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CONTROLLABILITY RESULTS FOR A NONLOCAL IMPULSIVE NEUTRAL STOCHASTIC FUNCTIONAL INTEGRO-DIFFERENTIAL EQUATIONS WITH DELAY AND POISSON JUMPS

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ABSTRACT. The current paper is concerned with the controllability of impulsive neutral stochastic delay partial functional integro-differential equations with Poisson jumps in Hilbert spaces. Sufficient conditions are established using the theory of resolvent operators developed by Grimmer [Resolvent operators for integral equations in Banach spaces, Trans. Amer. Math. Soc., 273(1982):333–349] combined with a fixed point approach for achieving the required result. An example is presented to illustrate the application of the obtained results.

KEYWORDS: Controllability, Resolvent operators, C_0-semigroup, impulsive integrodifferential equations, fixed point theory.
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1. Introduction

The theory of semigroups of bounded linear operators is closely related to solving differential and integrodifferential equations in Banach spaces. In recent years, this theory has been applied to a large class of nonlinear differential equations in Banach spaces. Using the method of semigroups, various types of solutions to semilinear evolution equations have been discussed by Pazy [22]. Various evolutionary processes from fields as diverse as physics, population dynamics, aeronautics, economics and engineering are characterized by the fact that they undergo abrupt changes of state at certain moments of time between intervals of continuous evolution. Because the duration of these changes are often negligible compared to the

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total duration of the process, such changes can be reasonably well approximated as being instantaneous changes of state, or in the form of impulses. These processes can be more suitably modeled by impulsive differential equations, which allow for discontinuities in the evolution of the state.
The development of the theory of functional differential equations with infinite delay heavily depends on a choice of a phase space. In fact, various phase spaces have been considered and each different phase space requires a separate development of the theory [13]. The common space is the phase space $\mathcal{B}$ proposed by Hale and Kato in [12], which is widely applied in functional differential equations with infinite delay. However, this phase space is not correct for the impulsive case. Generally, the theory of impulsive functional differential equations or inclusions is based on the phase space defined later (see [15]).

In many cases, deterministic models often fluctuate due to noise, which is random or at least appears to be so. Therefore, we must move from deterministic problems to stochastic ones. Taking the disturbances into account, the theory of differential inclusions has been generalized to stochastic functional differential inclusions (see [7, 6] and the references therein). The existence, uniqueness, stability, controllability and other quantitative and qualitative properties of solutions of stochastic evolution equations or inclusions have recently received a lot of attention (see [14] and the references therein). As one of the fundamental concepts in mathematical control theory, controllability plays an important role both in deterministic and stochastic control theory. Roughly speaking, controllability generally means that it is possible to steer a dynamical control system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. The controllability of nonlinear stochastic systems in infinite dimensional spaces has been extensively studied by several authors, see [21] and the references therein. Recently, Park, Balachandran, and Arthi [19] investigated the controllability of impulsive neutral integro-differential systems with infinite delay in Banach spaces using Schauder-type fixed point theorem. Arthi and Balachandran [2] established the controllability of damped second-order impulsive neutral functional differential systems with infinite delay by means of the Sadovskii fixed point theorem combined with a noncompact condition on the cosine family of operators. Very recently, also using Sadovski's fixed point theorem, Muthukumar and Rajivganthi [18] proved sufficient conditions for the approximate controllability of fractional order neutral stochastic integro-differential systems with nonlocal conditions and infinite delay.

Motivated by the previously mentioned works, in this paper, we will extend some such results of mild solution for the following neutral stochastic partial functional integrodifferential equations with infinite delay and Poisson jumps.

$$\begin{cases} d\left[x(t)-g\left(t,x_t,\int_0^t\sigma_1(t,s,x_s)ds\right)\right]=\left[A\left[x(t)-g\left(t,x_t,\int_0^t\sigma_1(t,s,x_s)ds\right)\right]\right.\\ \left.+f\left(t,x_t,\int_0^t\sigma_2(t,s,x_s)ds\right)\right]dt+\left[\int_0^t B(t-s)[x(s)-g\left(s,x_s,\int_0^s\sigma_1(s,\tau,x_\tau)d\tau\right)]ds\right]dt\\ +Cu(t)dt+\int_{-\infty}^t\sigma(t,s,x_s)dw(s)+\int_{\mathfrak{U}}\gamma(t,x(t-),v)d\tilde{N}(dt,dv), \quad t_k\neq t\in J:=[0,T],\\ \Delta x(t_k)=I_k(x_{t_k}), \quad k=\{1,\cdots,m\}=:\overline{1,m},\\ x(s)-q(x_{t_1},x_{t_2},\cdots,x_{t_n})(s)=\varphi(s)\in\mathcal{L}_2(\Omega,\mathcal{B}), \text{ for a.e } \ s\in J_0:=(-\infty,0], \end{cases} \tag{1.1}$$

where $0<t_1<t_2<\cdots<t_n<T, n\in\mathbb{N}; x(\cdot)$ is a stochastic process taking values in a real separable Hilbert space $\mathbb{H}$, $A: D(A)\subset\mathbb{H}\to\mathbb{H}$ is the infinitesimal generator of a strongly continuous semigroup on $\mathbb{H}, (B(t))_{t\geq 0}$ is a family of closed linear operator on $\mathbb{H}$ having the same domain $D(B)$ which contains the domain of A. The history $x_t: J_0\to\mathbb{H}, x_t(\theta)=x(t+\theta)$ for $t\geq 0$, belongs to the phase space $\mathcal{B}$, which will be described in Section 2. Assume that the mappings $f,g: J\times\mathcal{B}\times\mathbb{H}\to\mathbb{H}, \sigma: J\times J\times\mathcal{B}\to\mathcal{L}_2^0, \sigma_i: J\times J\times\mathcal{B}\to\mathbb{H}$, i =1, 2, $I_k:\mathcal{B}\to\mathbb{H}, k=\overline{1,m}, q:\mathcal{B}^n\to\mathcal{B}$, and $\gamma: J\times\mathbb{H}\times\mathfrak{U}\to\mathbb{H}$ are appropriate functions to be specified later. The control function $u(\cdot)$ takes values in $L^2(J,U)$ of admissible control functions for a separable Hilbert space U and C is a bounded linear operator from U into $\mathbb{H}$. Furthermore, let $0=t_0<t_1<\cdots<t_m<t_{m+1}=T$ be prefixed points, and $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$ represents the jump of the function x at time t_k with I_k, determining the size of the jump, where $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t=t_k$, respectively. The initial data $\varphi(t)=\{\varphi(t) : -\infty<t\leq 0\}$ is an $\mathcal{F}_0$-measurable $\mathcal{B}$-valued random variables independent of the Wiener process $\{w(t)\}$ and the Poisson point process $p(\cdot)$ with a finite second moment.

The aim of our paper is to study the controllability of nonlocal impulsive neutral stochastic functional integrodifferential equations with infinite delay and Poisson jumps in Hilbert spaces. The main techniques used here include the Banach contraction principle and techniques based on the use of a strongly continuous family of operators $R(t); t\geq 0$ defined on the Hilbert space $\mathbb{H}$ and called their resolvent (the precise definition will be given below). The resolvent operator is similar to semigroup operator for abstract diferential equations in Banach spaces.

The structure of this paper is as follows: in Section 2, we briefly present some basic notations, preliminaries, and assumptions. The main results in Section 3 are devoted to study the controllability for the system (1.1) with their proofs. An example is given in Section 4 to illustrate the theory.

2. Preliminaries

In this section, we briefly recall some basic definitions and results for stochastic equations in infinite dimensions. For more details on this section, we refer the reader to Da Prato and Zabczyk (1992)[24] and Protter (2004)[25]. Let $(\mathbb{H},\|\cdot\|_{\mathbb{H}},\langle\cdot,\cdot\rangle_{\mathbb{H}})$ and $(\mathbb{K},\|\cdot\|_{\mathbb{K}},\langle\cdot,\cdot\rangle_{\mathbb{K}})$ denote two real separable Hilbert spaces, with their vectors, norms, and their inner products, respectively. We denote by $\mathcal{L}(\mathbb{K};\mathbb{H})$ the set of all linear bounded operators from $\mathbb{K}$ into $\mathbb{H}$, which is equipped with the usual operator norm $\|\cdot\|$. In this paper, we use the symbol $\|\cdot\|$ to denote norms of operators regardless of the spaces potentially involved when no confusion possibly arises.

2.1. **Basic preliminaries on the stochastic integration and the abstract phase space.** Let $(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete filtered probability space satisfying the usual condition (i.e. it is right continuous and $\mathcal{F}_0$ contains all P-null sets). Let $w = (w(t))_{t\geq 0}$ be a Q-Wiener process defined on the probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ with the covariance operator Q such that $Tr(Q) < \infty$. We assume that there exists a complete orthonormal system $\{e_k\}_{k\geq 1}$ in $\mathbb{K}$, a bounded sequence of nonnegative real numbers λ_k such that $Qe_k = \lambda_k e_k$, $k = 1, 2, \cdots$, and a sequence of independent Brownian motions $\{\beta_k\}_{k\geq 1}$ such that

$$\langle w(t), e\rangle_{\mathbb{K}} = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\langle e_k, e\rangle_{\mathbb{K}}\beta_k(t), \quad e \in \mathbb{K}, t \geq 0.$$

Let $\mathcal{L}_2^0 = \mathcal{L}_2(Q^{1/2}\mathbb{K}; \mathbb{H})$ be the space of all Hilbert–Schmidt operators from $Q^{1/2}\mathbb{K}$ into $\mathbb{H}$ with the inner product $\langle \Psi, \phi\rangle_{\mathcal{L}_2^0} = Tr[\Psi Q\phi^*]$, where ϕ^* is the adjoint of the operator ϕ. Let $p = p(t), t \in D_p$ (the domain of p(t)) be a stationary $\mathcal{F}_t$-Poisson point process taking its value in a measurable space $(\mathfrak{U}, \mathcal{B}(\mathfrak{U}))$ with a σ-finite intensity measure $\lambda(dv)$ by $N(dt, dv)$ the Poisson counting measure associated with p, that is,

$$N(t, \mathfrak{U}) = \sum_{s\in D_p, s\leq t} \mathbb{I}_{\mathfrak{U}}(p(s));$$

for any measurable set $\mathfrak{U} \in \mathcal{B}(\mathbb{K} - \{0\})$, which denotes the Borel σ-field of $(\mathbb{K} - \{0\})$. Let

$$\tilde{N}(dt, dv) = N(dt, dv) - \lambda(dv)dt,$$

be the compensated Poisson measure that is independent of $w(t)$. Denote by $\mathcal{P}^2(J \times \mathfrak{U}; \mathbb{H})$ the space of all predictable mappings $\gamma : J \times \mathfrak{U} \to \mathbb{H}$ for which

$$\int_0^t \int_{\mathfrak{U}} \mathbb{E}\|\gamma(t, v)\|^2_{\mathbb{H}}\lambda(dv)dt < \infty.$$

We may then define the $\mathbb{H}$-valued stochastic integral $\int_0^t \int_{\mathfrak{U}} \gamma(t, v)\tilde{N}(dt, dv)$, which is a centered square integrable martingale. For the construction of this kind of integral, we can refer to Protter [25].

The collection of all strongly measurable, square-integrable $\mathbb{H}$-valued random variables, denoted by $\mathcal{L}_2(\Omega, \mathbb{H})$, is a Banach space equipped with norm $\|x\|_{\mathcal{L}_2} = (\mathbb{E}\|x\|^2)^{1/2}$. Let $\mathcal{C}(J, \mathcal{L}_2(\Omega, \mathbb{H}))$ be the Banach space of all continuous maps from J to $\mathcal{L}_2(\Omega, \mathbb{H})$, satisfying the condition $\sup_{t\in J} \mathbb{E}\|x(t)\|^2 < \infty$. An important subspace is given by $\mathcal{L}_2^0(\Omega, \mathbb{H}) = \{f \in \mathcal{L}_2(\Omega, \mathbb{H}) :$ f is $\mathcal{F}_0 -$ measurable$\}$. Further, let

$$\mathcal{L}_2^{\mathbb{F}}(0, T; \mathbb{H}) = \Big\{g : J \times \Omega \to \mathbb{H} : g \text{ is } \mathbb{F} - \text{progressively measurable and } \mathbb{E}\left(\int_J \|g(t)\|^2_{\mathbb{H}}dt\right) < \infty\Big\}.$$

Since the system (1.1) has impulsive effects, the phase space used in Balasubramaniam and Ntouyas [5] and Park et al. [21] cannot be applied to these systems. So, we need to introduce an abstract phase space $\mathcal{B}$, as follows:

Assume that $l : J_0 \to (0, +\infty)$ is a continuous function with $l_0 = \int_{J_0} l(t)dt < \infty$. For any $a > 0$, we define

$$\mathcal{B} := \Big\{\psi : J_0 \to \mathbb{H} : (\mathbb{E}\|\psi(\theta)\|^2)^{1/2} \text{ is a bounded and measurable function on } [-a, 0] \text{ and } \int_{J_0} l(s) \sup_{\theta\in[s,0]} (\mathbb{E}\|\psi(\theta)\|^2)^{1/2}ds < \infty\Big\}.$$

If $\mathcal{B}$ is endowed with the norm

$$\|\psi\|_{\mathcal{B}} = \int_{J_0} \sup_{\theta\in[s,0]} (\mathbb{E}\|\psi(\theta)\|^2)^{1/2} ds, \quad \forall\psi \in \mathcal{B},$$

then, it is clear that $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ is a Banach space (Hino, Murakami, & Naito, 1991).

Let $J_T = (-\infty, T]$. We consider the space

$$\mathcal{B}_T := \{x : J_T \to \mathbb{H} \text{ such that } x_k \in \mathcal{C}(J_k, \mathbb{H}) \text{ and there exist } x(t_k^-) \text{ and } x(t_k^+) \text{ with } x(t_k^-) = x(t_k^+), x(0) - q(x_{t_1}, x_{t_2}, \cdots, x_{t_n}) = \varphi \in \mathcal{B}, k = \overline{1,m}\},$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}], k = \overline{1,m}$. Set $\|\cdot\|_T$ be a seminorm in $\mathcal{B}_T$ defined by

$$\|x\|_T = \|\varphi\|_{\mathcal{B}} + \sup_{s\in J}(\mathbb{E}\|x(s)\|^2)^{1/2}, \quad x \in \mathcal{B}_T.$$

Now, recall the following useful lemma that appeared in Chang [9]

Lemma 2.1. *(Chang, [9]) Assume that $x \in \mathcal{B}_T$, then for $t \in J, x_t \in \mathcal{B}$. Moreover,*

$$l_0(\mathbb{E}\|x(t)\|^2)^{1/2} \leq \|x_t\|_{\mathcal{B}} \leq \|x_0\|_{\mathcal{B}} + l_0 \sup_{s\in[0,t]} (\mathbb{E}\|x(s)\|^2)^{1/2}.$$

2.2. **Partial integrodifferential equation in Banach space.** In the present section we recall some definitions, notations and properties needed in what follows. Let Z_1 and Z_2 be Banach spaces. We denote by $\mathcal{L}(Z_1, Z_2)$ the Banch space of bounded linear operators from Z_1 into Z_2 endowed with the operator norm and we abbreviate this notation to $\mathcal{L}(Z_1)$ when $Z_1 = Z_2$.

In what follows, $\mathbb{H}$ will denote a Banach space, A and B(t) are closed linear operators on $\mathbb{H}$. Y represents the Banach space $\mathcal{D}(A)$, the domain of operator A, equiped with the graph norm

$$\|y\|_Y := \|Ay\| + \|y\| \text{ for } \quad y \in Y.$$

The notation $C([0, +\infty); Y)$ stands for the space of all continuous function from $[0, +\infty)$ into Y. We then consider the following Cauchy problem

$$\begin{cases} v'(t) = Av(t) + \int_0^t B(t-s)v(s)ds \text{ for } \quad t \geq 0, \\ v(0) = v_0 \in \mathbb{H}. \end{cases} \tag{2.1}$$

Definition 2.2. ([11]) A resolvent operator of Eq.(2.1) is a bounded linear operator valued function $R(t) \in \mathcal{L}(\mathbb{H})$ for $t \geq 0$, satisfying the following propreties:

(i) R(0)=I and $\|R(t)\| \leq Ne^{\beta t}$ for some constant N and β.
(ii) For each $x \in \mathbb{H}$, R(t)x is strongly continuous for $t \geq 0$.
(iii) For $x \in Y$, $R(.)x \in C^1([0, +\infty); \mathbb{H}) \cap C([0, +\infty); Y)$ and

$$\begin{aligned} R'(t)x &= AR(t)x + \int_0^t B(t-s)xds \\ &= R(t)Ax + \int_0^t R(t-s)xds \text{ for } \quad t \geq 0. \end{aligned}$$

For additional details on resolvent operators, we refer the reader to [11, 10]. The resolvent operator plays an important role to study the existence of solutions and to establish a variation of constants for non-linear systems. For this reason, we need to know when the linear system(2.1) possesses a resolvent operator. Theorem 2.1 below provides a satisfactory answer to this problem.

In what follows we suppose the following assumptions:

(H1) A is the infinitesimal generator of a C_0-semigroup $(T(t))_{t\geq 0}$ on $\mathbb{H}$

(H2) For all $t \geq 0$, B(t) is a closed linear operator from $\mathcal{D}(A)$ to $\mathbb{H}$, and $B(t) \in \mathcal{L}(Y, \mathbb{H})$. For any $y \in Y$, the map $t \to B(t)y$ is bounded, differentiable and the derivative $t \to B^{'}(t)y$ is bounded uniformly continuous on $\mathbb{R}^{+}$.

Theorem 2.1. *([11]) Assume that hypotheses **(H1)** and **(H2)** hold. Then the Eq. (2.1) admits a resolvent operator* $(R(t))_{t\geq 0}$.

In the sequel, we recall some results on the existence of solutions for the following integro-differentiel equation

$$\begin{cases} v^{'}(t) = Av(t) + \int_0^t B(t-s)v(s)ds + q(t) \text{ for } \quad t \geq 0, \\ v(0) = v_0 \in \mathbb{H}. \end{cases} \tag{2.2}$$

where $q \ : \ [0, +\infty[\to \mathbb{H}$ is continuous function.

Definition 2.3. A continuous function $v : [0, +\infty) \to \mathbb{H}$ is said to be a strict solution of the Eq.(2.2) if

(i) $v \in C^1([0, +\infty); \mathbb{H}) \cap C([0, +\infty); Y)$,
(ii) v satisfies Eq.(2.2) for $t \geq 0$.

Remark 2.4. From this definition we deduce that $v(t) \in \mathcal{D}(A)$, and the function $B(t-s)v(s)$ is integrable, for all $t > 0$ and $s \in [0, +\infty)$.

Theorem 2.2. *([11]) Assume that hypotheses **(H1)** and **(H2)** hold. If v is a strict solution of the Eq.(2.2), then the following variation of constant formula holds*

$$v(t) = R(t)v_0 + \int_0^t R(t-s)q(s)ds \;\; \textit{for} \quad t \geq 0. \tag{2.3}$$

Accordingly, we can establish the following definiton.

Definition 2.5. A function $v : [0, +\infty) \to \mathbb{H}$ is called mild solution of the Eq.(2.2), for $v_0 \in \mathbb{H}$, if v satisfies the variation of constants formula (2.3).

The next theorem provides sufficient conditions ensuring the regularity of solutions of the Eq.(2.2).

Theorem 2.3. *Let $q \in C^1([0, +\infty); \mathbb{H})$ and v be defined by (2.3). If $v_0 \in \mathcal{D}(A)$, then v is a strict solution of the Eq.(2.2).*

Now, we give the definition of mild solution for (1.1).

Definition 2.6. An $\mathcal{F}_t$-adapted càdlàg stochastic process $x : J_T \to \mathbb{H}$ is called a mild solution of (1.1) on J_T if $x(0) - q(x_{t_1}, x_{t_2}, \cdots, x_{t_n})(0) = x_0 = \varphi \in \mathcal{B}$, satisfying $\varphi, q \in \mathcal{L}_2^0(\Omega, \mathbb{H})$; is such that the following conditions hold:

(i) $\{x_t : t \in J\}$ is a $\mathcal{B}$-valued stochastic process;
(ii) For arbitrary $t \in J, x(t)$ satisfies the following integral equation:

$$\begin{aligned} x(t) &= R(t)[x_0 + q(x_{t_1}, x_{t_2}, \cdots, x_{t_n})(0) - g(0, x_0, 0)] + g\left(t, x_t, \int_0^t \sigma_1(t, s, x_s)ds\right) \\ &+ \int_0^t R(t-s)Cu(s)ds + \int_0^t R(t-s)f\left(s, x_s, \int_0^s \sigma_2(s, \xi, x_\tau)d\tau\right)ds \\ &+ \int_0^t R(t-s)\int_{-\infty}^s \sigma(s, \tau, x_\tau)dw(\tau)ds + \int_0^t R(t-s)\int_{\mathfrak{U}} \gamma(t, x(t-), v)\tilde{N}(dt, dv) \\ &+ \sum_{0<t_k<t} R(t-s)I_k(x_{t_k}), \quad \text{and} \end{aligned} \tag{2.4}$$

(iii) $\Delta x(t_k) = I_k(x_{t_k}), k = \overline{1,m}$.

Definition 2.7. The system (1.1) is said to be controllable on the interval J_T, if for every initial stochastic process $\varphi \in \mathcal{B}$ defined on J_0 and $y_1 \in \mathbb{H}$; there exists a stochastic control $u \in L^2(J, U)$ which is adapted to the filtration $\{\mathcal{F}_t\}_{t\in J}$ such that the solution $x(\cdot)$ of the system (1.1) satisfies $x(T) = y_1$, where y_1 and T are the preassigned terminal state and time, respectively.

To prove our main results, we shall impose the following assumptions.

(H3) There exists positive constants M_R and M_{σ_1} such that for all $t, s \in J, x, y \in \mathcal{B}$

$$\|R(t)\|^2 \leq M_R;$$
$$\mathbb{E}\left\|\int_0^t [\sigma_1(t,s,x) - \sigma_1(t,s,y)]ds\right\|^2 \leq M_{\sigma_1}\|x-y\|^2_{\mathcal{B}}.$$

(H4) The function $g : J \times \mathcal{B} \times \mathbb{H} \to \mathbb{H}$ is continuous and there exists a positive constant M_g such that for all $t \in J, x_1, x_2 \in \mathcal{B}, y_1, y_2 \in \mathcal{L}_2(\Omega, \mathbb{H})$

$$\mathbb{E}\|g(t,x_1,y_1) - g(t,x_2,y_2)\|^2 \leq M_g(\|x_1 - x_2\|^2_{\mathcal{B}} + \mathbb{E}\|y_1 - y_2\|^2).$$

(H5) For each $(t,s) \in J \times J$, the function $\sigma_2 : J \times J \times \mathcal{B} \to \mathbb{H}$ is continuous and there exists a positive constant M_{σ_2} such that for all $t, s \in J, x, y \in \mathcal{B}$

$$\mathbb{E}\left\|\int_0^t [\sigma_2(t,s,x) - \sigma_2(t,s,y)]ds\right\|^2 \leq M_{\sigma_2}\|x-y\|^2_{\mathcal{B}}.$$

(H6) The function $f : J \times \mathcal{B} \times \mathbb{H} \to \mathbb{H}$ is continuous and there exists a positive constant M_f such that for all $t \in J, x_1, x_2 \in \mathcal{B}, y_1, y_2 \in \mathcal{L}_2(\Omega, \mathbb{H})$

$$\mathbb{E}\|f(t,x_1,y_1) - f(t,x_2,y_2)\|^2 \leq M_f(\|x_1 - x_2\|^2_{\mathcal{B}} + \mathbb{E}\|y_1 - y_2\|^2).$$

(H7) The functions $I_k, k \in \mathcal{C}(\mathcal{B}, \mathbb{H}), k = \overline{1,m}$ and there exist positive constants M_{I_k} and $\overline{M}_{I_k}$ such that for all $x, y \in \mathcal{B}$

$$\mathbb{E}\|I_k(x)\|^2 \leq M_{I_k};$$
$$\mathbb{E}\|I_k(x) - I_k(y)\|^2 \leq \overline{M}_{I_k}\|x-y\|^2_{\mathcal{B}}.$$

(H8) For each $\varphi \in \mathcal{B}, h(t) = \lim_{c\to\infty}\int_{-c}^0 \sigma(t,s,\varphi)dw(s)$ exists and continuous. Further, there exists a positive constant M_h such that

$$\mathbb{E}\|h(t)\|^2 \leq M_h.$$

(H9) The function $\sigma : J \times J \times \mathcal{B} \to \mathcal{L}(\mathbb{K}, \mathbb{H})$ is continuous and there exists positive constants $M_\sigma, \overline{M}_\sigma$ such that for all $s, t \in J$ and $x, y \in \mathcal{B}$

$$\mathbb{E}\|\sigma(t,s,x)\|^2_{\mathcal{L}_2^0} \leq M_\sigma;$$
$$\mathbb{E}\|\sigma(t,s,x) - \sigma(t,s,y)\|^2_{\mathcal{L}_2^0} \leq \overline{M}_\sigma\|x-y\|^2_{\mathcal{B}}.$$

(H10) The function $q : \mathcal{B}^n \to \mathcal{B}$ is continuous and there exist positive constants $M_q, \overline{M_q}$ such that for all $x, y \in \mathcal{B}, t \in J_0$

$$\mathbb{E}\|q(x_{t_1}, x_{t_2}, ..., x_{t_n})(t)\|^2 \leq M_q;$$
$$\mathbb{E}\|q(x_{t_1}, x_{t_2}, ..., x_{t_n})(t) - q(y_{t_1}, y_{t_2}, ..., y_{t_n})(t)\|^2 \leq \overline{M_q}\|x-y\|^2_{\mathcal{B}}.$$

(H11) The linear operator $W : L^2(J, U) \to L^2(\Omega, \mathbb{H})$ defined by

$$Wu = \int_J R(T-s)Cu(s)ds$$

has an induced inverse W^{-1} which takes values in $L^2(J, U)/Ker\,W$ (see Carimichel & Quinn, 1984) and there exist two positive constants M_C and M_W such that

$$\|C\|^2 \leq M_C \quad \text{and} \quad \|W^{-1}\|^2 \leq M_W.$$

(H12) The function $\gamma : J \times \mathbb{H} \times \mathfrak{U} \to \mathbb{H}$ is a Borel measurable function and satisfies the Lipschitz continuity condition, the linear growth condition, and there exists positive constants $M_\gamma, \overline{M}_\gamma$ such that for any $x, y \in \mathcal{L}_2^{\mathbb{F}}(0, T; \mathbb{H}), t \in J$

$$\mathbb{E}\left(\int_0^t \int_{\mathfrak{U}} \|\gamma(t, x(s-), v)\|_{\mathbb{H}}^2 \lambda(dv)ds\right) \vee \mathbb{E}\left(\int_0^t \int_{\mathfrak{U}} \|\gamma(t, x(s-), v)\|_{\mathbb{H}}^4 \lambda(dv)ds\right)^{1/2}$$
$$\leq M_\gamma \mathbb{E}\int_0^t (1 + \|x(s)\|_{\mathbb{H}}^2)ds;$$
$$\mathbb{E}\left(\int_0^t \int_{\mathfrak{U}} \|\gamma(t, x(s-), v) - \gamma(t, y(s-), v)\|_{\mathbb{H}}^2 \lambda(dv)ds\right)$$
$$\vee\, \mathbb{E}\left(\int_0^t \int_{\mathfrak{U}} \|\gamma(t, x(s-), v) - \gamma(t, y(s-), v)\|_{\mathbb{H}}^4 \lambda(dv)ds\right)^{1/2} \leq \overline{M}_\gamma \mathbb{E}\int_0^t \|x(s) - y(s)\|_{\mathbb{H}}^2 ds$$

3. Main Results

In this section, we shall investigate the controllability of nonlocal impulsive neutral stochastic functional integro-differential equations with infinite delay and Poisson jumps in Hilbert spaces.

The main result of this section is the following theorem.

Theorem 3.1. *Assume that the assumptions **(H1)**–**(H12)** hold. If* $\Xi < 1$ *and* $\Theta < 1$*, then the system (1.1) is controllable on* J_T*, where*

$$\Xi := 28(1 + 7T^2 M_W M_C)\left[2l_0^2\left(M_g(1 + 2M_{\sigma_1}) + T^2 M_R M_f(1 + 2M_{\sigma_2})\right) + T^2 M_\gamma \tilde{C}\right] + 7M_g$$
$$\Theta := \Big\{84 l_0^2 T^2 M_C M_W M_R^2 \bar{M}_q + 12 l_0^2 (1 + 7T^2 M_C M_R M_W)\,[M_g(1 + M_{\sigma_1})$$
$$+\; T^2 M_R M_f (1 + M_{\sigma_2}) + T^3 M_R \bar{M}_\sigma Tr(Q) + \frac{T\bar{M}_\gamma \tilde{C}}{2l_0^2} m M_R \sum_{k=1}^{m} \bar{M}_{I_k}\Big]\Big\}.$$

Proof. Using the assumption **(H9)**, for an arbitrary function $x(\cdot)$, we define the control process

$$u_x^T(t) = W^{-1}\Big\{x_1 - R(T)[x_0 + q(x_{t_1}, \cdots, x_{t_n}) - g(0, x_0, 0)] - g(T, x_T, \int_0^T \sigma_1(T, s, x_s)ds)$$
$$- \int_0^T R(T-s) f(s, x_s, \int_0^s \sigma_2(s, \tau, x_\tau)d\tau)ds - \int_0^T R(T-s)[h(s) + \int_0^s \sigma(s, \tau, x_\tau)dw(\tau)]ds$$
$$- \int_0^T R(T-s)\int_{\mathfrak{U}} \gamma(t, x(t-), v)\tilde{N}(dt, dv) - \sum_{0<t_k<T} R(T-t_k) I_k(x_{t_k})\Big\}(t) \tag{3.1}$$

Let's put (1.1) into a fixed point problem. Consider the operator $\Pi : \mathcal{B}_T \to \mathcal{B}_T$ defined by

$$\Pi x(t) = x_0 + q(x_{t_1}, \cdots, x_{t_n})(t), \quad t \in J_0;$$

$$\Pi x(t) = R(t)[x_0 + q(x_{t_1}, \cdots, x_{t_n})(0) - g(0, x_0, 0)] + g\left(t, x_t, \int_0^t \sigma_1(t, s, x_s)ds\right)$$
$$+ \int_0^t R(t-s)Cu_x^T(s)ds + \int_0^t R(t-s)f\left(s, x_s, \int_0^s \sigma_2(s, \tau, x_\tau)d\tau\right)ds$$
$$+ \int_0^t R(t-s)\left[h(s) + \int_0^s \sigma(s, \tau, x_\tau)dw(\tau)\right]ds + \int_0^t R(t-s)\int_{\mathfrak{U}} \gamma(t, x(t-), v)\tilde{N}(dt, dv)$$
$$+ \sum_{0<t_k<t} R(t-t_k)I_k(x_{t_k}), \quad \text{for a.e } t \in J.$$

In what follows, we shall show that using the control $u_x^T(\cdot)$, the operator Π has a fixed point, which is then a mild solution for system (1.1).

Clearly, $\Pi x(T) = y_1$.
For $\varphi \in \mathcal{B}$, we define $\tilde{\varphi}$ by

$$\tilde{\varphi}(t) = \begin{cases} x_0 + q(x_{t_1}, x_{t_2}, \cdots, x_{t_n})(t) & \text{if } t \in J_0, \\ R(t)[x_0 + q(x_{t_1}, x_{t_2}, \cdots, x_{t_n})(0)] & \text{if } t \in J. \end{cases}$$

then $\tilde{\varphi}(t) \in \mathcal{B}_T$. Set $x(t) = z(t) + \tilde{\varphi}(t), t \in J_T$.
It is easy to see that x satisfies (2.4) if and only if z satisfies $z_0 = 0$ and

$$z(t) = -R(t)g(0, x_0, 0) + g(t, z_t + \tilde{\varphi}_t, \int_0^t \sigma_1(t, s, z_s + \tilde{\varphi}_s)ds) + \int_0^t R(t-s)Cu_{z+\tilde{\varphi}}^T(s)ds$$
$$+ \int_0^t R(t-s)f(s, z_s + \tilde{\varphi}_s, \int_0^s \sigma_2(s, \tau, z_\tau + \tilde{\varphi}_\tau)d\tau)ds$$
$$+ \int_0^t R(t-s)[h(s) + \int_0^s \sigma(s, \tau, z_\tau + \tilde{\varphi}_\tau)dw(\tau)]ds + \sum_{0<t_k<t} R(t-t_k)I_k(z_{t_k} + \tilde{\varphi}_{t_k})$$
$$+ \int_0^t R(t-s)\int_{\mathfrak{U}} \gamma(t, z(t-) + \tilde{\varphi}(t-), v)\tilde{N}(dt, dv), \quad t \in J,$$

where $u_{z+\tilde{\varphi}}^T(t)$ is obtained from (3.1) by replacing $x_t = z_t + \tilde{\varphi}_t$

Let $\mathcal{B}_T^0 = \{y \in \mathcal{B}_T : y_0 = 0 \in \mathcal{B}\}$. For any $y \in \mathcal{B}_T^0$, we have

$$\|y\|_T = \|y_0\|_{\mathcal{B}} + \sup_{s\in J}(\mathbb{E}\|y(s)\|^2)^{1/2} = \sup_{s\in J}(\mathbb{E}\|y(s)\|^2)^{1/2},$$

and thus $(\mathcal{B}_T^0, \|\cdot\|_T)$ is a Banach space. Set $B_r = \{y \in \mathcal{B}_T^0 : \|y\|_T^2 \le r\}$ for some $r \ge 0$, then $B_r \subseteq \mathcal{B}_T^0$ is uniformly bounded, and for $u \in B_r$, by Lemma 2.1, we have

$$\begin{aligned} \|z_t + \tilde{\varphi}_t\|_{\mathcal{B}} &\le 2(\|z_t\|_{\mathcal{B}}^2 + \|\tilde{\varphi}_t\|_{\mathcal{B}}^2) \\ &\le 4(l_0^2 \sup_{s\in[0,t]} (\mathbb{E}\|z(s)\|^2 + \|z_0\|_{\mathcal{B}}^2 + l_0^2 \sup_{s\in[0,t]} (\mathbb{E}\|\tilde{\varphi}(s)\|^2 + \|\tilde{\varphi}_0\|_{\mathcal{B}}^2) \\ &\le 4l_0^2(r + 2M_R[\mathbb{E}\|\varphi(0)\|^2 + M_q) + 4\|\tilde{\varphi}\|_{\mathcal{B}}^2 \qquad (3.2) \\ &:= r^*. \end{aligned}$$

Consider the map $\bar{\Pi} : \mathcal{B}_T^0 \to \mathcal{B}_T^0$ defined by $\bar{\Pi}z(t) = 0$, for $t \in J_0$ and

$$\bar{\Pi}z(t) = -R(t)g(0, x_0, 0) + g(t, z_t + \tilde{\varphi}_t, \int_0^t \sigma_1(t, s, z_s + \tilde{\varphi}_s)ds) + \int_0^t R(t-s)Cu_{z+\tilde{\varphi}}^T(s)ds$$
$$+ \int_0^t R(t-s)f(s, z_s + \tilde{\varphi}_s, \int_0^s \sigma_2(s, \tau, z_\tau + \tilde{\varphi}_\tau)d\tau)ds$$
$$+ \int_0^t R(t-s)[h(s) + \int_0^s \sigma(s, \tau, z_\tau + \tilde{\varphi}_\tau)dw(\tau)]ds + \sum_{0<t_k<t} R(t-t_k)I_k(z_{t_k} + \tilde{\varphi}_{t_k})$$

$$+\int_0^t R(t-s)\int_{\mathfrak{U}}\gamma(t,z(t-)+\tilde{\varphi}(t-),v)\tilde{N}(dt,dv), \quad t\in J.$$

Obviously, the operator Π has a fixed point which is equivalent to prove that $\bar{\Pi}$ has a fixed point. Note that, by our assumptions, we infer that all the functions involved in the operator are continuous, therefore $\bar{\Pi}$ is continuous.

Let $z,\bar{z}\in\mathcal{B}^0_T$. From (3.1), by our assumptions, Hölder's inequality, the Doob martingale inequality, and the Burkholder-Davis-Gundy inequality for pure jump stochastic integral in Hilbert space (see Luo & Liu, 2008, [17]), Lemma 2.1, and in view of (3.2), for $t\in J$, we obtain the following estimates.

$$\begin{aligned}
&\mathbb{E}\|u^T_{z+\tilde{\varphi}}\|^2\\
&\leq 7M_W\left\{\mathbb{E}\|x_1\|^2+3M_R[\|x_0\|^2+M_q+C_2]+2M_g([2+2M_{\sigma_1}]r^*+2C_1)+C_2\right.\\
&+2TM_R(M_f([1+2M_{\sigma_2}]r^*+2C_3)+C_4)+2M_R(M_h+tTr(Q)M_\sigma)+T^2M_\gamma\tilde{C}(1+\frac{r^*}{l_0^2})\\
&\left.+mM_R\sum_{k=1}^m M_{I_k}\right\}:=\mathfrak{L},
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}\|u^T_{z+\tilde{\varphi}}(t)-u^T_{\bar{z}+\tilde{\varphi}}(t)\|^2\\
&\leq 12l_0^2M_W\left\{2M_R\bar{M}_q+M_g(1+M_{\sigma_1})+T^2M_RM_f(1+M_{\sigma_2})+T^3M_R\bar{M}_\sigma Tr(Q)\right.\\
&\qquad\left.+\frac{T\bar{M}_\gamma\tilde{C}}{2l_0^2}+mM_R\sum_{k=1}^m\bar{M}_{I_k}\right\}\sup_{s\in J}\mathbb{E}\|z(t)-\bar{z}(t)\|^2
\end{aligned}$$

where $\tilde{C}>0$ is a positive constant and

$$C_1:=T\sup_{(r,s)\in J\times J}\sigma_1^2(r,s,0),\quad C_2:=\sup_{(t,s)\in J\times\mathcal{B}}\|g(t,s,0)\|^2,$$
$$C_3:=T\sup_{(r,s)\in J\times J}\sigma_2^2(r,s,0),\quad C_4:=\sup_{t\in J}\|f(t,0,0)\|^2$$

Lemma 3.2. *Under the assumptions of Theorem 3.1, there exists $r>0$ such that $\bar{\Pi}(B_r)\subseteq B_r$.*

Proof. If this property is false, then for each $r>0$, there exists a function $z^r(\cdot)\in B_r$, but $\bar{\Pi}(z^r)\notin B_r$, i.e. $\|\bar{\Pi}(z^r)(t)\|^2>r$ for some $t\in J$. However, by our assumptions, Hölder's inequality and the Burkholder–Davis–Gundy inequality, we have

$$\begin{aligned}
&r<\mathbb{E}\|\bar{\Pi}(z^r)(t)\|^2\\
&\leq 7\left[2M_g([1+2M_{\sigma_1}]r^*+2C_1)+C_2+M_RC_2+2T^2M_R[(M_f[1+2M_{\sigma_2}]r^*+2C_3)+C_4]\right.\\
&\left.+T^2M_RM_C\mathfrak{L}+2T^2M_R(M_h+TTr(Q)M_\sigma)+TM_\gamma\tilde{C}(1+\frac{r^*}{l_0^2})+mM_R\sum_{k=1}^m M_{I_k}\right]\\
&\leq M^{**}+7(1+7T^2M_WM_C)\left[2\left(M_g(1+2M_{\sigma_1})+T^2M_RM_f(1+2M_{\sigma_2})\right)+\frac{T^2M_\gamma\tilde{C}}{l_0^2}\right]r^*\\
&+7M_gr^* \qquad (3.3)
\end{aligned}$$

where

$$M^{**}:=7\left(2C_1+C_2(1+M_R)+2T^2M_R[2C_3+C_4]+mM_R\sum_{k=1}^m M_{I_k}\right)$$

$$+\,49T^2M_WM_RM_C\left(\|x_1\|^2+3M_r[\|x_0\|^2+M_q+C_2]+2M_gC_1+C_2+2TM_R[2C_3+C_4]\right.$$
$$\left.+2M_R(M_h+TM_\sigma Tr(Q))+T^2M_\gamma\tilde{C}+mM_R\sum_{k=1}^{m}M_{I_k}\right) \tag{3.4}$$

Dividing both sides of (3.3) by r and noting that

$$\begin{aligned} r^* &= 4l_0^2(r+2M_R\mathbb{E}\|\varphi(0)\|^2+M_q)+4\|\tilde{\varphi}\|_{\mathcal{B}}^2 \\ &\longrightarrow \infty, \quad r\to\infty \end{aligned}$$

and taking the limit as $r\to\infty$, we obtain $1\le\Xi$ which contradicts our assumption. Thus, for some positive number $r, \bar{\Pi}(B_r)\subseteq B_r$. This completes the proof of Lemma 3.2. □

Lemma 3.3. *Under the assumptions of Theorem 3.1, $\bar{\Pi}:\mathcal{B}_T^0\to\mathcal{B}_T^0$ is a contraction mapping.*

Proof. Let $z,\bar{z}\in\mathcal{B}_T^0$. Then, by our assumptions, Hölder's inequality, Burkholder-Davis-Gundy's inequality, Lemma 2.1, and since $\|z_0\|_{\mathcal{B}}^2=0$ and $\|\bar{z}_0\|_{\mathcal{B}}^2=0$, for each $t\in J$, we see that

$$\begin{aligned} &\mathbb{E}\|(\bar{\Pi}z)(t)-(\bar{\Pi}\bar{z})(t)\|^2 \\ &\le 14\left\{M_g(1+M_{\sigma_1})+T^2M_RM_f(1+M_{\sigma_2})+T^3M_R\bar{M}_\sigma Tr(Q)+\frac{T\bar{M}_\gamma\tilde{C}}{2l_0^2}\right. \\ &\left.+mM_R\sum_{k=1}^{m}\bar{M}_{I_k}\right\}\sup_{s\in J}\mathbb{E}\|z(t)-\bar{z}(t)\|^2+7T^2M_RM_C\mathbb{E}\|u^T_{z+\tilde{\varphi}}(t)-u^T_{\bar{z}+\tilde{\varphi}}(t)\|^2 \\ &\le\Big\{84l_0^2T^2M_CM_WM_R^2\bar{M}_q+14l_0^2(1+7T^2M_CM_RM_W) \\ &\times\left[M_g(1+M_{\sigma_1})+T^2M_RM_f(1+M_{\sigma_2})T^3M_R\bar{M}_\sigma Tr(Q)+\frac{T\bar{M}_\gamma\tilde{C}}{2l_0^2}\right. \\ &\qquad\left.\left. mM_R\sum_{k=1}^{m}\bar{M}_{I_k}\right]\right\}\sup_{s\in J}\mathbb{E}\|z(t)-\bar{z}(t)\|^2 \end{aligned}$$

Taking the supremum over t, we obtain

$$\|(\bar{\Pi}z)(t)-(\bar{\Pi}\bar{z})(t)\|_T^2\le\Theta\|z-\bar{z}\|_T^2.$$

By our assumption, we conclude that $\bar{\Pi}$ is a contraction on $\mathcal{B}_T^0$. Thus, we have completed the proof of Lemma 3.3. □

On the other hand, by Banach fixed point theorem, there exists a unique fixed point $x(\cdot)\in\mathcal{B}_T^0$ such that $(\Pi x)(t)=x(t)$. This fixed point is then the mild solution of the system (1.1). Clearly, $x(T)=(\Pi x)(T)=y_1$. Thus, the system (1.1) is controllable on J_T. The proof for Theorem 3.1 is thus complete. □

Now, let us consider a special case for the system (1.1).

If $\gamma(t,x(t-),v)\equiv 0$, the system (1.1) becomes the following nonlocal impulsive neutral stochastic functional integrodifferential equations with infinite delay without

Poisson jumps

$$\begin{cases} d\left[x(t)-g\left(t,x_t,\int_0^t \sigma_1(t,s,x_s)ds\right)\right] = \left[A\left[x(t)-g\left(t,x_t,\int_0^t \sigma_1(t,s,x_s)ds\right)\right]\right. \\ \left. +f\left(t,x_t,\int_0^t \sigma_2(t,s,x_s)ds\right)\right]dt + \left[\int_0^t B(t-s)[x(s)-g\left(s,x_s,\int_0^s \sigma_1(s,\tau,x_\tau)d\tau\right)]ds\right]dt \\ +Cu(t)dt + \int_{-\infty}^t \sigma(t,s,x_s)dw(s) \ t_k \neq t \in J := [0,T], \\ \Delta x(t_k) = I_k(x_{t_k}), \quad k = \{1,\cdots,m\} =: \overline{1,m}, \\ x(0) - q(x_{t_1},x_{t_2},\cdots,x_{t_n}) = x_0 = \varphi \in \mathcal{B}, \text{ for a.e } \ s \in J_0 := (-\infty,0], \end{cases} \tag{3.5}$$

Corollary 3.4. *Assume that all assumptions of Theorem 3.1 hold except that* ***(H12)*** *and* Ξ, Θ *replaced by* $\hat{\Xi}, \hat{\Theta}$ *such that*

$$\hat{\Xi} := 24l_0^2(1+6T^2M_BM_RM_W)[M_g(1+2M_{\sigma_1})+T^2M_RM_f(1+M_{\sigma_1})]+6M_g$$

and

$$\hat{\Theta} := \Big\{72l_0^2T^2M_CM_WM_R\bar{M}_q + 12l_0^2(1+6T^2M_CM_RM_W) \times \left[M_g(1+M_{\sigma_1})+T^2M_RM_f(1+M_{\sigma_2})+T^3M_R\bar{M}_\sigma Tr(Q)+mM_R\sum_{k=1}^m \bar{M}_{I_k}\right]\Big\}$$

If $\hat{\Xi} < 1$ *and* $\hat{\Theta} < 1$, *then the system (3.5) is controllable on* J_T.

4. Application

In this section, the established previous results are applied to study the controllability of the stochastic nonlinear wave equation with infinite delay and Poisson jumps. Specifically, we consider the following controllability of nonlocal impulsive neutral stochastic functional integrodifferential equations with infinite delay and Poisson jumps of the form:

$$\begin{cases} \dfrac{\partial}{\partial t}\left[y(t,\xi) - G_1\left(t, y(t-\tau,\xi), \displaystyle\int_0^t g(t,s,y(s-\tau,\xi))ds\right)\right] \\ = \dfrac{\partial^2}{\partial \xi^2}\left[y(t,\xi) - G_1\left(t, y(t-\tau,\xi), \displaystyle\int_0^t g(t,s,y(s-\tau,\xi))ds\right)\right] dt \\ + \left[\displaystyle\int_0^t \Gamma(t-s)\left[y(s,\xi) - G_1\left(t, y(t-\tau,\xi), \int_0^t g(t,s,y(s-\tau,\xi))ds\right)\right] + b(\xi)u(t)\right] dt \\ \qquad + \left[g_1\left(t, y(t-\tau,\xi), \displaystyle\int_0^t g_2(t,s,y(s-\tau,\xi))ds\right)\right] dt \\ \qquad + \displaystyle\int_{-\infty}^t \delta(s-t)y(t,\xi)d\beta(s) + \int_{\mathfrak{U}} y(t-\xi)v\tilde{N}(dt,dv) \text{ for } t_k \neq t \in J, \xi \in [0,\pi], \\ \Delta y(t_k)(\xi) = \displaystyle\int_{-\infty}^{t_k} \eta_k(t_k - s)y(s,\xi)ds, \quad k = \{1,\cdots,m\} =: \overline{1,m}, \ \ \xi \in [0,\pi], \\ y(0,\xi) = y(t,\pi) = 0, \ \ t \in J, \\ y(t,\xi) - \displaystyle\sum_{i=1}^n \int_0^\pi p_i(\xi,\zeta)y(t,\zeta)d\zeta = \varphi(t,\xi), \quad t \in J_0, \qquad \xi \in [0,\pi], \end{cases} \tag{4.1}$$

where, $\gamma : \mathbb{R}^+ \to \mathbb{R}$ is a continuous function and $\beta(t)$ is a standard one-dimensional Wiener process in $\mathbb{H}$, defined on a stochastic basis $(\Omega, \mathcal{F}, P)$; $\mathfrak{U} = \{v \in \mathbb{R} : 0 < \|v\|_{\mathbb{R}} \leq a, a > 0\}$; $0 < t_1 < t_2 < \cdots < t_n < T, n \in \mathbb{N}$; $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} < T$ are prefixed numbers, and $\varphi \in \mathcal{B}$. Let $p = p(t), t \in D_p$ be a $\mathbb{K}$-valued σ-finite stationary Poisson point process (independent of $\beta(t)$) on a complete probability space with the usual condition $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$. Let $\tilde{N}(ds,dv) := N(ds,dv) - \lambda(dv)ds$, with the characteristic measure $\lambda(dv)$ on $\mathfrak{U} \in \mathfrak{B}(\mathbb{K} - \{0\})$. Assume that

$$\int_{\mathfrak{U}} v^2 \lambda(dv) < \infty.$$

To rewrite (4.1) into the abstract from of (1.1), we consider the space $\mathbb{H} = L^2([0,\pi])$ with the norm $\|\cdot\|$. Let $e_n(\xi) := \sqrt{\frac{2}{\pi}} \sin n\xi, n = 1,2,3,...$ denote the completed orthogonal basics in $\mathbb{H}$ and $\beta(t) = \sum_{n=1}^\infty \sqrt{\lambda_n}\beta_n(t)e_n, t \geq 0, \lambda > 0$, where $\{\beta_n(t)\}_{n\geq 0}$ are one-dimensional standard Brownian motions mutuallyi ndependent on a usual complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$.

Defined $A : \mathbb{H} \to \mathbb{H}$ by $A = \frac{\partial^2}{\partial \xi^2}$, with domain $D(A) = \mathbb{H}^2([0,\pi]) \cap \mathbb{H}_0^1([0,\pi])$, where
$\mathbb{H}_0^1([0,\pi]) = \{w \in L^2([0,\pi]), w(0) = w(\pi) = 0\}$ and $\mathbb{H}^2([0,\pi]) = \{w \in L^2([0,\pi]) : \frac{\partial w}{\partial z}, \frac{\partial^2 w}{\partial z^2} \in L^2([0,\pi])\}$.
Then,

$$Ax = \sum_{n=1}^\infty n^2 \langle x, e_n \rangle e_n, \quad x \in D(A), \tag{4.2}$$

It is well known that A is the infinitesimal generator of a strongly continuous semigroup on $\mathbb{H}$; thus, **(H1)** is true.
Let $\Gamma : \mathcal{D}(A) \subset \mathbb{H} \to \mathbb{H}$ be the operator defined by $\Gamma(t)(z) = B(t)Az$ for $t \geq 0$ and $z \in \mathcal{D}(A)$.

Now, we give a special $\mathcal{B}$-space. Let $l(s) = e^{2s}, s \leq 0$, then $l_0 = \int_{J_0} l(s)ds = \frac{1}{2}$ and define

$$\|\psi\|_{\mathcal{B}} = \int_{J_0} e^{2s} \sup_{\theta \in [s,0]} (\mathbb{E}\|\psi(\theta)\|^2)^{\frac{1}{2}} ds, \quad \forall \psi \in \mathcal{B}.$$

It follows from Hino et al. [13] that $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ is a Banach space. Additionally, we assume that the following conditions hold:

(1) Let $C \in \mathcal{L}(\mathbb{R}, \mathbb{H})$ be defined as

$$Cu(\xi) = b(\xi)u, \quad 0 \leq \xi \leq \pi, \quad u \in \mathbb{R}, \quad b(\xi) \in L^2([0,\pi])$$

(2) The linear operator $W : L^2(J,U) \to \mathbb{H}$ defined by

$$Wu = \int_J R(T-s)b(\xi)u(s)ds$$

is a bounded linear operator but not necessarily one-to-one. Let $KerW = \{u \in L^2(J,U) : Wu = 0\}$ be null space of W and $[KerW]^{\perp}$ be its orthogonal complement in $L^2(J,U)$. Let $W^* : [KerW]^{\perp} \to \text{Range}(W)$ be the restriction of W to $[KerW]^{\perp}, W^*$ is necessarily one-to-one operator. The inverse mapping theorem says that $(W^*)^{-1}$ is bounded since $[KerW]^{\perp}$ and Range(W)are Banach spaces. Since the inverse operator W^{-1} is bounded and takes values in $L^2(J,U)/KerW$, the assumption **(H11)** is satisfied.

(3) The functions $p_i : [0,\pi] \times [0,\pi] \to \mathbb{H}$ are $\mathcal{C}^2$-functions, for each $i = \overline{1,n}$.

(4) The function $\nu_1(\theta) \geq 0$ is continuous in $]-\infty, 0]$ satisfying

$$\int_{-\infty}^{0} \nu_1^2(\theta)d\theta < \infty, \qquad \left(\int_{-\infty}^{0} \frac{(\nu_1(s))^2}{l(s)}ds\right)^{\frac{1}{2}} < \infty.$$

(5) $b_2, b_3 : \mathbb{R} \to \mathbb{R}$ are continuous, and

$$\left(\int_{-\infty}^{0} \frac{(b_3(s))^2}{l(s)}ds\right)^{\frac{1}{2}} < \infty.$$

(6) The functions $\tilde{b}_2 : \mathbb{R} \to \mathbb{R}$ is continuous and $\tilde{b}_i : \mathbb{R} \to \mathbb{R},\ i = 1,3$ are continuous and there exist continuous functions $r_j : \mathbb{R} \to \mathbb{R}, j = 1,2,3,4$ such that

$$\begin{aligned} |\tilde{b}_1(t,s,x,y)| &\leq r_1(t)r_2(s)|y|; \quad (t,s,x,y) \in \mathbb{R}^4, \\ |\tilde{b}_3(t,s,x,y)| &\leq r_3(t)r_4(s)|y|; \quad (t,s,x,y) \in \mathbb{R}^4, \end{aligned}$$

with $\tilde{L}_1^b = \left(\int_{-\infty}^{0} \frac{(r_2(s))^2}{l(s)}ds\right)^{\frac{1}{2}} < \infty, \quad \tilde{L}_2^b = \left(\int_{-\infty}^{0} \frac{(r_4(s))^2}{l(s)}ds\right)^{\frac{1}{2}} < \infty.$

(7) The functions $d_i \in C(\mathbb{R}, \mathbb{R})$, and $L_{I_i} = \left(\int_{-\infty}^{0} d_i^2(s)ds\right)^{\frac{1}{2}} < \infty$, where $i = 1, \cdots, m$ are finite.

Let $\phi(\theta)(\xi)=\phi(\theta,\xi),(\theta,\xi)\in J\times\mathcal{B}$, with $\phi(\theta)x=\phi(\theta,x),(\theta,x)\in J_0\times[0,\pi]$. Let $y(t)(\xi)=y(t,\xi).g,\ f\ :J\times\mathcal{B}\times\mathbb{H}\to\mathbb{H},\sigma:J\times J\times\mathcal{B}\to\mathcal{L}_2^0,\gamma:J\times\mathbb{H}\times\mathfrak{U}\to\mathbb{H}$, and $I_k:\mathcal{B}\to\mathbb{H},k=\overline{1,m}$ be the operators defined by

$$\begin{aligned}
\sigma_1(t,s,\phi)(\tau) &= g(t,s,\phi(\theta,\tau)),\\
g(t,\phi,\int_0^t\sigma_1(t,s,\phi)ds)(\tau) &= G_1(t,\phi(\theta,\tau),\int_0^t g(t,s,\phi(\theta,\xi))ds),\\
&= \int_{-\infty}^0\nu_1\phi(\theta)(\xi)d\theta+\int_0^t\int_{-\infty}^0 b_2(t)b_3(l)\phi(l,\xi)dlds,\\
\sigma_2(t,s,\phi)(\tau) &= g_1(t,s,\phi(\theta,\xi)),\\
f(t,\phi,\int_0^t\sigma_2(t,s,\phi)ds)(\tau) &= g_1(t,\phi(\theta,\xi),\int_0^t g_2(t,s,\phi(\theta,\xi))ds),\\
&= \int_{-\infty}^0\tilde{b}_1(t,s,\xi,\phi(s,\xi))ds\\
&\quad+\int_0^t\int_{-\infty}^0\tilde{b}_2(t)\tilde{b}_3(s,l,\xi,\phi(l,\xi))dlds,\\
\sigma(t,s,\phi)(\tau) &= \delta(s-t)\phi(s)(\tau),\\
\gamma(t,\phi(\tau),v) &= \phi(\tau)v,\\
I_i(t,\phi)(\xi) &= \int_{-\infty}^0 d_i(t-s)\phi(\theta)\xi ds,
\end{aligned}$$

Moreover, if Γ is bounded and C^1 function such that Γ' is bounded and uniformly continuous, then **(H2)** is satisfied and hence, by Theorem 2.1, Eq. (1.1) has a resolvent operator $(R(t))_{t\geq0}$ on $\mathbb{H}$.
Thus it is easy to show that conditions (4) and (5) implies that g satisfies conditions in **(H4)**, in fact for any $\phi_1,\phi_2\in\mathcal{B},y_1,y_2\in\mathcal{L}_2(\Omega,L^2([0,\pi]))$, we have

$$\mathbb{E}\|g(t,\phi_1,y_1)-g(t,\phi_2,y_2)\|^2\leq M_g(\|x_1-x_2\|_{\mathcal{B}}^2+\mathbb{E}\|y_1-y_2\|^2),$$

where

$$M_g=[\gamma_g^1+T\|b_2\|_\infty\gamma_G^2]^2.$$

Similary we can verify that other assumptions are satisfied and therefore, by Theorem 3.1, we can conclude that the system (4.1) is controllable on J_T.

5. Conclusion

This paper has studied the controllability of a new class of impulsive nonlocal delayed stochastic functional integrodifferential equations of neutral type, driven by a Wiener process and Poisson jumps. Clearly, using the stocastic analysis theory, the resolvent operator theory in the sense of Grimmer, combinated with the Banach fixed point theory, we established the conditions for the controllability of the aforementioned system. Finally, we give an application to illustrate the obtained results.

There are two direct issues which require further study. We will study the conditions for the approximate controllability of the system (1.1). Also, we will investigate the optimal controls problems for the nonlocal integro-differential system.

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