 1, ..., *m*, (17)

$y_{ijt} \in \{0,1\}, x_{ijt} \geq 0, I_{it} \geq 0$; *i* = 1, ..., *n*; *j* = 1, ..., *r*; *t* = 1, ..., *m*.(18)

2.4 Developing EC-CLSPP in a form of MILP model

For membership function of trapezoidal fuzzy number, the lower and upper crisp values of the trapezoidal fuzzy number ($\tilde{B}$) at α = 0 and 1 at each corner points of a trapezoidal membership function, which are denoted by $\left(\tilde{B}\right)_0^L$, $\left(\tilde{B}\right)_1^L$, $\left(\tilde{B}\right)_1^U$ and $\left(\tilde{B}\right)_0^U$ (see Figure 1b), are defined as follows:

$h_{Lower/}\tilde{B} = \left(\tilde{B}\right)_1^L \alpha + \left(\tilde{B}\right)_0^L (1-\alpha)$ and $h_{Upper/}\tilde{B} = \left(\tilde{B}\right)_1^U \alpha + \left(\tilde{B}\right)_0^U (1-\alpha)$(20)

where $\mu_{\tilde{B}}(h) \epsilon$ [0,1] and h ϵ H.

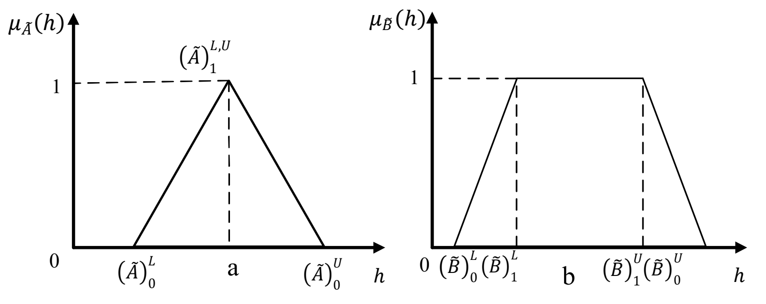

Figure 1 Membership function of (a) triangular fuzzy number, and (b) trapezoidal fuzzy number.

An Illustrative Numerical Example

1.Data Set

This section the numerical example of the production lot size model, where demand is a fuzzy variable, in form of F-CLSPP model is illustrated to be EC-CLSPP in a form of MILP model and to show how to solve this problem. The details of data set are as follows:

Assume that the beginning and ending stock of the planning horizon are both zero. The numbers of items, machines and periods are assumed to be 3 items (n = 3), 3 machines (r = 3), and 5 periods (m = 5), respectively, in which all parameters are randomly generated shown in Table 1. The capacity (in unit of time) of each machine and period are assumed to be 480 minutes (c_{jt} = 480). Let the demands for each item and period based on the experience of the decision maker be classified as three verbal levels, that are shown in Table 2. The membership functions of fuzzy demand are defined as triangular fuzzy number for item 1 and item 2, and as trapezoidal fuzzy number for item 3, which are shown in Figure 2-4, respectively.

Table 1 All parameters for numerical example.

i	s_{ij}			p_{ij}			f_{ij}			b_{ij}			h_i
	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	
1	200	400	600	5	5	5	8	9	8	2.7	2.7	2.7	0.8
2	500	500	500	3	3	3	5	5	5	2.82	2.82	2.82	0.9
3	500	800	400	2	3	3	1	1	1	1.4	1.5	1.4	0.2

Table 2 The demand for numerical example in the form of verbal demand.

t	$\tilde{d}_{it}$					
	i = 1		i = 2		i = 3	
	Verbal prediction	Triangular fuzzy number	Verbal prediction	Triangular fuzzy number	Verbal prediction	Trapezoidal fuzzy number
1	High	(200,250,300)	Low	(130,150,170)	Medium	(30,40,50,60)
2	Medium	(150,200,250)	High	(170,190,210)	Medium	(30,40,50,60)
3	Medium	(150,200,250)	Low	(130,150,170)	Medium	(30,40,50,60)
4	Low	(100,150,200)	Low	(130,150,170)	High	(50,60,70,80)
5	Low	(100,150,200)	Medium	(150,170,190)	High	(50,60,70,80)

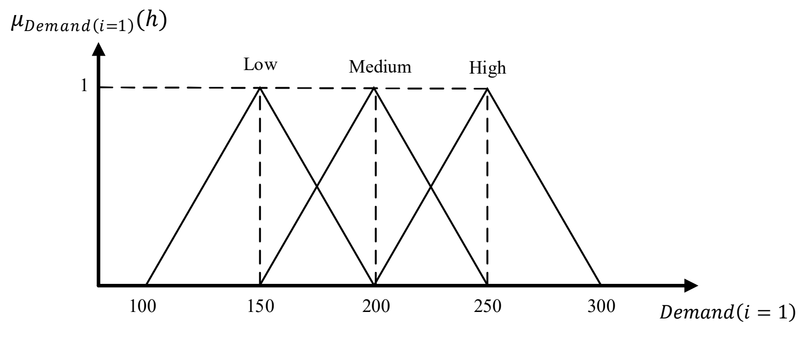

Figure 2 Membership function of fuzzy demand for item 1.

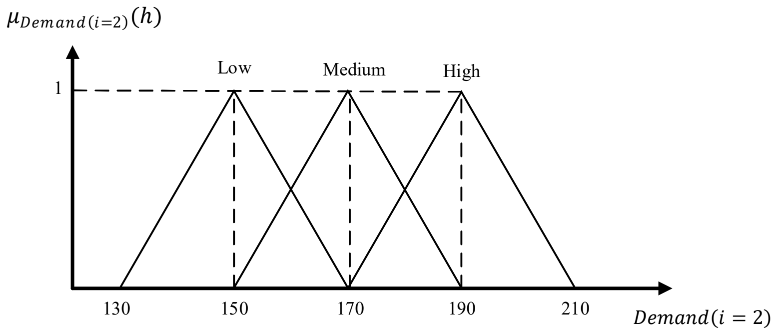

Figure 3 Membership function of fuzzy demand for item 2.

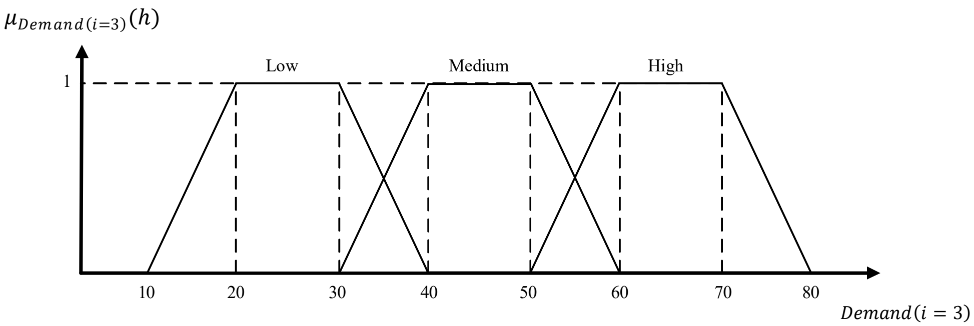

Figure 4 Membership function of fuzzy demand for item 3.

After using the possibility approach to transform F-CLSPP to the EC-CLSPP and using extension principle to develop EC-CLSPP in a form of MILP model, the fuzzy demand parameters with membership function can be modeled as the upper and lower crisp value at each α -cut and shown in Table 3-4.

Table 3 The lower crisp value of demand item 1-3 at each α-cut.

t	$(\tilde{d}_{it})^L_\alpha$		
	i = 1	i = 2	i = 3
1	$250\alpha + 200(1 - \alpha)$	$150\alpha + 130(1 - \alpha)$	$40\alpha + 30(1 - \alpha)$
2	$200\alpha + 150(1 - \alpha)$	$190\alpha + 170(1 - \alpha)$	$40\alpha + 30(1 - \alpha)$
3	$200\alpha + 150(1 - \alpha)$	$150\alpha + 130(1 - \alpha)$	$40\alpha + 30(1 - \alpha)$
4	$150\alpha + 100(1 - \alpha)$	$150\alpha + 130(1 - \alpha)$	$60\alpha + 50(1 - \alpha)$
5	$150\alpha + 100(1 - \alpha)$	$170\alpha + 150(1 - \alpha)$	$60\alpha + 50(1 - \alpha)$

Table 4 The upper crisp value of demand item 1-3 at each α-cut.

t	$(\tilde{d}_{it})^U_\alpha$		
	i = 1	i = 2	i = 3
1	$250\alpha + 300(1-\alpha)$	$150\alpha + 170(1-\alpha)$	$50\alpha + 60(1-\alpha)$
2	$200\alpha + 250(1-\alpha)$	$190\alpha + 210(1-\alpha)$	$50\alpha + 60(1-\alpha)$
3	$200\alpha + 250(1-\alpha)$	$150\alpha + 170(1-\alpha)$	$50\alpha + 60(1-\alpha)$
4	$150\alpha + 200(1-\alpha)$	$150\alpha + 170(1-\alpha)$	$70\alpha + 80(1-\alpha)$
5	$150\alpha + 200(1-\alpha)$	$170\alpha + 190(1-\alpha)$	$70\alpha + 80(1-\alpha)$

Table 5 Results of numerical examples.

	$\alpha = 0$			$\alpha = 0.1$			$\alpha = 0.2$			$\alpha = 0.3$		
ITEM1	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	100	0	0	105	0	0	125	0	0	115	0	0
t = 2	150	0	0	170	0	0	175	0	0	165	0	0
t = 3	175	0	0	175	0	0	175	0	0	165	0	0
t = 4	175	0	0	175	0	0	175	0	0	115	0	0
t = 5	0	0	0	0	0	0	0	0	0	115	0	0
ITEM2	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	0	122	0	0	126	0	0	130	0	0	134	0
t = 2	0	168	0	0	0	168	0	0	168	0	168	0
t = 3	0	130	0	0	132	0	0	134	0	0	136	0
t = 4	0	130	0	0	0	132	0	134	0	0	136	0
t = 5	150	0	0	0	152	0	154	0	0	0	156	0
ITEM3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	0	0	160	0	0	165	0	0	170	0	0	170
t = 2	0	0	0	0	0	0	0	0	0	0	0	0
t = 3	0	0	0	0	0	0	0	0	0	0	0	0
t = 4	0	0	0	0	0	0	0	0	0	0	0	0
t = 5	0	0	0	0	0	0	0	0	0	0	0	0
	$\alpha = 0.4$			$\alpha = 0.5$			$\alpha = 0.6$			$\alpha = 0.7$		
ITEM1	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	120	0	0	125	0	0	140	0	0	155	0	0
t = 2	170	0	0	175	0	0	175	0	0	175	0	0
t = 3	170	0	0	175	0	0	175	0	0	175	0	0
t = 4	120	0	0	125	0	0	130	0	0	135	0	0
t = 5	120	0	0	125	0	0	130	0	0	135	0	0
ITEM2	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	0	138	0	0	142	0	0	146	0	0	150	0
t = 2	0	0	168	0	168	0	0	0	168	0	0	168
t = 3	0	138	0	0	0	140	0	142	0	0	0	144

t = 4	0	138	0	0	140	0	0	0	142	0	0	144
t = 5	150	0	158	0	160	0	154	162	0	0	0	164
ITEM3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	0	0	180	0	0	185	0	0	190	0	0	195
t = 2	0	0	0	0	0	0	0	0	0	0	0	0
t = 3	0	0	0	0	0	0	0	0	0	0	0	0
t = 4	0	0	0	0	0	0	0	0	0	0	0	0
t = 5	0	0	0	0	0	0	0	0	0	0	0	0

	$\alpha = 0.8$			$\alpha = 0.9$			$\alpha = 1.0$		
ITEM1	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	170	0	0	165	0	0	65	135	0
t = 2	175	0	0	175	0	0	175	0	0
t = 3	175	0	0	165	175	0	175	0	0
t = 4	140	0	0	0	0	0	150	0	0
t = 5	140	0	0	145	0	0	150	0	0
ITEM2	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	0	154	0	0	158	0	0	0	162
t = 2	0	0	168	0	168	0	0	168	0
t = 3	0	146	0	0	0	148	0	150	0
t = 4	0	0	146	0	148	0	0	0	152
t = 5	0	0	166	0	168	0	154	0	168
ITEM3	MC1	MC2	MC3	MC1	MC2	MC3	MC1	MC2	MC3
t = 1	0	0	200	0	0	205	0	0	210
t = 2	0	0	0	0	0	0	0	0	0
t = 3	0	0	0	0	0	0	0	0	0
t = 4	0	0	0	0	0	0	0	0	0
t = 5	0	0	0	0	0	0	0	0	0

2.Computing Results

After using the possibility approach to transform F-CLSPP to the EC-CLSPP and using extension principle to develop EC-CLSPP in a form of MILP model, this model were formed in term of corner points for all membership functions and can be solved with basic software. The optimization software Gurobi Optimization for AMPL (Free Academic License) was used to find optimal solutions. The computing results of this numerical examples are shown in Table 5 and Figure 5. The results in table 5 provide the production quantity of each item on each machine in each period at each acceptable possible levels (α). From Fig. 5, the trend of total cost of solving this model withα = 0, 0,1, 0.2, ..., 1 is an increasing trend. It means that when changing possible levels will affect the plan of production quantity and increase total costs when higher possible levels are required.

3.Decision making process

Production planning can make decisions from verbal information demand from experienced decision makers under the acceptable possible levels. For example, the zero of the acceptable possible level (α = 0) will be chosen if the decision makers are sure in their verbal data.

Conclusion

In this paper, the F-CLSPP model was proposed for production planning when demand come from the experience of the decision maker, that it is called fuzzy demand. This case often occurs in SMEs business or new product production planning. The problem studied involves the production of multiple items in a single-stage lot size problem with capacity of time to produce of each machine in the setting of unrelated parallel machines. This problem was formulated in the F-CLSPP model and then it was transformed to the EC-CLSPP model by using the CCP concept and possibility approach. Then the extension principle was used to convert the fuzzy constraints to equivalent crisp constraints. As a result, this model is in a form of MILP model and can be solved by using basic software. Gurobi Optimization for AMPL was used to find the optimal solution effectively and provide useful information including total cost, production quantity, and the inventory level. Moreover, the results of numerical example showed that when changing possible levels will affect the plan of production quantity and increase total costs when higher possible levels are required. Therefore, the production planners can make decisions under the possible level depend on confidence in information data.

Acknowledgment

This research is part of a research project to study and to design a fuzzy approach to determine production lot size for capacitated single-stage production process with fuzzy demand, which received research funding from the Faculty of Engineering revenue budget from Srinakharinwirot University in 2020 (Contract Number 193/2020).

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