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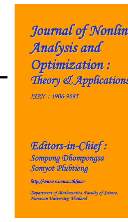

ON SOFT FIXED POINT OF PICARD-MANN HYBRID ITERATIVE SEQUENCES IN SOFT NORMED LINEAR SPACES

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ABSTRACT. In the present paper, we contribute to the development of soft set theory by introducing soft contractive-like operators and soft Picard-Mann hybrid iterative sequences. We then show that the soft Picard-Mann hybrid iterative sequences converges strongly to the unique soft fixed point for the class of soft contractive-like operators. Our results are generalization and improvement of several results on iterative schemes in literature.

KEYWORDS : soft unique fixed point; soft contractive-like operators; soft Picard-Mann hybrid iterative sequences; soft normed linear spaces.
AMS Subject Classification: 47H10, 54H25

1. INTRODUCTION

Mathematical tools have been used in the study of behaviour of different parts of systems and their subsystems. This behaviour are either usually certain or uncertain in nature. In 1999, Molodtsov [16] introduced a new concept called soft set as a mathematical tool for dealing with uncertainties arising in problems in different areas of mathematical sciences. Chief among them are problems in computer science, economics, engineering, medical sciences, and physics. He argued that soft set provides better tool for handling uncertainty than fuzzy set because of its non-restrictive parametrization and is easily applicable to real life problems.

The concept of soft topology on soft set was initiated by Cagman et al. in [6] and some important properties of soft topological spaces were considered. In 2012, Das and Samanta [7] introduced the concept of real soft set and soft real number and explained their properties. In 2013, Das and Samanta [8] also introduced the concept of soft metric using the notion in [7], they hence proved that each soft metric space is a toplological space. Wardowski [22], introduced a new notion of soft element of a soft set and establish its natural relation with soft operations and

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soft objects in soft topological spaces. They defined in a different way than in the literature, a soft mapping transforming a soft set into a soft set and provided basic properties of such mappings using the notion of soft element. They obtained the natural first fixed point results in the soft set theory using the new approach to soft mappings. Abbas et al. [1] in 2015, initiated their notion of soft contraction mapping based on the theory of soft elements of soft metric spaces and proved interesting results on fixed point of such mappings including soft Banach contraction principle.

Fixed point iterative sequences are designed to be applied in solving equations arising in physical formulation but there is no systematic study of numerical aspects of these iterative sequences. The reader can see [3, 4, 21] and other literature for contributions to research on numerical iterative schemes for approximating fixed points. Here, we shall employ the concepts of [2] and [22] and prove soft fixed point results for soft Picard-Mann hybrid iterative sequences using a soft norm version of contractive-like operators. Numerical examples will also be presented to back up our results.

We will now consider some of these schemes as they are relevant to this work. Let (X, d) be a metric space and $T : X \to X$ be a self map of X. Assume that $F_T = \{p \in X : T_p = p\}$ is the set of fixed points of T. For $x_0 \in X$, the sequence $\{x_n\}_{n=1}^{\infty}$ defined by

$$x_{n+1} = Tx_n, \quad n \geq 0, \tag{1.1}$$

is called the Picard iterative scheme [21].

Let $(E, ||.||)$ be a real normed linear space and $T : E \longrightarrow E$ a self map of E. For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, n \geq 0, \tag{1.2}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$ is called the Mann iterative scheme [15].

If $\alpha_n = 1$ in (1.2), we have the Picard iterative scheme (1.1).

Rhoades [18, 20] perhaps for the first time used computer programs to compare the rate of convergence Mann and Ishikawa iterative procedures. He illustrated the difference in the rate of convergence for increasing and decreasing functions through examples.

These various results are worth emulating. In 2013, Khan [11], gave a different perspective to iteration procedure, he introduced the following Picard-Mann hybrid iterative scheme for a single nonexpansive mapping T. For any initial point $x_0 \in E$ the sequence $\{x_n\}_{n=0}^{\infty}$ is defined by

$$\begin{aligned} x_{n+1} &= Ty_n \\ y_n &= (1 - \alpha_n)x_n + \alpha_n Tx_n, n \geq 0, \end{aligned} \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1].

He showed that the hybrid scheme (Picard-Mann scheme (1.3)) converges faster than all of Picard (1.1), Mann (1.2) and Ishikawa [13] iterative schemes in the sense of Berinde [5] for contractions. He also proved strong convergence and weak convergence theorems with the help of his iterative process (1.3)for the class of nonexpansive mappings in general Banach spaces and applied it to obtain results in uniformly convex Banach spaces. Motivated by the work of Khan [11], we prove strong convergence of Picard-Mann iterative scheme for a general class of operators in a real normed space.

Osilike [17] proved several stability results which are generalizations and extensions of most of the results of Rhoades [19] using the following contractive definition: for each $x, y \in X$, there exist $a \in [0, 1)$ and $L \geq 0$ such that

$$d(Tx, Ty) \leq ad(x, y) + Ld(x, Tx). \tag{1.4}$$

In 2003, Imoru and Olatinwo [12] proved some stability results using the following general contractive definition : for each $x, y \in X$, there exist $\delta \in [0, 1)$ and a monotone increasing function $\varphi : R^+ \to R^+$ with $\varphi(0) = 0$ such that

$$d(Tx, Ty) \leq \delta d(x, y) + \varphi(d(x, Tx)). \tag{1.5}$$

Definition 1.1. [16] Let U be an universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\epsilon \in A$, $F(\epsilon)$ may be consider as the set of ϵ-approximate element of the soft set (F, A).

Definition 1.2. [10] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

(i) $A \subseteq B$
(ii) for all $\epsilon \in A$, $F(\epsilon) \subseteq G(\epsilon)$. We write $(F, A)\tilde{\subset}(G, B)$. (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

Definition 1.3. [9] Two soft sets (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 1.4. [9] The complement of a soft set (F, A) is denoted by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$, for all $\alpha \in A$.

Definition 1.5. [14] A soft set (F, E) over U is said to be an absolute soft set denoted by $\check{U}$ if $\forall \epsilon \in E$, $F(\epsilon) = U$.

Definition 1.6. [14] A soft set (F, E) over U is said to be a null soft set denoted by ϕ if $\forall \epsilon \in E$,$F(\epsilon) = \emptyset$.

Definition 1.7. [7] Let X be a non-empty set and E be a non-empty parameter set. Then a function $\epsilon : E \to X$ is said to be a soft element of X. A soft element ϵ of X is said to belongs to a soft set A of X, which is denoted by $\epsilon\tilde{\in}A$, if $\epsilon(e) \leq A(e)$, $\forall e \in E$. Thus for a soft set A of X with respect to the index set E, we have $A(e) = \epsilon(e), \epsilon\tilde{\in}A, e \in E$.

It is to be noted that every singleton soft set (a soft set (F, E) for which $F(e)$ is a singleton set, $\forall e \in E$) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall e \in E$.

Definition 1.8. [7] Let R be the set of real numbers and $B(R)$ the collection of all non-empty bounded subsets of R and A taken as a set of parameters. Then a mapping $F : A \to B(R)$ is called a soft real set. It is denoted by (F, A). If specifically (F, A) is a singleton soft set, then after identifying (F, A) with the corresponding soft element, it will be called a soft real number. We use notations $\tilde{r}$, $\tilde{s}$, $\tilde{t}$ to denote soft real numbers whereas $\bar{r}$, $\bar{s}$, $\bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r, \forall \lambda \in A$. For instance, $\bar{0}$ is the soft real number where $\bar{0}(\lambda) = 0, \forall \lambda \in A$.

Definition 1.9. [8] Let U be a universe, A be a non-empty subset of parameters and $\check{U}$ an absolute soft set, i.e $F(\epsilon) = U$ for all $\epsilon \in A$, where $(F, A) = \check{U}$. Let $SP(\tilde{U})$ be any nonempty set of soft elements of a soft set (F, A) and $R(A)^*$ be a set of all soft real sets. A mapping $d : SP(\tilde{U}) \times SP(\tilde{U}) \to R(A)^*$ is said to be a soft metric on the soft set $\tilde{U}$ if d satisfies the following axioms:
$(M1)$. $d(\tilde{x}, \tilde{y}) \tilde{\geq} \bar{0}, \forall \tilde{x}, \tilde{y} \in \tilde{U}$.
$(M2)$. $d(\tilde{x}, \tilde{y}) = \bar{0} \iff \tilde{x} = \tilde{y}$.
$(M3)$. $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x}), \forall \tilde{x}, \tilde{y} \in \tilde{U}$.
$(M4)$. $d(\tilde{x}, \tilde{y}) \tilde{\leq} d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y}), \forall \tilde{x}, \tilde{y}, \tilde{z} \in \tilde{U}$.

The soft set $\tilde{U}$ endowed with the soft metric d is called a soft metric space and is denoted by $(\tilde{U}, d, A)$ or $(\tilde{U}, d)$. $(M1), (M2), (M3)$ and $(M4)$ are said to be soft metric axioms.

Definition 1.10. [8] Let $\{\tilde{x}_n\}$ be a sequence of soft elements in a soft metric space $(\tilde{U}, d)$. The sequence $\{\tilde{x}_n\}$ is said to be convergent in $(\tilde{U}, d)$ if there is a soft element $\tilde{x} \in \tilde{U}$ such that $d(\tilde{x}_n, \tilde{x}) \to \bar{0}$ as $n \to \infty$. This means for every $\tilde{\epsilon} \tilde{>} \bar{0}$ chosen arbitrarily, there exists a natural number $N = N(\tilde{\epsilon})$, such that $\bar{0} \tilde{<} d(\tilde{x}_n, \tilde{x}) \tilde{<} \tilde{\epsilon}$ whenever $n > N$. We denote this by

$$\lim_{n \to \infty} \tilde{x}_n = \tilde{x}.$$

Proposition 1.11. [8] *The limit of a sequence $\{\tilde{x}_n\}$ in a soft metric space $(\tilde{U}, d)$, if it exists is unique.*

Definition 1.12. [8] A sequence $\{\tilde{x}_n\}$ of soft point in a soft metric space $(\tilde{U}, d)$ is said to be a Cauchy sequence in $(\tilde{U}, d)$ if for each $\tilde{\epsilon} \tilde{>} \bar{0}$, there exists an $m \in N$ such that $d(\tilde{x}_i, \tilde{x}_j) \tilde{<} \tilde{\epsilon}$ for all $i, j \geq m$. That is $d(\tilde{x}_i, \tilde{x}_j) \to \bar{0}$ as $i, j \to \infty$.

Proposition 1.13. [8] *Every convergent sequence $\{\tilde{x}_n\}$ in a soft metric space $(\tilde{U}, d)$ is a Cauchy sequence.*

Definition 1.14. [8] A soft metric space $(\tilde{U}, d)$ is called complete if every Cauchy sequence in it converges to some soft point of $\tilde{U}$.

Definition 1.15. [9] Let V be a vector or linear space over a field K and A a set of parameters. A soft set (F, A) where $F : A \to P(V)$ is called a soft vector or linear space over V. It is denoted by $\tilde{V}$.

Definition 1.16. [9] Let V be a vector or linear space over a field K and A a set of parameters. Let G be a soft set over V. Now G is said to be a soft vector space or a soft linear space of V over K if $G(\lambda)$ is vector or linear subspace of V for every $\lambda \in A$.

Definition 1.17. [9] Let N be the absolute soft Linear Space, i.e. $\check{N}(\lambda) = N$ for every $\lambda \in A$ and $SE(\check{N})$ be any nonempty set of soft elements of absolute soft Linear Space and $R(A)^*$ be a set of all soft real sets. Then a mapping $\|.\| : SE(\check{N}) \to R(A)^*$ is said to be a soft norm on the soft vector space $\check{N}$ if $\|.\|$ satisfies the following conditions: For all $\tilde{x}, \tilde{y} \in \check{N}$,

N1. $\|\tilde{x}\| \geq 0$,
N2. $\|\tilde{x}\| = 0 \iff x = 0$,
N3. $\|\tilde{\alpha}\tilde{x}\| = |\tilde{\alpha}|\|\tilde{x}\|$ for every soft scalar $\tilde{\alpha}$,

N4. $\|\tilde{x}+\tilde{y}\|\tilde{\leq}\|\tilde{x}\|+\|\tilde{y}\|$.

The soft vector space $\check{N}$ with the soft norm $\|.\|$ on it is called a soft normed linear space and denoted by $(\check{N},\|.\|,A)$ or $(\check{N},\|.\|)$. A complete soft normed linear space is a soft Banach space.

Theorem 1.18. [1] *Let $(\check{N},\|.\|,A)$ be a soft Banach space with a finite set A. Suppose the soft mapping $T:\check{N}\to\check{N}$ satisfies:*

$$\|T(\tilde{x})-T(\tilde{y})\|\leq\tilde{a}\|\tilde{x}-\tilde{y}\|,$$

for all $\tilde{x},\tilde{y}\in SP(\check{N})$ where $\bar{0}\leq\tilde{a}<\bar{1}$. Then T has a unique fixed point.

Theorem 1.19. [2] *Let $(\tilde{U},d,A)$ be complete soft metric space with a finite set A. Suppose the soft mapping $f:\tilde{U}\to\tilde{U}$ satisfies*

$$d(f(\tilde{x}),f(\tilde{y}))\leq cd(\tilde{x},\tilde{y}),$$

for all $\tilde{x},\tilde{y}\in SP(\tilde{U})$ where $\bar{0}\tilde{\leq}\tilde{c}\tilde{<}\bar{1}$. then f has a unique fixed point, that is, there exists a unique soft point $\tilde{x}$ such that $f(\tilde{x})=\tilde{x}$.

Theorem 1.20. [2] *Let $(\tilde{U},d,A)$ be complete soft metric space with a finite set A, suppose the soft mapping $f:\tilde{U}\to\tilde{U}$ satisfies*

$$d(f(\tilde{x}),f(\tilde{y}))\leq c[d(\tilde{x},f(\tilde{x}))+d(\tilde{y},f(\tilde{y}))]$$

for all $\tilde{x},\tilde{y}\in SP(\tilde{U})$ where $\bar{0}\tilde{\leq}\tilde{c}\tilde{<}\frac{\bar{1}}{2}$. then f has a unique fixed point.

Definition 1.21. [9] Let $(\tilde{N},\|.\|,A)$ be a soft normed linear space and $\tilde{r}\tilde{>}\bar{0}$ be soft real numbers. Then $B(\tilde{x},\tilde{r})$, $\bar{B}(\tilde{x},\tilde{r})$ and $S(\tilde{x},\tilde{r})$ are called soft open ball, soft closed ball and soft sphere respectively, where
(a) $B(\tilde{x},\tilde{r})=\{\tilde{y}\tilde{\in}\tilde{N}:\|\tilde{x}-\tilde{y}\|\tilde{<}\tilde{r}\}\subset SE(\tilde{N})$,
(b) $\bar{B}(\tilde{x},\tilde{r})=\{\tilde{y}\tilde{\in}\tilde{N}:\|\tilde{x}-\tilde{y}\|\tilde{\leq}\tilde{r}\}\subset SE(\tilde{N})$,
(c) $\tilde{S}(\tilde{x},\tilde{r})=\{\tilde{y}\tilde{\in}\tilde{N}:\|\tilde{x}-\tilde{y}\|\tilde{=}\tilde{r}\}\subset SE(\tilde{N})$.

Definition 1.22. [9] A sequence of soft element $\{\tilde{x}_n\}$ in a soft normed linear space $(\tilde{N},\|.\|,A)$ converges to a soft element $\tilde{x}$ if $\|\tilde{x}_n-\tilde{x}\|\to\bar{0}$ as $n\to\infty$. This means for every $\tilde{\epsilon}\tilde{>}\bar{0}$ chosen arbitrarily, there exists a natural number $N=N(\tilde{\epsilon})$, such that $\bar{0}\tilde{\leq}\|\tilde{x}_n-\tilde{x}\|\tilde{<}\tilde{\epsilon}$, whenever $n>N$. i.e $n>N$ implies $\tilde{x}_n\tilde{\in}B(\tilde{x},\tilde{\epsilon})$. $\tilde{x}$ is said to be the limit of the sequence $\{\tilde{x}_n\}$ as $n\to\infty$.

Example 1.23. [9] Let's consider the set R of all real number endowed with the usual norm $\|.\|$ and $(R,\|.\|,A)$ a soft normed space generated by the crisp norm $\|.\|$ where A is a non-empty set of parameters. Let $(Y,A)\subset R$ such that $Y(\lambda)=(0,1]$ in a real line, $\forall\lambda\in A$. Let's choose a sequence $\{\tilde{x}_n\}$ of soft element of (Y,A) where $\tilde{x}_n(\lambda)=\frac{1}{n},\forall n\in N,\lambda\in A$. Then there is a number $\tilde{x}\in(Y,A)$ such that $\tilde{x}_n\to\tilde{x}$ in $(Y,\|.\|,A)$. However, the sequence $\{\tilde{y}_n\}$ of soft element of (Y,A) where $\tilde{y}_n(\lambda)=\frac{1}{2},\forall n\in N,\lambda\in A$ is convergent in $(Y,\|.\|,A)$ and converges to $\frac{\bar{1}}{2}$.

Proposition 1.24. [9] *The limit of a sequence $\{\tilde{x}_n\}$ in a soft normed linear space, if it exists is unique.*

Definition 1.25. [9] A sequence $\{\tilde{x}_n\}$ of a soft element in a soft normed linear space $(\tilde{N},\|.\|)$ is said to be bounded if the set $\{\|\tilde{x}_n-\tilde{x}_m\|:n,m\in N\}$ of real numbers is bounded. i.e. if there exists $\tilde{M}\tilde{\geq}\bar{0}$ such that

$$\|\tilde{x}_n-\tilde{x}_m\|\tilde{\leq}\tilde{M}\quad\forall n,m\in N.$$

Definition 1.26. [9] A sequence $\{\tilde{x}_n\}$ of soft element in a soft normed linear space $(\tilde{N}, \|.\|, A)$ is said to be a Cauchy sequence in $\tilde{N}$ if for every $\tilde{\epsilon}\tilde{>}\bar{0}$, there exists an $m \in N$ such that $\|\tilde{x}_i - \tilde{x}_j\|\tilde{<}\tilde{\epsilon}$ for all $i, j \geq m$. That is $\|\tilde{x}_i - \tilde{x}_j\| \to \bar{0}$ as $i, j \to \infty$.

Proposition 1.27. [9] *Every convergent sequence $\{\tilde{x}_n\}$ in a soft normed linear space is Cauchy and every Cauchy sequence is bounded.*

Definition 1.28. [9] A soft subset (Y, A) with $Y(\lambda)\neq\emptyset, \ \forall\lambda \in A$ in a soft normed linear space $(\tilde{N}\|.\|, A)$ is said to be bounded if there exists a soft real number $\bar{k}$ such that $\|\tilde{x}\|\tilde{\leq}\bar{k}, \ \forall\tilde{x}\tilde{\in}(Y, A)$.

Definition 1.29. [9] A soft normed linear space $(\tilde{N}, \|.\|, A)$ is called complete if every Cauchy sequence in it converges to a soft element of $\tilde{N}$.

Definition 1.30. [9] Let $(\tilde{N}, \|.\|, A)$ be a soft normed linear space. Then
(i) If $\tilde{x}_n \to \tilde{x}$ and $\tilde{y}_n \to \tilde{y}$, then $\tilde{x}_n + \tilde{y}_n \to \tilde{x} + \tilde{y}$
(ii) If $\tilde{x}_n \to \tilde{x}$ and $\tilde{\lambda}_n \to \tilde{\lambda}$, then $\tilde{\lambda}_n\tilde{x}_n \to \tilde{\lambda}\tilde{x}$
(iii) If $\{\tilde{x}_n\}$ and $\{\tilde{y}_n\}$ are Cauchy sequences in $\tilde{N}$ and $\{\tilde{\lambda}_n\}$ is a Cauchy sequence of soft scalars, then $\{\tilde{x}_n + \tilde{y}_n\}$ and $\{\tilde{x}_n + \tilde{\lambda}_n\}$ are also Cauchy sequences in $\tilde{N}$.

Definition 1.31. Let $(\tilde{N}, \|.\|, A)$ be a soft complete normed linear space with a finite set A. Suppose the soft mapping $f : \check{N} \to \check{N}$ satisfies

$$\|T(\tilde{x}) - T(\tilde{y})\| \leq \bar{b}[\|\tilde{x} - T(\tilde{x})\| + \|\tilde{y} - T(\tilde{y})\|]$$

for all $\tilde{x}, \tilde{y} \in SP(\check{N})$ where $\bar{0}\tilde{\leq}\bar{b}\tilde{<}\frac{\bar{1}}{2}$. then T has a unique fixed soft point. This is called soft Kannan contractive mapping.

Definition 1.32. Let $(\tilde{N}, \|.\|, A)$ be a soft complete normed linear space with a finite set A. Suppose the soft mapping $T : \check{N} \to \check{N}$ satisfies

$$\|T(\tilde{x}) - T(\tilde{y})\| \leq \bar{c}[\|\tilde{x} - T(\tilde{y})\| + \|\tilde{y} - T(\tilde{x})\|]$$

for all $\tilde{x}, \tilde{y} \in SP(\check{N})$ where $\bar{0}\tilde{\leq}\bar{c}\tilde{<}\frac{\bar{1}}{2}$. then T has a unique fixed soft point. This is called soft Chaterjea contractive mapping.

Proposition 1.33. *Let $(\tilde{N}, \|.\|, A)$ be a soft complete normed linear space with a finite set A . Suppose the soft mapping $f : \check{N} \to \check{N}$ satisfies:*
(SZ_1). $\|T(\tilde{x}) - T(\tilde{y})\| \leq \tilde{a}\|\tilde{x} - \tilde{y}\|$,
(SZ_2). $\|T(\tilde{x}) - T(\tilde{y})\| \leq \tilde{b}[\|\tilde{x} - T(\tilde{x})\| + \|\tilde{y} - T(\tilde{y})\|]$,
(SZ_3). $\|T(\tilde{x}) - T(\tilde{y})\| \leq \tilde{c}[\|\tilde{x} - T(\tilde{y})\| + \|\tilde{y} - T(\tilde{x})\|]$, *for all $\tilde{x}, \tilde{y} \in SP(\check{N})$ where $\bar{0}\tilde{\leq}\tilde{a}\tilde{<}\bar{1}$, $\bar{0}\tilde{\leq}\tilde{b}\tilde{<}\frac{\tilde{1}}{2}$ and $\bar{0}\tilde{\leq}\tilde{c}\tilde{<}\frac{\tilde{1}}{2}$. Then T has a unique fixed soft point if at least one of the conditions above is true. This is called soft Zamfirescu contractive mapping. It is the soft space version of the contractive mapping of Zamfirescu [23] in literature.*

We will now show that every soft Zamfirescu operator T satisfies the inequalities: $\|T(\tilde{x}) - T(\tilde{y})\| \leq \bar{\delta}\|\tilde{x} - \tilde{y}\| + 2\delta\|\tilde{x} - T(\tilde{x})\|$, where $\tilde{\delta} = max\{\tilde{a}, \frac{\tilde{b}}{1-\tilde{b}}, \frac{\tilde{c}}{1-\tilde{c}}\}\tilde{<}\tilde{1}$.

Consider (SZ_1):
$\|T(\tilde{x}) - T(\tilde{y})\| \leq \tilde{a}\|\tilde{x} - \tilde{y}\|$.

Consider (SZ_2):

$$\begin{aligned} \|T(\tilde{x}) - T(\tilde{y})\| &\leq \tilde{b}[\|\tilde{x} - T(\tilde{x})\| + \|\tilde{y} - T(\tilde{y})\|] \\ &\leq \tilde{b}[\|\tilde{x} - T(\tilde{x})\| + \|\tilde{y} - \tilde{x} + \tilde{x} - T(\tilde{x}) + T(\tilde{x}) - T(\tilde{y})\|] \\ &\leq 2\tilde{b}\|\tilde{x} - T(\tilde{x})\| + \tilde{b}\|\tilde{x} - \tilde{y}\| + \tilde{b}\|T(\tilde{x}) - T(\tilde{y})\| \end{aligned}$$

$$\leq \quad \frac{\tilde{b}}{1-\tilde{b}}\|\tilde{x}-\tilde{y}\| + \frac{\tilde{2}\tilde{b}}{1-\tilde{b}}\|\tilde{x}-T(\tilde{x})\|.$$

Consider (SZ_3):

$$\begin{aligned}
\|T(\tilde{x})-T(\tilde{y})\| &\leq \tilde{c}[\|\tilde{x}-T(\tilde{y})\|+\|\tilde{y}-T(\tilde{x})\|] \\
&\leq \tilde{c}[\|\tilde{x}-T(\tilde{x})+T(\tilde{x})-T(\tilde{y})\|+\|\tilde{y}-\tilde{x}+\tilde{x}-T(\tilde{x})\|] \\
&\leq 2\tilde{c}\|\tilde{x}-T(\tilde{x})\|+\tilde{c}\|\tilde{x}-\tilde{y}\|+\tilde{c}\|T(\tilde{x})-T(\tilde{y})\| \\
&\leq \frac{\tilde{c}}{1-\tilde{c}}\|\tilde{x}-\tilde{y}\| + \frac{\tilde{2}\tilde{c}}{1-\tilde{c}}\|\tilde{x}-T(\tilde{x})\|.
\end{aligned}$$

Denote $\tilde{\delta} = max\{\tilde{a}, \frac{\tilde{b}}{1-\tilde{b}}, \frac{\tilde{c}}{1-\tilde{c}}\}$. By (SZ_1), (SZ_2) and (SZ_3), we get

$$\|T(\tilde{x})-T(\tilde{y})\| \leq \bar{\delta}\|\tilde{x}-\tilde{y}\| + 2\delta\|\tilde{x}-T(\tilde{x})\|,$$

where $\bar{0}\tilde{\leq}\tilde{\delta}\tilde{<}\bar{1}$. If $\tilde{L} = 2\tilde{\delta}$, we obtain:

$$\|T(\tilde{x})-T(\tilde{y})\| \leq \bar{\delta}\|\tilde{x}-\tilde{y}\| + \tilde{L}\|\tilde{x}-T(\tilde{x})\|.$$

Suppose $\tilde{\varphi}(t) = \tilde{L}t$, we get

$$\|T(\tilde{x})-T(\tilde{y})\| \leq \bar{\delta}\|\tilde{x}-\tilde{y}\| + \tilde{\varphi}(\|\tilde{x}-T(\tilde{x})\|),$$

where $\tilde{\varphi} : R(A)^* \to R(A)^*$ is a monotone increasing function with $\tilde{\varphi}(\bar{0}) = \bar{0}$. This ends the proof.

We now consider some iterative schemes in a soft normed linear space.
Let $(\tilde{N}, \|.\|, A)$ be a soft normed linear space with A, a finite set and $f : \check{N} \to \check{N}$ a soft self mapping of $\check{N}$. Define $F_T = \{\tilde{q}\tilde{\in}\check{N} : T\tilde{q} = \tilde{q}\}$ to be the set of fixed point of T. For $\tilde{x}_0\tilde{\in}\check{N}$, the sequence $\{\tilde{x}_n\}_{n=0}^{\infty}$ defined by

$$\tilde{x}_{n+1} = T\tilde{x}_n, \tag{1.6}$$

$n\tilde{\geq}\bar{0}$ is called the soft Picard iterative scheme.

For $\tilde{x}_0\tilde{\in}\check{N}$, the sequence $\{\tilde{x}_n\}_{n=0}^{\infty}$ defined by

$$\tilde{x}_{n+1} = (1-\tilde{\alpha}_n)\tilde{x}_n + \tilde{\alpha}_n T\tilde{x}_n, \tag{1.7}$$

$n\tilde{\geq}\bar{0}$, where $\{\tilde{\alpha}_n\}_{n=1}^{\infty}$ is a soft real sequence in $[\bar{0}, \bar{1}]$ is called the soft Mann iterative scheme.

For $\tilde{x}_0\tilde{\in}\check{N}$, the sequence $\{\tilde{x}_n\}_{n=0}^{\infty}$ defined by

$$\begin{aligned}
\tilde{x}_{n+1} &= T\tilde{y}_n, \\
\tilde{y}_n &= (1-\tilde{\alpha}_n)\tilde{x}_n + \tilde{\alpha}_n T\tilde{x}_n,
\end{aligned} \tag{1.8}$$

$n\tilde{\geq}\bar{0}$, where $\{\tilde{\alpha}_n\}_{n=1}^{\infty}$ is a soft real sequence in $[\bar{0}, \bar{1}]$ is called the soft Picard-Mann hybrid iterative scheme.

We shall need the following lemma in proving our result.

Lemma 1.34. [5] *Let δ be a real number satisfying $0 \leq \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ a sequence of positive numbers such that $lim_{n\to\infty}\epsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying $u_{n+1} \leq \delta u_n + \epsilon_n$, n=0,1,2,..., we have $lim_{n\to\infty}u_n = 0$.*

2. MAIN RESULTS

Theorem 2.1. *Let $(\tilde{N}, \|.\|, A)$ be a soft normed linear space with a finite set A and $T : \tilde{N} \to \tilde{N}$ be a soft self mapping satisfying the soft contractive-like condition*

$$\|T\tilde{x} - T\tilde{y}\| \leq \tilde{\delta}\|\tilde{x} - \tilde{y}\| + \tilde{\varphi}(\|\tilde{x} - T\tilde{x}\|), \tag{2.1}$$

for each $\tilde{x}, \tilde{y} \in SP(\tilde{N})$, $\bar{0}\tilde{\leq}\tilde{\delta} < \bar{1}$ and $\tilde{\varphi}$ is a monotone increasing function with $\tilde{\varphi}(\bar{0}) = \bar{0}$. For arbitrary $\tilde{x}_0\tilde{\in}\tilde{N}$, let $\{\tilde{x}_n\}_{n=0}^{\infty}$ be the soft Picard-Mann hybrid iterative scheme defined by (1.8), where $\{\tilde{\alpha}_n\}_{n=0}^{\infty}$ is a soft real sequence in $[\bar{0}, \bar{1}]$. Then
(i) T defined by (2.1) has a unique soft fixed point $\tilde{q}$;
(ii) the soft Picard-Mann hybrid iterative scheme (1.8) converges strongly to $\tilde{q}$ of T.

Proof. We shall first show that T has a unique fixed point.
Suppose $\tilde{q}_1, \tilde{q}_2\tilde{\in}\tilde{F}_T$ such that $\tilde{q}_1\tilde{\neq}\tilde{q}_2$

$$\begin{aligned} \|\tilde{q}_1 - \tilde{q}_2\| &= \|T\tilde{q}_1 - T\tilde{q}_2\| \\ &\leq \tilde{\delta}\|\tilde{q}_1 - \tilde{q}_2\| + \tilde{\varphi}(\|\tilde{q}_1 - T\tilde{q}_1\|) \\ &\leq \tilde{\delta}\|\tilde{q}_1 - \tilde{q}_2\| + \tilde{\varphi}(0) \\ &\leq \tilde{\delta}\|\tilde{q}_1 - \tilde{q}_2\|. \end{aligned}$$

Thus, $(1 - \tilde{\delta})\|\tilde{q}_1 - \tilde{q}_2\| \leq \bar{0}$, which implies $\|\tilde{q}_1 - \tilde{q}_2\| \leq \bar{0}$.
That is, $\tilde{q}_1 = \tilde{q}_2$. Thus, T has a unique fixed point $\tilde{q}$.

Next, we prove that $\lim_{n\to\infty} \tilde{x}_n = \tilde{q}$. That is, we show that the soft Picard-Mann hybrid iterative sequence converges strongly to $\tilde{q}$ of T.

$$\begin{aligned} \|\tilde{x}_{n+1} - \tilde{q}\| &= \|T\tilde{y}_n - T\tilde{q}\| \\ &\leq \tilde{\delta}\|\tilde{y}_n - \tilde{q}\| + \tilde{\varphi}(\|\tilde{q} - T\tilde{q}\|) \\ &\leq \tilde{\delta}\|\tilde{y}_n - \tilde{q}\| \\ &\leq \tilde{\delta}[(1 - \tilde{\alpha}_n)\|\tilde{x}_n - \tilde{q}\| + \tilde{\alpha}_n\|T\tilde{x}_n - \tilde{q}\|] \\ &\leq \tilde{\delta}[(1 - \tilde{\alpha}_n)\|\tilde{x}_n - \tilde{q}\| + \tilde{\delta}\tilde{\alpha}_n\|\tilde{x}_n - \tilde{q}\|] \\ &\leq \tilde{\delta}[1 - \tilde{\alpha}_n(\bar{1} - \tilde{\delta})]\|\tilde{x}_n - \tilde{q}\|. \end{aligned}$$

Hence, $\lim_{n\to\infty} \|\tilde{x}_n - \tilde{q}\| = \bar{0}$.
Since, $\tilde{\delta}[1 - \tilde{\alpha}_n(\bar{1} - \tilde{\delta})] \to \bar{0}$ as $n \to \infty$. Therefore $\{\tilde{x}_n\}_{n=0}^{\infty}$ converges strongly to a soft fixed point $\tilde{q}$. □

Theorem 2.2. *Let $(\tilde{N}, \|.\|, A)$ be a soft normed linear space with a finite set, A and $T : \tilde{N} \to \tilde{N}$ be a soft self mapping satisfying the soft contractive-like condition*

$$\|T\tilde{x} - T\tilde{y}\| \leq \tilde{\delta}\|\tilde{x} - \tilde{y}\| + \tilde{\varphi}(\|\tilde{x} - T\tilde{x}\|), \tag{2.2}$$

for each $\tilde{x}, \tilde{y} \in SP(\tilde{N})$, $\bar{0}\tilde{\leq}\tilde{\delta} < \bar{1}$. For arbitrary $\tilde{x}_0\tilde{\in}\tilde{N}$, let $\{\tilde{x}_n\}_{n=0}^{\infty}$ be the soft Mann iterative scheme defined by (1.7), where $\{\tilde{\alpha}_n\}_{n=0}^{\infty}$ is a soft real sequence in $[\bar{0}, \bar{1}]$. Then
(i) T defined by (2.2) has a unique soft fixed point $\tilde{q}$;
(ii) the soft Mann iterative scheme (1.7) converges strongly to $\tilde{q}$ of T.

Proof. The proof is similar to that of Theorem 2.1. □

Theorem 2.2 leads to the following corollary:

Corollary 2.3. *Let $(\tilde{N}, \|.\|, A)$ be a soft normed linear space with a finite set A and $T : \tilde{N} \to \tilde{N}$ be a soft self mapping satisfying the soft contractive-like condition*

$$\|T\tilde{x} - T\tilde{y}\| \leq \tilde{\delta}\|\tilde{x} - \tilde{y}\| + \tilde{\varphi}(\|\tilde{x} - T\tilde{x}\|), \tag{2.3}$$

for each $\tilde{x}, \tilde{y} \in SP(\tilde{N})$, $\bar{0}\tilde{\leq}\tilde{\delta} < \bar{1}$. For arbitrary $\tilde{x}_0\tilde{\in}\tilde{N}$, let $\{\tilde{x}_n\}_{n=0}^{\infty}$ be the soft Picard iterative scheme defined by (1.6). Then
(i) T defined by (2.3) has a unique soft fixed point $\tilde{q}$;
(ii) the soft Picard iterative scheme (1.6) converges strongly to $\tilde{q}$ of T.

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Competing Interest

The authors declare that there are no competing interest.

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