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SOLUTION OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS INVOLVING MIXED PARTIAL DERIVATIVE BY ELZAKI SUBSTITUTION METHOD

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ABSTRACT. In this paper we have to find the solution of nonlinear partial differential equation involving mixed partial derivatives by Elzaki Substitution method. Elzaki transform is applied and then nonlinear term handled with the help of Adomian polynomial. We get exact solution of nonlinear partial differential equations involving mixed partial derivatives. It is believed that this work will make it easy to study the nonlinear partial differential equations involving mixed partial derivatives arising different areas of research and innovation. Therefore the current method can be extended for the solution of higher order nonlinear problems. We give illustration through three problems.

KEYWORDS: Elzaki transform, Elzaki substitution method, Nonlinear Partial differential equations, Adomian polynomial.
AMS Subject Classification: 35A22, 35F20, 35F25.

1. Introduction

The study of exact solutions of nonlinear partial differential equation is very important in the study of nonlinear physical systems. In the literature, some of nonlinear partial differential equations solved by different integral transform like Laplace transform, Fourier transform, Hankel transform, Sumudu transform, Elzaki transform with Fundamental properties [13]. To solve partial differential equations very effective method is Laplace transform. After Laplace transform, in 1993 Watugula in [10] proposed a new integral transform named the Sumudu transform and used in control engineering problem. Nonlinear Fractional Partial Differential Equations Using the ElzakiTransform Method and the Homotopy Perturbation Method [11].

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R. I. Nuruddeen, et.al reviewed decomposition methods [12]. T. M. Elzaki and Salih M. Elzaki [3] showed Connection between Laplace transform and Elzaki transform. ADM coupled with Laplace transform method [6, 8, 7], Sumudu Decomposition Method [9], homotopy perturbation & Elzaki transform method [4]. T. M. Elzaki proposed fundamental properties of Elzaki transform [5]. Duality relations of Kamal transform with Laplace , Laplace-Carson, Aboodh, Sumudu, Elzaki transform and some useful integral transforms [1, 2]. The present study extends the idea of Laplace Substitution Method [7] for the solution of nonlinear partial differential equations involving mixed partial derivatives. This paper investigates the exact solution to the nonlinear partial differential equation involving mixed partial derivatives by Elzaki Substitution method. The paper is organized as follows. The basic definition and some relevant properties are noted in section 2. Methodology formation of Elzaki Substitution Method is done in section 3. Finally applications are discussed in section 4 and conclusion in section 5.

2. Basic definition and properties of Elzaki Transform:

Definition 2.1. If $f(t)$ is continuous function and for all $t \geq 0$ in a region $(-1)^j \times [0,\ \infty)$, then Elzaki transform for kernel having exponential function is defined as

$$E\left[f\left(t\right)\right] = T\left(v\right) = v\int_0^\infty f\left(t\right)e^{\frac{-t}{v}}\ dt\ and\ v \in \left(-k_1,\ k_2\right);\ k_1, k_2 > 0 \tag{2.1}$$

Definition 2.2. If $E\left[f\left(t\right)\right] = T\left(v\right)$ then $F(t)$ is called the inverse Elzaki trnasform of $T(v)$ and it is defined as

$$F(t) = E^{-1}\{T(v)\} \tag{2.2}$$

where E^{-1} is the inverse Elzaki transform operator.

The Elzaki Transform and its Inverse [13, 1, 2] of fundamental functions are given as follows

Sr. No.	Elzaki Transform	Inverse Elzaki Transform
1	$E\left\{(t^n)\right\} = n!v^{(n+2)}; n = 0, 1, 2, ..., n$	$E^{(-1)}v^n = \frac{y^{(n-2)}}{(n-2)!}, n = 2, 3, 4, \cdots$
2	$E\left\{(e^a y)\right\} = \frac{v^2}{(1-av)}$	$E^{(-1)}\left\{\frac{v^2}{(1-av)}\right\} = e^{ay}$
3	$E\left\{(\sin ay)\right\} = \frac{(av)^3}{(1+a^2v^2)}$	$E^{(-1)}\left\{\frac{v^3}{(1+a^2v^2)}\right\} = \frac{1}{asinay}$
4	$E\left\{(\cos ay)\right\} = \frac{(av)^2}{(1+a^2v^2)}$	$E^{(-1)}\left\{\frac{v^2}{(1+a^2v^2)}\right\} = \frac{1}{acosay}$
5	$E\left\{(\sinh ay)\right\} = \frac{(av)^3}{(1-a^2v^2)}$	$E^{(-1)}\left\{\frac{v^3}{(1-a^2v^2)}\right\} = \frac{1}{asinhay}$
6	$E\left\{(\cosh ay)\right\} = \frac{(av)^2}{(1-a^2v^2)}$	$E^{(-1)}\left\{\frac{v^2}{(1-a^2v^2)}\right\} = \frac{1}{acoshay}$

Table 1. Some Elzaki and its Inverse Transform

2.1. Properties.

The properties are illustrated using definition of Elzaki transform [11]. We mention some in the following.

(i) If $f\left(t\right) = \sum_{n=0}^{\infty} a_n t^n$ then $E\left[f\left(t\right)\right] = T\left(v\right) = \sum_{n=0}^{\infty} n!v^{n+2}$

(ii) If $f\left(t\right) = tf(t)$ then $E\left\{tf\left(t\right)\right\} = v^2\frac{dT}{dv} - vT(v)$

(iii) If $f\left(t\right) = t^2 f(t)$ then $E\left\{t^2 f\left(t\right)\right\} = v^4\frac{d^2T}{dv^2}$

(iv) If $f\left(t\right) = f'\left(t\right)$ then $E\left[f'\left(t\right)\right] = T'\left(v\right) = \frac{T(v)}{v} - v\ f(0)$

(v) If $f\left(t\right) = f^{(n)}\left(t\right)$ then $E\left[f^{(n)}\left(t\right)\right] = T^{(n)}\left(v\right) = \frac{T(v)}{v^n} - \sum_{k=0}^{n-1} v^{2-n-k} f^{(k)}\left(0\right), n \geq 1$.

2.2. **Properties for Partial Differential Equation using Elzaki transform:** Let $u(x,\ t)$ be a function of two independent variables x *and* t then

(i) E[u(x,t)] = T(x,v)
(ii) $\mathrm{E}\left[\frac{\partial \mathrm{u(x,t))}}{\partial \mathrm{t}}\right] = \frac{1}{v}\ \mathrm{T}\,(x,v)$ - vu$\,(x,0)$
(iii) $\mathrm{E}\left[\frac{\partial \mathrm{u}(x,\mathrm{t}))}{\partial x}\right] = \frac{d[T(x,v)]}{dx}$
(iv) $E\left[\frac{\partial^2 u(x,t))}{\partial t^2}\right] = \frac{1}{v^2}T(x,v) - u(x,0) - \frac{v(\partial u(x,0))}{\partial t}$
(v) $E\left[\frac{\partial^2 u(x,t))}{\partial x^2}\right] = \frac{d^2[T(x,v)]}{dx^2}$
(vi) $E\left[\frac{(\partial^n u(x,t))}{(\partial t^n)}\right] = \frac{1}{v^n}T(x,v) - \frac{u(x,0)}{v^{(n-2)}} - \frac{1}{v^{(n-3)}}\frac{(\partial u(x,0))}{\partial t} - \ldots - v\frac{(\partial^{(n-1)}u(x,0))}{(\partial t^{(n-1)})}$.

2.3. **Properties for Partial Differential Equation using Elzaki transform:** Let $u(x,\ t)$ be a function of two independent variables x *and* t then

3. Elzaki Substitution Method

In this section, we extend the proposed Elzaki Substitution method to solve nonlinear partial differential equation involving mixed order partial derivative. To illustrate the basic idea of [8], we consider a general nonlinear nonhomogeneous partial differential equation with initial conditions as

$$Lu\,(x,\ t) + Ru\,(x,\ t) + Nu\,(x,\ t) = h(x,\ t) \tag{3.1}$$

$$u\,(x,\ 0) = f\,(x)\,,\ u_t\,(0,\ t) = g(t) \tag{3.2}$$

Where $L = \frac{\partial^2}{\partial x \partial t}$, $Ru(x,\ t)$ is the remaining term contains linear partial derivatives, $Nu(x,\ t)$ is nonlinear term and $h(x,\ t)$ is the source term
We get equation (3.1) as

$$\frac{\partial^2 u\,(x,\ t)}{\partial x \partial t} + Ru\,(x,\ t) + Nu\,(x,\ t) = h(x,\ t)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t}u\,(x,\ t)\right) + Ru\,(x,\ t) + Nu\,(x,\ t) = h(x,\ t) \tag{3.3}$$

Substituting $\frac{\partial u}{\partial t} = U$ in (3.3), we get

$$\frac{\partial U}{\partial x} + Ru\,(x,\ t) + Nu\,(x,\ t) = h(x,\ t) \tag{3.4}$$

Taking Elzaki transform on both sides w.r.t. x,

$$\frac{1}{v}E_x\,[U(x,\ t)] - vU\,(0,\ t) + E_x\,[Ru\,(x,\ t) + Nu\,(x,\ t)] = E_x\,[h(x,\ t)]$$

$$E_x\,[U(x,\ t)] = v^2 u_t\,(0,\ t) + vE_x\,[h(x,\ t)] - vE_x\,[Ru\,(x,\ t) + Nu\,(x,\ t)]$$

Using initial conditions

$$E_x\,[U(x,\ t)] = v^2 g(t) + vE_x\,[h(x,\ t)] - vE_x\,[Ru\,(x,\ t) + Nu\,(x,\ t)]$$

Taking Inverse Elzaki transform on the both sides w. r. t. x, we get

$$U(x,\ t) = {E_x}^{-1}\left[v^2 g(t)\right] + {E_x}^{-1}\left[vE_x\,[h(x,\ t)]\right] - {E_x}^{-1}\left[vE_x\,[Ru\,(x,\ t) + Nu\,(x,\ t)]\right]$$

Resubstituting the value of $U\,(x,\ t) = \frac{\partial u(x,\ t)}{\partial t}$, we get

$$\frac{\partial u(x,\ t)}{\partial t} = g\,(t) + {E_x}^{-1}\left[vE_x\,[h(x,\ t)]\right] - {E_x}^{-1}\left[vE_x\,[Ru\,(x,\ t) + Nu\,(x,\ t)]\right] \tag{3.5}$$

Taking Elzaki transform of equation (3.5) & Using initial conditions, we get

$$E_t\,[u(x,\ t)] = v^2 f\,(x) + v\left[E_t\left[g\,(t) + {E_x}^{-1}\left[vE_x\,[h(x,\ t)]\right] - {E_x}^{-1}\left[vE_x\,[Ru\,(x,\ t) + Nu\,(x,\ t)]\right]\right]\right]$$

Taking inverse Elzaki transform on both sides w. r. t. t, we get

$$u\left(x,\ t\right)=f\left(x\right)+{E_t}^{-1}\left[v\left[E_t\left[g\left(t\right)+{E_x}^{-1}\left[vE_x\left[h(x,\ t)\right]\right]-{E_x}^{-1}\left[vE_x\left[Ru\left(x,\ t\right)+Nu\left(x,\ t\right)\right]\right]\right]\right]\right] \tag{3.6}$$

Here the solution of the nonlinear term obtained through Adomian Polynomial then it is using in series as $u\left(x,\ t\right)=\sum_{n=0}^{\infty}u_n(x,t)$. We get the exact or approximate solution through (3.6).

4. Applications:

Example 4.1. Solve the nonlinear partial differential equation

$$\frac{\partial^2 u}{\partial x\partial y}-u\frac{\partial u}{\partial x}=0$$

having initial conditions $u\left(x,\ 0\right)=x,\ u_y\left(0,\ y\right)=0$

Solution:
We can write the given p. d. e. as

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)=u\frac{\partial u}{\partial x}=uu_x$$

Substitute $\frac{\partial u}{\partial y}=U(x,\ y)$

$$\frac{\partial U}{\partial x}=uu_x$$

Taking Elzaki transform on both sides w. r. t. x and using initial conditions, we get

$$E_x\left(U(x,\ y)\right)=vE_x[uu_x]$$

Taking inverse Elzaki transform on both sides w. r. t. x & Re-substitute $\frac{\partial u}{\partial y}=U(x,\ y)$, we get

$$\frac{\partial u}{\partial y}={E_x}^{-1}[vE_x\left[uu_x\right]]$$

Taking Elzaki transform on both sides w. r. t. y and using initial conditions, we get

$$E_y\left(u\left(x,\ y\right)\right)=v^2x+vE_y\left[{E_x}^{-1}[vE_x\left[uu_x\right]]\right]$$

Taking inverse Elzaki transform on both the sides w. r. t. y, we get

$$u\left(x,y\right)=x+{E_y}^{-1}\left[vE_y\left[{E_x}^{-1}[vE_x\left[uu_x\right]]\right]\right]$$

We find solution which is in series form

$$u\left(x,\ y\right)=\sum_{n=0}^{\infty}u_n(x,\ y)$$

Here nonlinear terms appear in the equation. We can find by using Adomial polynomial.
Now consider

$$u\frac{\partial u}{\partial x}=\sum_{n=0}^{\infty}A_n$$

Where A_n is Adomian polynomial and use to find $u_0,u_1,\ u_2,\ \ldots,\ u_n,\ n\geq 0$
Calculate terms as

$$A_0=u_0u_{0x}$$

$$A_1=u_0u_{1x}+u_1u_{0x}$$

$$A_2=u_0u_{2x}+u_1u_{1x}+u_2u_{0x}$$

.
.
.

etc
Use in the above equation, we get

$$\sum_{n=0}^{\infty} u_n(x,\ y) = x + {E_y}^{-1}\left[vE_y\left[{E_x}^{-1}[vE_x\left[\sum_{n=0}^{\infty} A_n\right]]\right]\right]$$

On comparing both the sides we get

$$u_0\left(x,\ y\right) = x$$

$$u_1\left(x,\ y\right) = {E_y}^{-1}\left[vE_y\left[{E_x}^{-1}[vE_x\left[A_0\right]]\right]\right]$$

$$u_1\left(x,\ y\right) = \frac{x^2y}{2}$$

$$u_2\left(x,\ y\right) = {E_y}^{-1}\left[vE_y\left[{E_x}^{-1}[vE_x\left[A_1\right]]\right]\right]$$

$$u_2\left(x,\ y\right) = \frac{x^3y^2}{\left(2\right)^2}$$

Similarly $u_3\left(x,\ y\right) = \frac{x^4y^3}{\left(2\right)^3}$

.
.
.

etc
Substituting the values in the series. We get the solution

$$u\left(x,\ y\right) = u_0\left(x,\ y\right) + u_1\left(x,\ y\right) + u_2\left(x,\ y\right) + u_3\left(x,\ y\right) + \ldots$$

$$u\left(x,\ y\right) = x + \frac{x^2y}{2} + \frac{x^3y^2}{\left(2\right)^2} + \frac{x^4y^3}{\left(2\right)^3} + \ldots$$

$$u\left(x,\ y\right) = x\sum_{n=0}^{\infty}\left(\frac{xy}{2}\right)^n$$

This is geometric series and it is convergent if $|xy| < 2$ for all $(x,\ y)$ belongs to the domain of $u\left(x,\ y\right)$

$$u\left(x,\ y\right) = \frac{2x}{2 - xy},\ |xy| < 2$$

This is exact solution of the nonlinear partial differential equation. Following figure 4.1 in the intervals.

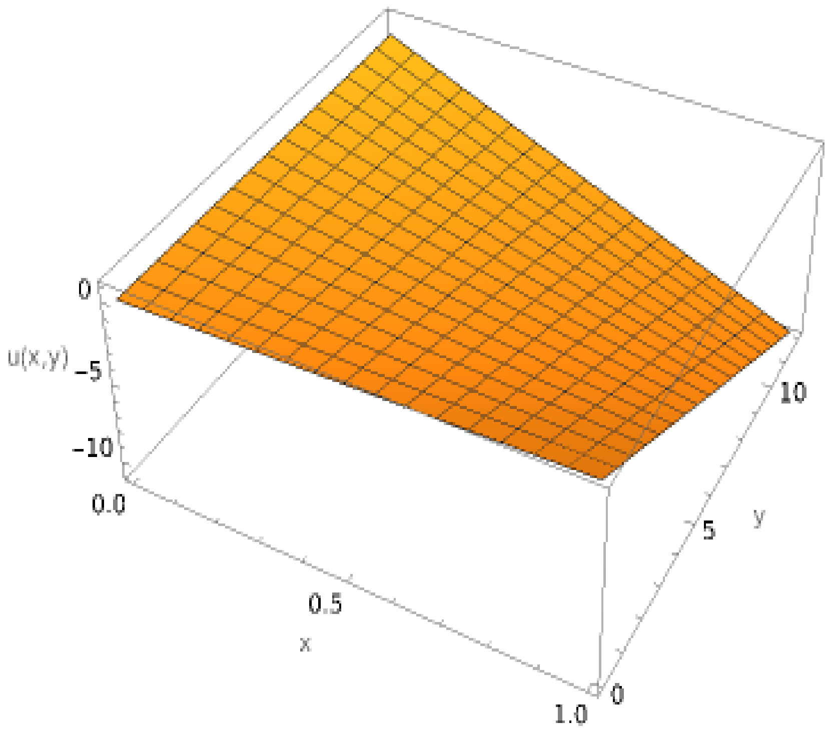

Fig 4.1

Example 4.2. Solve the nonlinear non-homogeneous partial differential equation

$$\frac{\partial^2 u}{\partial y \partial x} - \left(\frac{\partial u}{\partial x}\right)^2 + u^2 = e^x$$

with initial conditions $u\,(0,\ y) = y,\ u_x\,(x,\ 0) =\ 0.$

Solution:

We can rewrite equation as

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) - \left(\frac{\partial u}{\partial x}\right)^2 + u^2 = e^x$$

Substituting $\frac{\partial u}{\partial x} = U(x,\ y)$

$$\frac{\partial U}{\partial y} - U^2 + u^2 = e^x$$

Taking Elzaki transform of above equation on both sides w. r. t. y and Using initial condition we get

$$E\,(U\,(x,\ y)) = v^3 e^x + vE_y\left[U^2 - u^2\right]$$

Taking inverse Elzaki transform on the both sides w. r. t. y & Re-substituting $\frac{\partial u}{\partial x} = U(x,\ y)$

$$\frac{\partial u}{\partial x} = ye^x + {E_y}^{-1}\left[vE_y\left[U^2 - u^2\right]\right]$$

Taking Elzaki transform on the both sides w. r. t. x and Using initial condition we get,

$$E\,(u\,(x,\ y)) = y\left(\frac{v^2}{1-v}\right) - vE_x\left[{E_y}^{-1}\left[vE_y\left(U^2 - u^2\right)\right]\right]$$

Taking inverse Elzaki transform on the both sides w. r. t. x, and Using initial condition we get

$$u(x,\ y) = ye^x - {E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left(U^2 - u^2\right)\right]\right]\right]$$

We find solution which is in series form

$$u(x,\ y) = \sum_{n=0}^{\infty} u_n(x,\ y)$$

Here two nonlinear terms appear in the equation. We can find by using Adomial polynomial.

Now consider $\left(\frac{\partial u}{\partial x}\right)^2 = \sum_{n=0}^{\infty} A_n$ and $u^2 = \sum_{n=0}^{\infty} B_n$.

Where A_n and B_n are Adomian polynomials of components $u_0, u_1,\ u_2,\ \ldots,\ u_n,\ n \geq 0$

Calculate terms as

$$A_0 = {u_{0x}}^2$$
$$A_1 = 2u_{0x}u_{1x}$$
$$A_2 = u_{0x}u_{2x} + {u_{1x}}^2$$
$$\vdots$$

etc

Similarly

$$B_0 = {u_0}^2$$
$$B_1 = 2u_0u_1$$
$$B_2 = u_0u_2 + {u_1}^2$$
$$\vdots$$

etc

Substituting the above equation, we get

$$\sum_{n=0}^{\infty} u_n(x,\ y) = ye^x - {E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left(U^2 - u^2\right)\right]\right]\right]$$

$$\sum_{n=0}^{\infty} u_n(x,\ y) = ye^x - {E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left(\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right)\right]\right]\right]$$

On comparing both sides we get

$$u_0(x,\ y) = ye^x$$
$$u_1(x,\ y) = -{E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left({u_{0x}}^2 - {u_0}^2\right)\right]\right]\right]$$
$$= -{E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left(0^2 - 0^2\right)\right]\right]\right]$$
$$u_1(x,\ y) = 0$$
$$u_2(x,\ y) = -{E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left(2u_{0x}u_{1x} - 2u_0u_1\right)\right]\right]\right]$$
$$= -{E_y}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left(0 - 0\right)\right]\right]\right]$$
$$u_2(x,\ y) = 0$$

And hence we get $u_1(x,\ y) = 0$, $u_2(x,\ y) = 0$, … $u_n(x,\ y) = 0$. Using the values we get,

$$u(x,\ y) = ye^x$$

This is exact solution of the nonlinear partial differential equation. Following figure 4.2 in the intervals is.

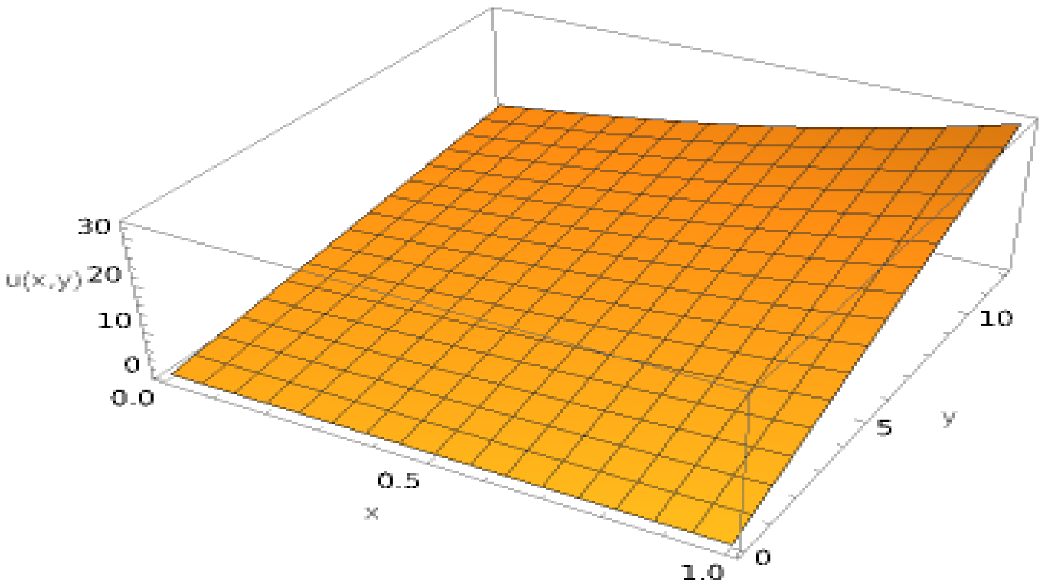

Fig 4.2

Example 4.3. Solve the nonhomogeneous partial differential equations

$$\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^2 = 2y + y^4$$

with initial conditions $u\left(0,\ y\right) = ay,\ u_x\left(x,\ 0\right) = 0$

Solution:
Consider $\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^2 = 2y + y^4$
Substituting $\frac{\partial u}{\partial x} = U$, we get

$$\frac{\partial U}{\partial y} + \frac{\partial U}{\partial x} + \left[U\right]^2 = 2y + y^4$$

Taking Elzaki transform w. r. t. y on the both sides and using initial conditions, we get

$$E_y\left[U(x,\ y)\right] + vE_y\left[U^2\right] = 2v^4 + 4!v^7$$

Taking Inverse Elzaki transform on both sides w. r. t. y and Re-substituting $\frac{\partial u}{\partial x} = U$, we get

$$\frac{\partial u}{\partial x} + {E_y}^{-1}\left[vE_y\left[\frac{\partial U}{\partial x} + U^2\right]\right] = y^2 + \frac{1}{5}y^5$$

Taking Elzaki transform on both sides w.r.t. x and using initial conditions, we get

$$E_x\left[u(x,\ y)\right] = ayv^2 + \left(y^2 + \frac{1}{5}y^5\right)v^3 - vE_x\left[{E_y}^{-1}\left[vE_y\left[U^2\right]\right]\right]$$

Taking inverse Elzaki transform on both sides w. r. t. x, we get

$$u\left(x,\ y\right) = ay + x\left(y^2 + \frac{1}{5}y^5\right) - {E_x}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left[\left(\frac{\partial u}{\partial x}\right)^2\right]\right]\right]\right]$$

We know in this substitution method, the solution obtained in the series form as $u\,(x,\ y) =\ \sum_{n=0}^{\infty} u_n(x,\ y)$
A non linear term appear in the above equation can be decompose it by using Adomian polynomial A_n

$$\sum_{n=0}^{\infty} A_n = \left(\frac{\partial u}{\partial x}\right)^2$$

We get,

$$A_0 = {u_{0x}}^2$$

$$A_1 = 2u_{0x}\ u_{1x}$$

$$A_2 = u_{0x}u_{2x} + {u_{1x}}^2$$

$$\vdots$$

etc
Substituting the values in the equation

$$\sum_{n=0}^{\infty} u_n(x,\ y) = ay + xy^2 + \frac{xy^5}{5} - {E_x}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left[\sum_{n=0}^{\infty} A_n\right]\right]\right]\right]$$

On comparing on the both sides of above equation, we get

$$u_0\,(x,\ y) = ay + xy^2$$

$$u_1\,(x,\ y) = \frac{xy^5}{5} - {E_x}^{-1}\left[vE_x\left[{E_y}^{-1}\left[vE_y\left[\sum_{n=0}^{\infty} A_n\right]\right]\right]\right]$$

$$u_1\,(x,\ y) = 0$$

$$u_2\,(x,\ y) = 0$$

$$\vdots$$

$$u_n\,(x,\ y) = 0$$

On substituting values, we get the solution

$$u\,(x,\ y) = ay + xy^2$$

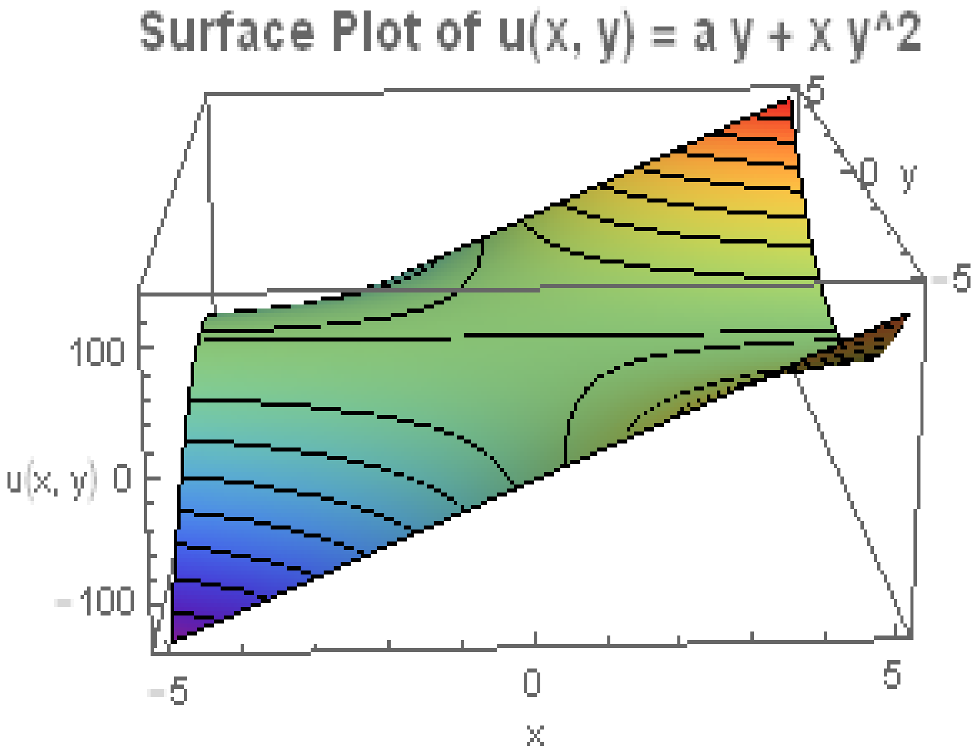

Fig 4.3

5. Conclusion

In this work we solved nonlinear partial differential equations with mixed partial derivatives, we have successfully examined the Elzaki Substitution Method in this study. Adomian polynomials are used to efficiently control the nonlinear components, enabling a methodical and practical approach to obtaining accurate solutions. A number of illustrative cases are used to demonstrate the method's capacity to produce precise results. The Elzaki Substitution Method demonstrated accuracy and dependability in each of the three nonlinear examples. The accuracy and relevance of the findings produced are further supported by the MATHEMATICA program, which has also been used to present graphical representations of the answers. The Elzaki Substitution Method has a number of benefits over the conventional Laplace Substitution Method, including easier application and simpler calculation, especially when dealing with complicated nonlinearities. Because such equations are common in the domains of science and engineering, it is therefore a useful tool for researchers.

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