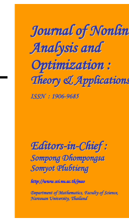

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OPTIMAL PRICING POLICY FOR MANUFACTURER AND RETAILER USING ITEM PRESERVATION TECHNOLOGY FOR DETERIORATING ITEMS

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ABSTRACT. This paper optimizes the selling price, the replenishment cycle and lot size of deteriorating seasonal items. The seasonal items become useless or completely deteriorated after sale season. The production of manufacturer and stock of retailer are affected by deterioration and assuming it is reduced by preservation technology investment. Manufacturer and retailer both are invested on a preservation technology under revenue sharing. We studied the effect of preservation technology on profit of manufacture as well as for retailer. Retailer can change the strategy by reducing the selling price to generate the excess demand for limited time duration. This paper aims to develop a continuous supply chain inventory model by optimizing the selling price of seasonal items. We optimized the profit by reducing price for stock dependent price sensitive demand and have shown that the profit function is concave function of selling price. The model is simulated and illustrated with numerical examples.

KEYWORDS : Inventory, stock and price dependent demand, optimal profit, deterioration, replenishment cycle.
AMS Subject Classification: 90B05, 90B30, 90B50

1. INTRODUCTION

In the real life, deterioration of product is a common phenomenon and always occur in the nature. There are many items in the nature that deteriorate significantly such as fruits, vegetable, milks, meat fresh foods, perfumes, alcohols soft drinks gasoline etc. Also demand of such items are just for a limited time horizon such type of items known as seasonal items. Recently more and more items become deteriorating nature and seasonal simultaneously because of the business competition, instant and rapid change in the technology. Hence this will become a very difficult problem to determine the inventory if the item is both deteriorating and seasonal.

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The preservation play a vital role for decreasing deterioration rate and for increasing seasonal item's life. Recently the effective use of preservation technology become essential for manufacturers and retailers both, to improve the customer service level, increase the business profit and reduce the economic losses. In the last few years, market surveys indicated that the demand rate of most of the inventory depends on the stock levels, and influenced by selling prices. In this paper we studied the pricing strategy of deteriorating seasonal items, optimize lot size, and replenishment cycle when deterioration rate is controlled by preservation technology investment. The decision variables of this model are the market demand, the production rate, the ordering frequency. Ghare and Schrader [19], are the first mathematician who developed an inventory model in which they considered deterioration as exponential decreasing function, after then remarkable works have been done on deteriorating inventory modeling. Liu and Lian [14], as well as Nandakumar and Morton [20], studied deteriorating items which have fixed life time. Jain and silver [13] as well as Kalpakam and Sapana [24], developed inventory models for random life deteriorated items.

In the deteriorating inventory modeling deterioration plays a vital role. Raffat, Wolfe and Eldin [7], developed an inventory model with constant demand rate and finite replenishment rate for deteriorating items. Heng, Labban and Linn [12], considering exponential decay in inventory model with constant demand and finite replenishment rate. The constant demand rate is a uniform deterministic demand rate but in the real life demand rate is not always constant. Time varying demand in inventory modeling have been developed mostly considering either linearly increasing/decreasing $D(t) = (\alpha + \gamma t)$, $\alpha > 0$, $\gamma \neq 0$ or exponentially increasing/decreasing $D(t) = \alpha . e^{at}$, $\alpha > 0$, $a \neq 0$. Haiping and Wang [31], suggested a model in which they considered time proportional demand and find optimum order quantity for deteriorating items. Xu and Wang [11], presented a model in which they consider linearly time varying demand for exponentially deteriorating items. Giri and Chaudhuari [2], developed a model in which they consider deterioration rate demand rate and costs were assumed to variable of time. Jalan and Chaudhuari [1] as well as Chakraborty, Giri and Chaudhuari [25], considered deterioration rate as two parameter and three parameter weibull distribution with instantaneous supply in their inventory models.

In analysis of market survey, it is observed that in the supermarket customers attraction is based on a large pile of goods. Hence displaying each of items in large quantities may be generate extra demand. But due to large quantity of items there may be arise problem of spacing of each item and also requirement of large scale investment. The situation become more critical when the displayed items is in nature of deteriorating. Due to this reason research attracted on inventory modeling in which demand consider as stock and price dependent. Sarkar, Mukherjee and Balan [4] as well as Datta and Pal [26], developed an inventory model assuming stock and selling price dependent demand for deteriorating items with and without shortages. Kim [5], developed price dependent inventory model for considering constant rate of deterioration with infinite rate of replenishment. Wee [10], studied the joint pricing and replenishment policy for deteriorating inventory with price elastic demand rate in the decline market for time dependent deterioration.

Samanta and Roy [9], developed a continuous production control inventory model for deteriorating items with shortages. They consider deterioration rate is very small, demand and production rate is constant. Ilkeong, Giri and Byungsung [16], developed an inventory model for amelioration/deteriorating items with time varying demand pattern finite planing horizon, taking into account the effects of inflation and time value of money. Ouyang, Wu, and Cheng [15], studied an economical inventory model for deteriorating items with exponential decreasing demand. In this model shortages are allowed assuming backlogging rate is a function of waiting time for the next period.

Uthayakumar and Parvathi [22], described an inventory model in which they assumed demand is stock dependent and deterioration rate is nonlinear function of time. They also assumed that the retailer adopts the trade credit policy offered by supplier. Jain, Rathore and Sharma [17], presented an economical production quantity model for deteriorating items in which they consider price and stock dependent demand with considering shortages. Jain and Kumar [23], presented an economical order quantity model in which he consider ramp type demand, starting with and without two parameter weibull distribution deterioration rate $z(t) = \alpha\beta t^{\beta-1}$ where $\alpha(0 < \alpha << 1)$ is the scale parameter and $(\beta > 0)$ is the shape parameter.

Cachon and Lariviere [8], developed supply chain coordination with revenue sharing contracts model, in this revenue sharing contract, a retailer pays to supplier a wholesale price for each unit that he purchased and also pays a percentage of the revenue that generates retailer. Shukla and Khedlekar [6], presented time and price dependent with varying holding cost inventory model for deteriorating items in this model they considered the demand as a parametric dependent linear function of time and price both. The coefficient of time parameter and coefficient of price parameter are examined simultaneously and proved that time is dominating variable over price in term of earning more profit. It is also proved that deterioration of items in the inventory is one of the most sensitive parameter to look in to besides many others. Khedlekar and Namdeo [27], developed an inventory model for stock and price dependent demand with deterioration, but there is no explanation about deteriorating rate of products. He, Wang and Lai [33], developed production inventory model in which they consider deteriorating properties of products

Giri and Bardhan [3], presented an integrated single-manufacturer single retailer supply chain model for deteriorating item. In this model demand function is assumed to be the function of on hand stock and price furthermore manufacturer and retailer are in an agreement of lot for lot policy.The proposed model is developed under the contract that the retailer offers the manufacturers a percentage of revenue(s), he generates by selling a lot. Palani and Maragatham [21], proposed a deterministic inventory model for exponential deteriorating items in which demand rate and holding cost are quadratic and linear function of time. They also consider that the deterioration is controlled by using preservation technology investment.

Yang, Wee, Chunge and Huang [18], developed a piecewise production inventory model for a multi market deteriorating product with time varying and price varying sensitive demand. He and Huange [32], studied a kind of deteriorating products whose deterioration can be controlled by investing on the preservation efforts. Study considers the seasonal and deterioration properties simultaneously, Demand rate is assumed to be decreasing linear function of selling price and assuming resultant deterioration is decreasing exponential function of cost of preservation technology investment per unit time.

Mishra [30], developed an integrated single-retailer and single-supplier inventory model for deteriorating items under revenue sharing on preservation technology investment in which he considered the demand rate is a non negative power function of selling price and stock level,and production rate is constant. He also proposed the manufacturer offers to the retailers for a percentage of revenue sharing on preservation technology investment. Numerical and graphical illustration is given by him. Khedlekar, Namdeo and Nigwal [28], introduced disruption factor in production inventory modeling considering with shortages and time proportional demand, Khedlekar, Shukla and Namdeo [29], designed pricing strategies for declining market demand of deteriorating item introducing item preservation technology. In the previous study in this field, demand function as a price sensitive, stock dependent and constant production rate is considered by researchers. This paper presents a model in which demand rate as a exponential decreasing function of t as well as price and stock dependent production rate taken as a linear function of t and also considered preservation technology investment factor for deteriorating items.

2. Notations and Assumptions

Following notation are used in this model.

- c Purchase cost per unit for retailer and selling cost for manufacturer,
- p Retail price per unit items,
- C_s Compiling cost per lot for manufacturer,
- C_p Production cost per unit items,
- C_0 The ordering cost per order of the retailer,
- C_d Deteriorating cost per cycle [Value of deteriorated products per unit],
- κ Cost coefficient of investment in the preservation technology,
- C_h Unit inventory holding cost per unit time,
- ξ Preservation technology cost for reducing deterioration rate in order preserve the product
- θ The deterioration rate,
- ρ Consequent deterioration rate $\rho = \theta e^{-\eta\xi}$,
- $D(p, I(t))$ Market Demand rate at time t; $D(p, I(t)) = \alpha e^{-at} - \beta p + \phi I(t)$ where α demand sensitive parameter, β price sensitive parameter, and ϕ stock sensitive parameter,
- q_m Production rate which is linear function of t; we assume $q_m = q + rt$; where r is the production sensitive parameter,
- q Production scale,
- δ The subsidy proportion provided by manufacturer to the retailer for preservation technology investment,
- T The length of cycle time,
- Q Initial lot-size during a cycle of length T,
- $I_m(t)$ Inventory level at time t for the manufacturer, $0 \leq t \leq T$,
- $I_r(t)$ Inventory level at time t for the retailer, $0 \leq t \leq T$,
- AP_R Total profit per unit time for the retailer,
- AP_M Total profit per unit time for the manufacturer,
- NTP Average Total profit per unit time under integrated system,

The following assumption are made in this model:

- Market Demand of product is $D(p, I(t))$ at unit time t; we assumed demand function $D(p, I(t)) = \alpha e^{-at} - \beta p + \phi I(t)$, is nonnegative exponential function of t as well as price and stock level, where α is initial demand and β is price sensitive parameter, ϕ is stock sensitive parameter, and $\alpha > 0$, $a \geq 0$, $\beta > 0$, $\phi \geq 0$,
- Holding cost and deterioration cost are constant,
- Production rate is linear increasing function $q_m = q + rt$,

- Partly or wholly deteriorated products have no value and there is no holding cost for them.
- The deterioration rate is controlled by preservation and products which are fully preserved by preservation technology,
- Preservation technology investment is shared by manufacturer and retailer for reducing the deterioration rate, and sharing rate is δ.
- The proportion of reduced deterioration rate, $\rho = \theta e^{-\eta\xi}$, is concave increasing function of ξ,
- The deterioration cost due to deterioration and holding cost for both manufacturer and retailer are same,
- The lead time is zero, and replenishment rate is finite, however the planning horizon is finite.
- In the finite time horizon T it is considered that $e^{-aT} \approx e^{-at}$ because a is taken as very small.

3. Proposed Model for Retailer

According to the assumptions the retailer receives the stock initially from the manufacturer, at time t, $0 \le t \le T$. The rate of change in inventory level for retailer is equal to demand rate and deterioration rate. Thus the following first order nonlinear differential equation representing the inventory status at any time t

$$\begin{aligned} \frac{dI_r(t)}{dt} + \rho I_r(t) &= -D(p, I_r(t)), \quad \text{where} \quad 0 \le t \le T. \\ &= -(\alpha e^{-at} - \beta p + \phi I_r(t)) \end{aligned} \tag{3.1}$$

with boundary condition $I_r(t) = Q$, at $t = 0$ and $I_r(t) = 0$, at $t = T$
Now we derived the average profit function of retailer during a replenishment cycle interval $[0, T]$.
The average profit for retailer can be formulated as
Average Profit $= \frac{1}{T}$[Sales Revenue-Purchase Cost-Ordering Cost-Inventory Holding Cost-Deterioration Cost-Preservation Technology Investment Cost]
Equation (3.1) leads to

$$I_r(t) = \left(1 - e^{(\rho+\phi)(T-t)}\right)\left(\frac{\beta p}{\rho+\phi} - \frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \tag{3.2}$$

The initial order lot size for retailer at time $t = 0$, where $t \in [0, T]$ is

$$I_r(0) = Q = \left(1 - e^{(\rho+\phi)T}\right)\left(\frac{\beta p}{\rho+\phi} - \frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \tag{3.3}$$

The total sales revenue in replenishment cycle time $[0, T]$ can be formulated as

$$SR_r = p\int_0^T D(p, I_r(t))dt$$

$$\begin{aligned} SR_r = \phi p &\left(T + \frac{1}{\rho+\phi} - \frac{e^{(\rho+\phi)T}}{(\rho+\phi)}\right)\left(\frac{\beta p}{\rho+\phi} - \frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \\ &+ \frac{p\alpha}{a}\left(1 - e^{-aT}\right) - \beta p^2 T \end{aligned} \tag{3.4}$$

Purchase cost of retailer is

$$PC_r = c.Q$$

$$PC_r = c\left(1 - e^{(\rho+\phi)T}\right)\left(\frac{\beta p}{\rho+\phi} - \frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \tag{3.5}$$

The inventory holding cost $IHCr$ is

$$IHC_r = h\int_0^T I_r(t)dt$$

$$IHCr = h\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{(\rho+\phi)}\right)\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \tag{3.6}$$

Deterioration cost DC_r in the interval of length $[0,T]$ is

$$DC_r = C_d\theta e^{-\eta\xi}\int_0^T I_r(t)dt$$

$$DC_r = C_d\theta e^{-\eta\xi}\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \tag{3.7}$$

Preservation technology investment cost $PTIC_r$ is

$$PTIC_r = (1-\delta)\kappa\xi T \tag{3.8}$$

The ordering cost is given by

$$OC_r = C_o \tag{3.9}$$

Hence the average profit function for retailer per unit time is

$$\begin{aligned} AP_r = {} & \frac{p\alpha}{Ta}(1-e^{-aT})-\beta p^2-\frac{1}{T}C_o-(1-\delta)\kappa\xi \\ & +\frac{\zeta}{T}\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \\ & -\frac{c}{T}\left(1-e^{(\rho+\phi)T}\right)\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \end{aligned} \tag{3.10}$$

where $\zeta = \left(\phi p - h - C_d\theta e^{-\eta\xi}\right)$

4. Proposed Model for Manufacturer

Manufacturer supply the quantity at rate q_m to the retailer. At time t, on hand inventory of manufacturer is $I(t)$. Due to preservation technology the reduced deterioration value is $\rho I_m(t)$. Thus the differential equation will be

$$\begin{aligned} \frac{dI_m(t)}{dt}+\rho I_m(t) &= q_m \quad \text{, where } \; t_s\le t\le T. \\ &= q+rt \end{aligned} \tag{4.1}$$

with boundary condition $I_m(t_s)=0$, at $t=t_m$ and $I_m(t)=Q$, at $t=T$
Now we derived the net profit function for the manufacturer during a replenishment cycle of length $[0,T]$.
The net profit function for manufacturer can be formulated as
Average Profit= $\frac{1}{T}$[Sales Revenue - Production Cost - Raw Material Ordering Cost - Holding Cost - Deterioration Cost - Preservation Technology Investment Cost]
Equation (3.11) leads to

$$I_m(t) = \left(\frac{q}{\rho}-\frac{r}{\rho^2}\right)\left(e^{\rho(t_s-t)}\right)+\frac{r}{\rho}\left(t-t_s\left(e^{\rho(t_s-t)}\right)\right) \tag{4.2}$$

Sales income in the cycle $[0,T]$ is

$$SR_m = c.Q$$

$$SR_m = c.\left(1-e^{(\rho+\phi)T}\right)\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right) \tag{4.3}$$

The production cost of product for manufacturer is

$$PC_m = C_p \int_{t_s}^{T} (q + rt)dt$$

$$PC_m = C_p(T - t_s)\left(q + \frac{r}{2}(T - t_s)\right) \tag{4.4}$$

Raw material ordering cost per lot for manufacturer is

$$RMOC = C_s \tag{4.5}$$

Preservation Technology Investment Cost is

$$PTIC = \delta\kappa\xi T \tag{4.6}$$

The Deterioration cost product per production cycle for manufacturer is

$$DC_m = C_d\theta e^{-\eta\xi} \int_{t_s}^{T} I_m(t)dt$$

$$\begin{aligned} DC_m = \omega\left(\frac{q}{\rho} - \frac{r}{\rho^2}\right)(T - t_s) + \omega\left(\frac{q}{\rho} - \frac{r}{\rho^2}\right)\frac{1}{\rho}\left(e^{\rho(t_s - T)} - 1\right) \\ + \omega\frac{r}{\rho}\left(\frac{T^2}{2} - \frac{t_s^2}{2}\right) - \omega\frac{rt_s}{\rho^2}\left(e^{\rho(t_s - T)} + 1\right) \end{aligned} \tag{4.7}$$

where, $\omega = C_d\theta e^{-\eta\xi}$
The holding cost of product per production cycle for manufacturer is

$$HC_m = h \int_{t_s}^{T} I_m(t)dt$$

$$\begin{aligned} HC_m = h.\left(\frac{q}{\rho} - \frac{r}{\rho^2}\right)(T - t_s) + h\left(\frac{q}{\rho} - \frac{r}{\rho^2}\right)\frac{1}{\rho}\left(e^{\rho(t_s - T)} - 1\right) \\ + h\frac{r}{\rho}\left(\frac{T^2}{2} - \frac{t_s^2}{2}\right) - h\frac{rt_s}{\rho^2}\left(e^{\rho(t_s - T)} + 1\right) \end{aligned} \tag{4.8}$$

Hence the average profit function for manufacturer per time unit is given by,

$$\begin{aligned} AP_m = \frac{c}{T}.\left(1 - e^{(\rho+\phi)T}\right)\left(\frac{\beta p}{\rho + \phi} - \frac{\alpha e^{-aT}}{\rho + \phi - a}\right) - \frac{C_p}{T}(T - t_s)\left(q + \frac{r}{2}(T - t_s)\right) \\ - \psi\left(\frac{q}{\rho} - \frac{r}{\rho^2}\right)(T - t_s) - \frac{\psi}{\rho}\left(\frac{q}{\rho} - \frac{r}{\rho^2}\right)\left(1 - e^{\rho(T - t_s)}\right) - \frac{\psi r}{\rho}\left(\frac{T^2}{2} - \frac{t_s^2}{2}\right) \\ - \delta\kappa\xi - \frac{C_s}{T} + \frac{\psi r t_s}{\rho^2}\left(1 - e^{\rho(T - t_s)}\right) \end{aligned} \tag{4.9}$$

where, $\psi = \left(h + C_d\theta e^{-\eta\xi}\right)$

5. Total Profit Function

In this article we have considered that the manufacturer and retailer both are work together as a single unit and for reducing the deterioration rate of items they both are invest on preservation technology with revenue sharing. To find total profit of whole supply chain unit we formulate the total average profit function of whole supply chain inventory system as,

$$NTP = AP_r + AP_m$$

$$\begin{aligned} NTP = & \frac{p\alpha}{Ta}(1-e^{-aT}) - \beta p^2 - \frac{C_p}{T}(T-t_s)\left(q+\frac{r}{2}(T-t_s)\right) - \frac{C_0}{T} - \frac{C_s}{T} - \kappa\xi \\ & + \frac{1}{T}\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right) - \psi\left(\frac{q}{\rho}-\frac{r}{\rho^2}\right)(T-t_s) \\ & - \frac{\psi}{\rho}\left(\frac{q}{\rho}-\frac{r}{\rho^2}\right)\left(1-e^{\rho(T-t_s)}\right) - \psi\frac{r}{\rho}\left(\frac{T^2}{2}-\frac{t_s^2}{2}\right) - \frac{\psi r t_s}{\rho^2}\left(1-e^{\rho(T-t_s)}\right) \end{aligned}$$

where, $\psi = \left(h + C_d\theta e^{-\eta\xi}\right)$ and $\zeta = \left(\phi p - h - C_d\theta e^{-\eta\xi}\right)$

Proposition 5.1. *There exist an unique optimal selling price p^* for stock dependent demand, net total profit function NTP(T, p) is maximum for fixed time horizon T and preservation cost ξ.*

Proof. The first order partial derivative of the net profit function is

$$\begin{aligned} \frac{\partial NTP(T,p)}{\partial p} = & \frac{\alpha}{Ta}(1-e^{-aT}) - 2\beta p - \frac{1}{T}\zeta\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)\frac{\beta p}{\rho+\phi} \\ & + \phi\left(\frac{\beta p}{\rho+\phi}-\frac{\alpha e^{-aT}}{\rho+\phi-a}\right)\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right) \end{aligned} \tag{5.1}$$

Where, $\zeta = \left(\phi p - h - C_d\theta e^{-\eta\xi}\right)$
If p^* is a optimal value p, then $\frac{\partial NTP(T,p)}{\partial p}$ must be equal to zero
i.e.

$$\frac{\partial NTP(T,p)}{\partial p} = 0$$

Solve for the optimal price p^*
we have

$$p^* = \frac{\frac{\alpha}{a}(1-e^{-aT}) - \left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)\left(\frac{\beta(h+C_d\theta e^{-\eta\xi})}{\rho+\phi}+\frac{\phi\alpha e^{-aT}}{\rho+\phi-a}\right)}{2\beta T - \frac{2\beta\phi}{\rho+\phi}\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)} \tag{5.2}$$

for maximum value NTP(T, p) at point $p = p^*$, we have

$$\frac{\partial^2 NTP(T,p)}{\partial p^2} = -2\beta T + \left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)\frac{2\beta p}{\rho+\phi} < 0 \tag{5.3}$$

for $\beta > 0$ and $\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right) < 0$, because $\left(T+\frac{1}{\rho+\phi} < \frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)$ □

Proposition 5.2. *For fixed ξ, there exist an optimal solution ($T^*.p^*$) that maximize the net profit function NTP(T, p), and also it is unique.*

Special case

When we consider demand function as a price sensitive only (i.e. $\phi = 0$) then the Equation (5.1) reduces to the following form

6. Total Profit Function

Therefore, the total average profit function whole inventory supply chain system is,

$$NTP = AP_r + AP_m$$

$$\begin{aligned} NTP = & \frac{p\alpha}{Ta}(1-e^{-aT}) - \beta p^2 - \frac{C_p}{T}(T-t_s)\left(q+\frac{r}{2}(T-t_s)\right) - \frac{C_0}{T} - \frac{C_s}{T} - \kappa\xi \\ & - \frac{1}{T}\sigma\left(T+\frac{1}{\rho}-\frac{e^{\rho T}}{\rho}\right)\left(\frac{\beta p}{\rho}-\frac{\alpha e^{-aT}}{\rho-a}\right) - \psi\left(\frac{q}{\rho}-\frac{r}{\rho^2}\right)(T-t_s) \\ & - \frac{\psi}{\rho}\left(\frac{q}{\rho}-\frac{r}{\rho^2}\right)\left(e^{\rho(t_s-T)}-1\right) - \psi\frac{r}{\rho}\left(\frac{T^2}{2}-\frac{t_s^2}{2}\right) - \frac{\psi r t_s}{\rho^2}\left(e^{\rho(t_s-T)}+1\right) \end{aligned}$$

where, $\psi = \left(h + C_d\theta e^{-\eta\xi}\right)$ and $\sigma = \left(h + C_d\theta e^{-\eta\xi}\right)$

Proposition 6.1. *There exist an unique optimal selling price p_1^* at which the net total profit function NTP (T, p) is maximum for fixed time horizon T and preservation cost ξ. where p_1^* is a price of unit item for special case.*

Proof. The first order partial derivative of the net profit function are

$$\frac{\partial NTP(T,p)}{\partial p} = \frac{\alpha}{Ta}(1-e^{-aT}) - 2\beta p - \frac{1}{T}\zeta\left(T+\frac{1}{\rho}-\frac{e^{\rho T}}{\rho}\right)\frac{\beta}{\rho} \tag{6.1}$$

Where, $\zeta = -\left(h + C_d\theta e^{-\eta\xi}\right)$

If p_1^* is a optimal value of p, then $\frac{\partial NTP(T,p)}{\partial p}$ must be equal to zero

i.e.

$$\frac{\partial NTP(T,p)}{\partial p} = 0$$

Solve for the optimal price p_1^*

we have

$$p_1^* = \frac{\frac{\alpha}{a}(1-e^{-aT}) - \left(T+\frac{1}{\rho}-\frac{e^{\rho T}}{\rho}\right)\frac{\beta}{\rho}(h+C_d\theta e^{-\eta\xi})}{2\beta T} \tag{6.2}$$

at point $p = p_1^*$, NTP(T, p) has maximum value if

$$\frac{\partial^2 NTP(T,p)}{\partial p^2} = -2\beta T < 0 \tag{6.3}$$

for $\beta > 0$. □

Proposition 6.2. *For fixed ξ, there exist an optimal solution ($T^*.p_1^*$), that maximize the net profit function NTP(T, p), and also it is unique.*

Theorem 6.1. *If p is selling price of a product with stock dependent and price sensitive demand and p_1^* is a selling price of a product with price sensitive demand, than for $aT \geq 0$, p^* is always less than equal to p_1^*.*

Proof. For this we will prove that p_1^*-$p^* > 0$

From the prepositions 1 and 3 we have

$$\left[\frac{\alpha}{a}(1-e^{-aT}) - \Delta_1\pi\frac{\beta}{\rho}\right]\left[2\beta T - \frac{2\beta\phi}{\rho+\phi}\Delta_2\right]$$

$$-2\beta T\left[\frac{\alpha}{a}(1-e^{-aT}) - \Delta_2\left(\frac{\beta\pi}{\rho+\phi}+\frac{\phi\alpha e^{-aT}}{\rho+\phi-a}\right)\right] > 0$$

$$or -\frac{\alpha}{a}(1-e^{-aT})\frac{2\beta\phi}{\rho+\phi}\Delta_2+\Delta_1\pi\frac{\beta}{\rho}\frac{2\beta\phi}{\rho+\phi}\Delta_2$$
$$-2\beta T\Delta_2\left(\frac{\beta\pi}{\rho+\phi}+\frac{\phi\alpha e^{-aT}}{\rho+\phi-a}\right)>0 \quad (6.4)$$

since $\pi=(h+C_d\theta e^{-\eta\xi})>0$, $\Delta_1=\left(T+\frac{1}{\rho}-\frac{e^{\rho T}}{\rho}\right)<0$
and $\Delta_2=\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)<0, \quad \forall\, aT\in R^+$,
therefore, from (6.4) p_1^*-$p>0$.
Where, $\pi=(h+C_d\theta e^{-\eta\xi})$, $\Delta_1=\left(T+\frac{1}{\rho}-\frac{e^{\rho T}}{\rho}\right)$,
and $\Delta_2=\left(T+\frac{1}{\rho+\phi}-\frac{e^{(\rho+\phi)T}}{\rho+\phi}\right)$. □

Corollary 6.3. *Net total profit $NTP(T,p)$ is a increasing function with respect to selling price p, i.e.*
if $p_1^*>p$, then

$$NTP(T,p_1^*)>NTP(T,p).$$

Example 6.4. In case of stock dependent and price sensitive demand the numerical example are as follow; $\alpha=65$, $\beta=0.5$, $\kappa=0.5$, $\xi=0.1$, $h=0.09$, $\theta=0.1$, $\phi=0.13$, $a=0.01$, $\eta=0.5$, $q=8$, $r=0.5$, $C=10$, $C_m=15$, $C_p=2$, $C_s=1.5$, $C_d=1.25$, then $p=18$, $T=1.47$, Net Total Profit= 1072.18

Example 6.5. (for special case)
In case of only price sensitive demand the numerical example are as follow; $\alpha=65$, $\beta=0.5$, $\kappa=0.5$, $\xi=0.1$, $h=0.09$, $\theta=0.1$, $\phi=0$, $a=0.01$, $\eta=0.5$, $q=8$ $r=0.5$, $C=10$ $C_m=15$, $C_p=2$, $C_s=1.5$, $C_d=1.25$,then $p=93.39$, $T=1.47$, Net Total Profit= 1642.66

In view of above numerical examples case second is more profitable in place of case first. In the second demand pattern manufacturer and retailer both are save the bulk revenue which are required initial investment on spacing, preservation investment and deterioration.

7. Sensitive Analysis

If the initial demand increases, the consumed order quantity, net profit and selling price of item (figure 1) increases sharply, thou time span (figure 2) decreased marginally. This reveals that initial demand boost the profit of manufacturer and as well as of retailer. If the stock dependent parameter ϕ increases then time horizon T reduces sharply (figure 3). This reveals that for highly stock dependent items keep the time horizon less as possible, and accordingly orders frequently. Moreover the net profit of manufacturer and retailer is sensitive on stock dependent parameter, for certain value of ϕ (0.01 to 0.025) the profit of both increases marginally, but thereafter $\phi\geq 0.025$, manufacturer's profit is constant and retailer's profit decreases sharply. Then, there exist an optimal value of ϕ that maximize the profit function (see figure 5)

Since β is a price sensitive parameter of demand function, and in this supply chain model, retailer may decide their items price for maximizing the total profit. On the basis of above statement for fixed $\beta=0.03$, $\alpha=10$, $\kappa=0.5$, $\xi=0.1$, $h=0.09$, $\theta=0.02$, $\phi=0.13$, $a=0.01$, $\eta=0.5$, $q=8$, $r=0.5$, $c=10$, $C_m=15$, $C_p=2$, $C_r=1.5$, $C_d=1.25$, then at the value of decision variables are $T=0.795$, $p=38$, NTP (Net total profit) = 320.10 (figure 4).

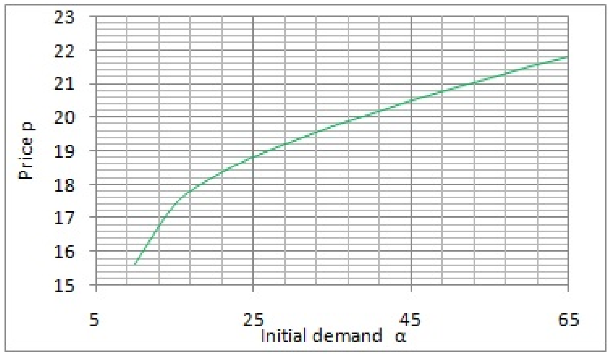

FIGURE 1. Effect of initial demand α on price p

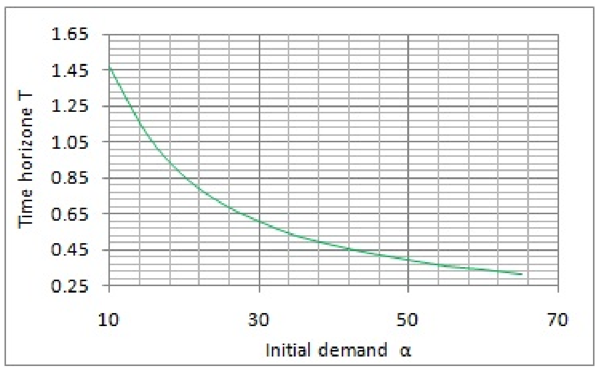

FIGURE 2. Effect of initial demand α on time horizon T

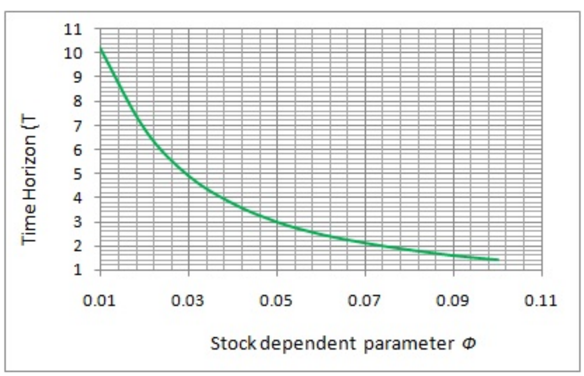

FIGURE 3. Effect of stock dependent parameter ϕ on time horizon T

Selling price of the item is highly sensitive on parameter β. Therefor those items which are highly price sensitive (for greater value of β) would be sustain in market with fluctuating price (figure 6). However high selling price does not guarantee to earn more profit (figure 4), for even business setup an optimal selling price exists for this profit is optimal for manufacturer as well as retailer. As per (figure 4) optimal selling price is $p = 38$ and for this sum of the manufacturer and retailer profit 320.10 is optimal. On increasing the parameter β, net profit NTP is

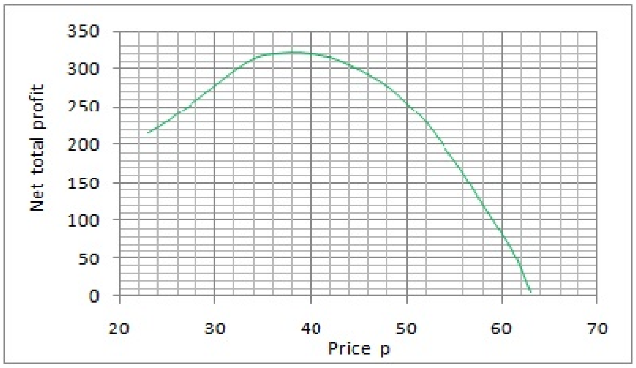

FIGURE 4. The effect of price p on NTP

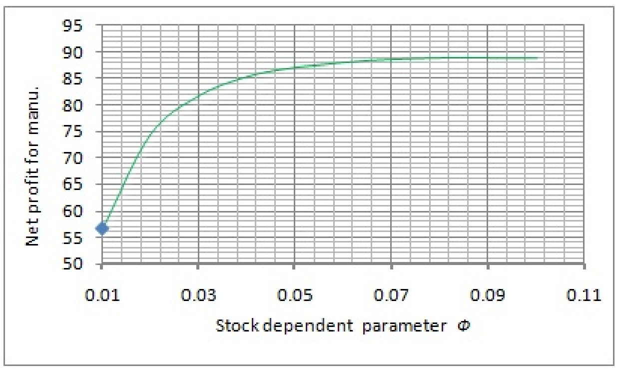

FIGURE 5. Effect of stock dependent parameter ϕ on AP_m

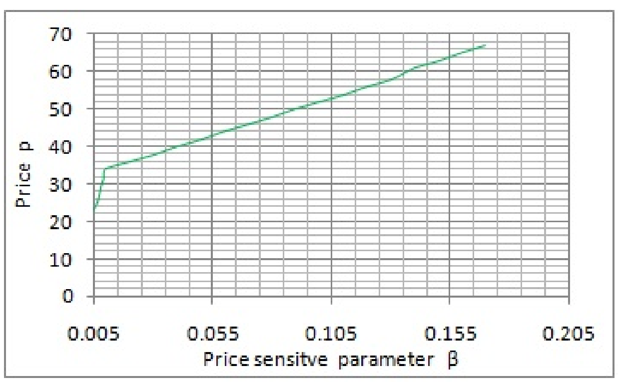

FIGURE 6. Effect of price sensitive parameter β on price p

also increases but after certain value of $\beta = 0.03$, NTP decreases.

Thus we have observed from (figure 6) selling price is linearly proportional to the parameter β and there exists an optimal price for highest profit. The above phenomenon follows for parameter β also. Hence there exists an optimal value of β that maximize the profit of manufacturer and retailer (figure 8). Above phenomenon is also applicable for time cycle, therefore there exists an optimal value of T

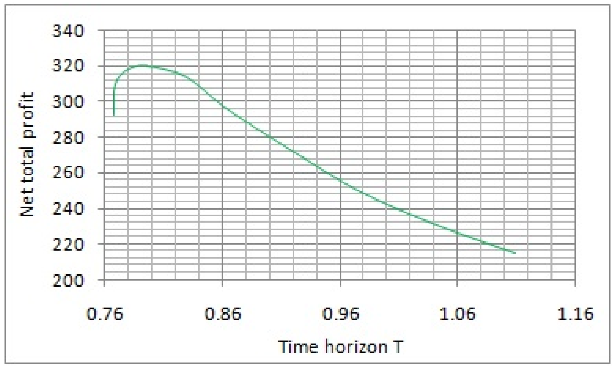

FIGURE 7. Effect of time horizon T on NTP

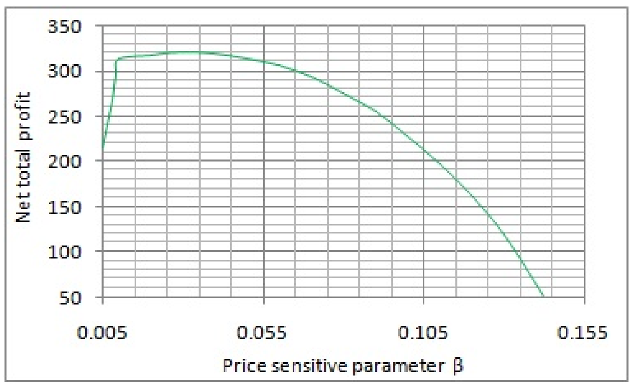

FIGURE 8. Effect of price sensitive parameter β on net total profit

(see figure 7) that maximize, the net total profit

In this research study it is found that when the production rate is time dependent linear function of t then the deterioration factor is more effective on the profit of manufacturer therefor if the deterioration rate is very low the profit of manufacturer is proportionally large and if the deterioration rate is high the profit of manufacturer is proportionally small i.e. the profit of manufacturer is inversely proportional to deteriorating rate.

8. CONCLUSION

The paper contains an inventory supply chain model for deteriorating seasonal items in which the deterioration rate can be controlled by investing on the preservation technology. By analysis, we have observed that for price sensitive and stock dependent demand pattern, deteriorating nature of products is more effective of the profit of manufacturer, therefore production management must provide to the retailer for a percentage of revenue sharing on preservation technology. We also observe that price dependent demand is more profitable than price and stock dependent demand. For a business setup we have found optimum time, price and time cycle to obtain maximum profit. It is advised to retailer to order in small lot size and small time cycle to obtain maximum profit, because the deterioration highly influence the model output. One can extend the model for multi supply chain and also for multi products. Also one can formulate the model in fuzzy environment.

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