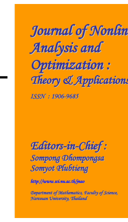

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GENERALIZED VARIATIONAL-LIKE INCLUSION PROBLEM INVOLVING $(H(.,.),\eta)$-MONOTONE OPERATORS IN BANACH SPACES

MOHD IQBAL BHAT[1] AND BISMA ZAHOOR[2]

[1]Department of Mathematics, South Campus, University of Kashmir, Anantnag-192101, India
[2]Department of Mathematics, University of Kashmir, Srinagar-190006, India

ABSTRACT. In this paper, we consider the generalized variational-like inclusion problem involving $(H(.,.),\eta)$-monotone operators in Banach spaces. Using proximal operator technique, we prove the existence of solution and suggest an iterative algorithm for solving the generalized variational-like inclusion problem. Also, we discuss the convergence analysis of the iterative algorithm. The results presented in this paper improve and generalize many known results in the literature.

KEYWORDS : $(H(.,.),\eta)$-monotone operator; Generalized η-proximal operator; Generalized variational-like inclusion problem; Iterative algorithm; Convergence analysis.
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1. PRELIMINARIES AND BASIC RESULTS

Throughout this paper unless or otherwise stated, X is a real Banach space with dual space $X^\star$, $\langle\cdot,\cdot\rangle$ is the dual pair between X and $X^\star$, 2^X denote the family of all the nonempty subsets of X. The normalized duality mapping $J : X \longrightarrow 2^{X^\star}$ is defined by

$$J(u) = \{f \in X^\star : \langle f, u\rangle = \|f\|\|u\|,\ \|f\| = \|u\|\},\ \forall u \in X.$$

A selection of the duality mapping J is a single-valued mapping $j : X \longrightarrow X^\star$ satisfying $j(u) \in J(u)$ for each $u \in X$.
Further, $J^\star : X^\star \longrightarrow X^{\star\star}$ be the normalized duality mapping on $X^\star$ defined by

$$J^\star(v) = \{f \in X^{\star\star} : \langle f, v\rangle = \|f\|\|v\|,\ \|f\| = \|v\|\},\ \forall v \in X^\star,$$

where $X^{\star\star}$ is a dual space of $X^\star$. Furthermore, $j^\star$ denotes a selection of $J^\star$
If $X \equiv \mathbf{H}$, a Hilbert space, then J and $J^\star$ are the identity mappings on $\mathbf{H}$.

[1]*Corresponding author.*
Email address : iqbal92@gmail.com.
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Let $CB(X)$ denotes the family of all nonempty closed and bounded subsets of X; $D(\cdot,\cdot)$ is the Hausdorff metric on $CB(X)$ defined by

$$D(A,B) = \max\Big\{\sup_{u\in A} d(u,B), \sup_{v\in B} d(A,v)\Big\},\ A,B \in CB(X).$$

The following concepts and results are needed in the sequel:

Lemma 1.1 (10). *Let X be a complete metric space, $T : X \longrightarrow CB(X)$ be a set-valued mapping. Then for any $\epsilon > 0$ and for any $u, v \in X$, $x \in T(u)$, there exists $y \in T(v)$ such that*

$$d(x,y) \le (1+\epsilon)D(T(u),T(v)),$$

where D is the Hausdorff metric on $CB(X)$.

Definition 1.2. Let $T : X \longrightarrow X^\star$; $A, B : X \longrightarrow X$, $N : X \times X \longrightarrow X, H : X \times X \longrightarrow X^\star$ and $\eta : X \times X \longrightarrow X$ be single-valued mappings. Then $\forall u, v, \cdot \in X$

(i) T is monotone, if

$$\Big\langle Tu - Tv, u - v\Big\rangle \ge 0.$$

(ii) T is strictly monotone, if

$$\Big\langle Tu - Tv, u - v\Big\rangle > 0,$$

and equality holds if and only if $u = v$.

(iii) T is α-strongly monotone, if there exists a constant $\alpha > 0$ such that

$$\Big\langle Tu - Tv, u - v\Big\rangle \ge \alpha\|u - v\|^2.$$

(iv) T is γ-Lipschitz continuous, if there exists a constant $\gamma > 0$ such that

$$\|Tu - Tv\| \ \le\ \gamma\|u - v\|.$$

(v) T is η-monotone, if

$$\Big\langle Tu - Tv, \eta(u,v)\Big\rangle \ge 0.$$

(vi) T is strictly η-monotone, if

$$\Big\langle Tu - Tv, \eta(u,v)\Big\rangle > 0,$$

and equality holds if and only if $u = v$.

(vii) A is said to be δ-strongly accretive, if there exists a constant $\delta > 0$ and $j(u - v) \in J(u - v)$ such that

$$\Big\langle Au - Av, j(u - v)\Big\rangle \ge \delta\|u - v\|^2,$$

where J is the normalized duality mapping.

(viii) $N(\cdot,\cdot)$ is l_1-Lipschitz continuous in the first argument, if there exists a constant $l_1 > 0$ such that

$$\|N(u,\cdot) - N(v,\cdot)\| \le l_1\|u - v\|.$$

(ix) $N(\cdot,\cdot)$ is l_2-Lipschitz continuous in the second argument, if there exists a constant $l_2 > 0$ such that

$$\|N(\cdot,u) - N(\cdot,v)\| \le l_2\|u - v\|.$$

(x) $H(A,\cdot)$ is α_1-strongly η-monotone with respect to A, if there exists a constant $\alpha_1 > 0$ such that

$$\Big\langle H(Au,\cdot) - H(Av,\cdot), \eta(u,v)\Big\rangle \ge \alpha_1\|u - v\|^2.$$

(xi) $H(\cdot, B)$ is α_2-relaxed η-monotone with respect to B, if there exists a constant $\alpha_2 > 0$ such that

$$\Big\langle H(\cdot, Bu) - H(\cdot, Bv), \eta(u, v)\Big\rangle \geq -\alpha_2\|u - v\|^2.$$

(xii) $H(\cdot, \cdot)$ is h_1-Lipschitz continuous with respect to A, if there exists a constant $h_1 > 0$ such that

$$\|H(Au, \cdot) - H(Av, \cdot)\| \;\leq\; h_1\|u - v\|.$$

(xiii) $H(\cdot, \cdot)$ is h_2-Lipschitz continuous with respect to B, if there exists a constant $h_2 > 0$ such that

$$\|H(\cdot, Bu) - H(\cdot, Bv)\| \;\leq\; h_2\|u - v\|.$$

(xiv) η is τ-Lipschitz continuous, if there exists a constant $\tau > 0$ such that

$$\|\eta(u, v)\| \;\leq\; \tau\|u - v\|.$$

Remark 1.3. If X is a Hilbert space, $\eta(u, v) = u - v, \forall u, v \in X$, then (x) and (xi) of Definition 1.2 reduces to (i) and (ii) of Definition 1.2, respectively in [12].

Definition 1.4. Let $M : X \longrightarrow 2^{X^\star}$ be a multi-valued mapping, $H : X \longrightarrow X^\star$ and $\eta : X \times X \longrightarrow X$ be single-valued mappings. Then:

(i) M is monotone, if

$$\langle x - y, u - v\rangle \geq 0,\ \forall\, u, v \in X,\ x \in M(u), y \in M(v).$$

(ii) M is η-monotone, if

$$\langle x - y, \eta(u, v)\rangle \geq 0,\ \forall\, u, v \in X,\ x \in M(u), y \in M(v).$$

(iii) M is strictly η-monotone, if

$$\langle x - y, \eta(u, v)\rangle > 0,\ \forall\, u, v \in X,\ x \in M(u), y \in M(v),$$

and equality holds if and only if $u = v$.

(iv) M is λ-strongly η-monotone, if there exists a constant $\lambda > 0$ such that

$$\langle x - y, \eta(u, v)\rangle \geq \lambda\|u - v\|^2,\ \forall\, u, v \in X,\ x \in M(u), y \in M(v).$$

(v) M is m-relaxed η-monotone, if there exists a constant $m > 0$ such that

$$\langle x - y, \eta(u, v)\rangle \geq -m\|u - v\|^2,\ \forall\, u, v \in X,\ x \in M(u), y \in M(v).$$

(vi) M is maximal monotone, if M is monotone and

$$(J + \lambda M)(X) = X^\star,\ \forall\, \lambda > 0,$$

where J is the normalized duality mapping.

(vii) M is maximal η-monotone, if M is η-monotone and

$$(J + \lambda M)(X) = X^\star,\ \forall\, \lambda > 0.$$

(viii) M is H-monotone, if M is monotone and

$$(H + \lambda M)(X) = X^\star,\ \forall\, \lambda > 0.$$

(ix) M is H-η-monotone, if M is m-relaxed η-monotone and $(H + \lambda M)(X) = X^\star,\ \forall\, \lambda > 0$.

Definition 1.5. For all $u, v, \cdot \in X$, a mapping $F : X \times X \times X \longrightarrow X^\star$ is said to be ϵ_1-Lipschitz continuous with respect to first argument, if there exists a constant $\epsilon_1 > 0$ such that

$$\|F(u, \cdot\,, \cdot) - F(v, \cdot\,, \cdot)\| \leq \epsilon_1\|u - v\|.$$

Similarly, we can define Lipschitz continuity of F in other arguments.

Lemma 1.6 (11). *Let X be a real Banach space and $J : X \longrightarrow 2^{X^\star}$ be the normalized duality mapping. Then, for all $u, v \in X$,*

$$\|u+v\|^2 \leq \|u\|^2 + 2\Big\langle v, j(u+v)\Big\rangle,\ \forall j(u+v) \in J(u+v).$$

2. $(H(.,.),\eta)$-Monotone Operator and Formulation of the Problem

Definition 2.1. Let X be a Banach space with the dual space $X^\star$. Let $H : X \times X \longrightarrow X^\star$, $\eta : X \times X \longrightarrow X,\ A, B : X \longrightarrow X$ be single-valued mappings. Then the set-valued mapping $M : X \longrightarrow 2^{X^\star}$ is said to be $(H(.,.),\eta)$-monotone with respect to A and B, if M is m-relaxed-η-monotone and $(H(A,B)+\rho M)(X) = X^\star,\ \forall \rho > 0$.

Remark 2.2. (i) If $H(Au, Bu) = Hu,\ \forall u \in X$, then Definition 2.1 reduces to the definition of H-η-monotone operators considered in [8]. It follows that this class of operators in Banach spaces provides a unifying framework for the class of η-subdifferential operators, maximal monotone operators, maximal η-monotone operators, H-monotone operators, (H,η)-monotone operators, G-η-monotone operators, A-monotone operators, A-η-monotone operators in Hilbert spaces and H-monotone operators, H-η-monotone operators, A-monotone operators in Banach spaces . We remark that $(H(.,.),\eta)$-monotone operator in Banach spaces acts from X to $X^\star$.
(ii) If $X \equiv \mathbf{H}$, a Hilbert space, $m = 0$ and $\eta(u,v) = u-v,\ \forall u, v \in \mathbf{H}$, then Definition 2.1 reduces to M-monotone operator studied in [12].

Now we give some properties of $(H(.,.),\eta)$-monotone operator.

Theorem 2.3. *Let $A, B : X \longrightarrow X,\ \eta : X \times X \longrightarrow X$, and $H : X \times X \longrightarrow X^\star$ be single-valued mappings and $H(A,B)$ be α-strongly η-monotone with respect to A, β-relaxed η-monotone with respect to B and $\alpha > \beta$. Let $M : X \longrightarrow 2^{X^\star}$ be $(H(.,.),\eta)$-monotone operator with respect to A and B. If $\Big\langle x-y, \eta(u,v)\Big\rangle \geq 0,\ \forall\ (v,y) \in Graph\ (M)$, then $(u,x) \in Graph(M)$, where $Graph\ (M) = \Big\{(a,b) \in X \times X : b \in M(a)\Big\}$.*

Theorem 2.4. *Let $A, B : X \longrightarrow X,\ \eta : X \times X \longrightarrow X$ and $H : X \times X \longrightarrow X^\star$ be single-valued mappings and $H(A,B)$ be α-strongly η-monotone with respect to A, β-relaxed η-monotone with respect to B and $\alpha > \beta$. Let $M : X \longrightarrow 2^{X^\star}$ be $(H(.,.),\eta)$-monotone operator with respect to A and B. Then $(H(A,B)+\rho M)^{-1}$ is a single-valued mapping for $0 < \rho < \dfrac{\alpha-\beta}{m}$.*

Based on Theorem 2.4, we define the generalized η-proximal operator associated with $(H(A,B),\eta)$-monotone operator as under:

Definition 2.5. Let $A, B : X \longrightarrow X,\ \eta : X \times X \longrightarrow X$ and $H : X \times X \longrightarrow X^\star$ be single-valued mappings and $H(A,B)$ be α-strongly η-monotone with respect to A, β-relaxed η-monotone with respect to B and $\alpha > \beta$. Let $M : X \times X \longrightarrow 2^{X^\star}$ be $(H(.,.),\eta)$-monotone operator with respect to A and B. Then the generalized η-proximal operator $J^{H(.,.),\eta}_{M(\cdot,z),\rho} : X \longrightarrow X$ for fixed $z \in X$ is defined by

$$J^{H(.,.),\eta}_{M(\cdot,z),\rho}(u) = \Big(H(A,B) + \rho M(\cdot,z)\Big)^{-1}(u),\ \forall u \in X.$$

Remark 2.6. The generalized η-proximal operator associated with $(H(.,.),\eta)$-monotone operator include as special cases the corresponding proximal operators associated with maximal monotone operators, η-subdifferential operators, maximal η-monotone operators, H-monotone operators, (H,η)-monotone operators, G-η-monotone operators, A-monotone operators, A-η-monotone operators.

One of the important properties of generalized η-proximal operator is its Lipschitz continuity which is as under:

Theorem 2.7. *Let $A, B : X \longrightarrow X$, $\eta : X \times X \longrightarrow X$ and $H : X \times X \longrightarrow X^\star$ be single-valued mappings and $H(A,B)$ be α-strongly η-monotone with respect to A, β-relaxed η-monotone with respect to B and $\alpha > \beta$. Let $M : X \times X \longrightarrow 2^{X^\star}$ be $(H(.,.),\eta)$-monotone operator with respect to A and B. Then the generalized η-proximal operator $J^{H(.,.),\eta}_{M(\cdot,z),\rho} : X \longrightarrow X$ for fixed $z \in X$ is k-Lipschitz continuous, where $k = \dfrac{\tau}{\alpha-\beta-m\rho}$, that is*

$$\|J^{H(.,.),\eta}_{M(\cdot,z),\rho}(u) - J^{H(.,.),\eta}_{M(\cdot,z),\rho}(v)\| \leq k\|u-v\|,\ \forall u,v \in X.$$

Now we formulate our main problem:

Let X be a real Banach space. Let $S,T,G : X \longrightarrow CB(X)$ be set-valued mappings, $N,H : X\times X \longrightarrow X^\star$, $\eta : X\times X \longrightarrow X, F : X\times X\times X \longrightarrow X^\star$ and $A,B,p,g : X \longrightarrow X$ be single-valued mappings. Let $M : X \times X \longrightarrow 2^{X^\star}$ be set-valued mapping such that for fixed $z \in G(X), M(.,z) : X \times X \longrightarrow 2^{X^\star}$ is an $(H(.,.),\eta)$-monotone operator with respect to A and B and $Range(g-p) \cap \mathrm{dom}(M(.,z)) \neq \emptyset$. For any given $f \in X^\star$, we consider the following generalized variational-like inclusion problem (in short, GVLIP): Find $u \in X,\ x \in S(u), y \in T(u), z \in G(u)$ such that

$$\theta^\star \in N\Big(g(x),A(y)\Big) + F(u,u,z) + M\Big((g-p)(u),z\Big) + f, \tag{2.1}$$

where $\theta^\star$ is the zero element in $X^\star$.

We remark that if $g - p \equiv I$ and $f \equiv 0$, then GVLIP (2.1) reduces to a variational inclusion of finding $u \in X$, $x \in S(u), y \in T(u), z \in G(u)$ such that

$$\theta^\star \in N\Big(g(x),A(y)\Big) + F(u,u,z) + M(u,z). \tag{2.2}$$

Variational inclusion (2.2) is an important generalization of variational inclusions considered by many researchers including [12,15]. For applications of such variational inclusions, see [7,8].

If $F = p = f \equiv 0$, $g \equiv I$ and $X \equiv \mathbf{H}$, a Hilbert space, then GVLIP (2.1) reduces to a generalized mixed quasi-variational-like inclusion involving $(H(\cdot,\cdot),\eta)$-monotone operators in a Hilbert space: Find $u \in \mathbf{H}$, $x \in S(u), y \in T(u), z \in G(u)$ such that

$$\theta^\star \in N\Big(x,A(y)\Big) + M(u,z). \tag{2.3}$$

Variational inclusion (2.3) is an important generalization of variational inclusions considered by Kazmi and Bhat [4,5].

We remark that for the suitable choice of mappings $A,B,S,T,G,N,H,F,M,\eta,g,p$ and the underlying space X, GVLIP (2.1) reduces to different classes of new and already known systems of variational inclusions/inequalities considered by many researchers including [6,9,13,15,18] and the related references cited therein.

3. Existence of Solution, Iterative Algorithm and Convergence Analysis

First, we give the following technical result:

Lemma 3.1. *Let $X, A, B, S, T, G, N, H, F, M, \eta, g, p$ be same as in GVLIP (2.1). Then (u, x, y, z) where $x \in S(u), y \in T(u), z \in G(u)$ is the solution of GVLIP (2.1) if and only if*

$$(g-p)(u) = J^{H(.,.),\eta}_{M(.,z),\rho}\Big[H\Big(A((g-p)(u)), B((g-p)(u))\Big) - \rho\Big\{N\Big(g(x), A(y)\Big) + F(u,u,z) + f\Big\}\Big],$$

and $J^{H(.,.),\eta}_{M(.,z),\rho}(u) = \Big(H(A,B) + \rho M(.,z)\Big)^{-1}(u)$ is the generalized η-proximal operator and $\rho > 0$ is a constant.

The above result along with Nadler's Theorem (Lemma 1.1) allow us to suggest the following iterative algorithm for solving GVLIP (2.1).

Iterative Algorithm 3.2. *For any arbitrary chosen $u_0 \in X,\ x_0 \in S(u_0),\ y_0 \in T(u_0)$ and $z_0 \in G(u_0)$, compute the sequences $\{u_n\}, \{x_n\}, \{y_n\}$ and $\{z_n\}$ by the iterative schemes such that*

$$\begin{aligned}
(g-p)(u_{n+1}) &= J^{H(.,.),\eta}_{M(.,z_n),\rho}\Big[H\Big(A((g-p)(u_n)), B((g-p)(u_n))\Big) \\
&\quad -\rho\Big\{N\Big(g(x_n), A(y_n)\Big) + F(u_n, u_n, z_n) + f\Big\}\Big], \\
x_n \in S(u_n) : &\quad \|x_{n+1} - x_n\| \le \Big(1 + (1+n)^{-1}\Big)D\Big(S(u_{n+1}), S(u_n)\Big); \\
y_n \in T(u_n) : &\quad \|y_{n+1} - y_n\| \le \Big(1 + (1+n)^{-1}\Big)D\Big(T(u_{n+1}), T(u_n)\Big); \\
z_n \in G(u_n) : &\quad \|z_{n+1} - z_n\| \le \Big(1 + (1+n)^{-1}\Big)D\Big(G(u_{n+1}), G(u_n)\Big).
\end{aligned}$$

for all $n = 0, 1, 2, \cdots$.

If $\rho = 1,\ g - p \equiv I$ and $f \equiv 0$, then the Iterative Algorithm 3.2 reduces to the following iterative algorithm.

Iterative Algorithm 3.3. *For any arbitrary chosen $u_0 \in X,\ x_0 \in S(u_0),\ y_0 \in T(u_0)$ and $z_0 \in G(u_0)$, compute the sequences $\{u_n\}, \{x_n\}, \{y_n\}$ and $\{z_n\}$ by the iterative schemes such that*

$$u_{n+1} = J^{H(.,.),\eta}_{M(.,z_n)}\Big[H\Big(A(u_n), B(u_n)\Big) - \Big\{N\Big(g(x_n), A(y_n)\Big) + F(u_n, u_n, z_n)\Big\}\Big],$$

$$\begin{aligned}
x_n \in S(u_n) : &\quad \|x_{n+1} - x_n\| \le \Big(1 + (1+n)^{-1}\Big)D\Big(S(u_{n+1}), S(u_n)\Big); \\
y_n \in T(u_n) : &\quad \|y_{n+1} - y_n\| \le \Big(1 + (1+n)^{-1}\Big)D\Big(T(u_{n+1}), T(u_n)\Big); \\
z_n \in G(u_n) : &\quad \|z_{n+1} - z_n\| \le \Big(1 + (1+n)^{-1}\Big)D\Big(G(u_{n+1}), G(u_n)\Big).
\end{aligned}$$

We remark that Iterative Algorithm 3.3 gives the approximate solution to the variational inclusion (2.2).

Now, we prove the following theorem which ensures the convergence of iterative sequences generated by the Iterative Algorithm 3.2 to the solution of GVLIP 2.1.

Theorem 3.4. *Let X be a real Banach space. Let $S, T, G : X \longrightarrow CB(X)$ be α_1-D-Lipschitz, α_2-D-Lipschitz, α_3-D-Lipschitz continuous mappings, respectively. Let $N : X \times X \longrightarrow X^\star$ be l_1-Lipschitz continuous and l_2-Lipschitz continuous with* *respect to first and second arguments, respectively.* $\eta : X \times X \longrightarrow X$ *be* τ*-Lipschitz continuous,* $F : X \times X \times X \longrightarrow X^\star$ *be* β_j*-Lipschitz continuous with respect to* j*th argument, for* $j = 1, 2, 3$ *and* $A, B, p, g : X \longrightarrow X$ *be single-valued mappings such that* g *is* r_1*-Lipschitz continuous,* A *is* r_2*-Lipschitz continuous,* $(g - p)$ *is* s*-Lipschitz continuous and* $(g - p - I)$ *is* λ*-strongly accretive. Let* $H : X \times X \longrightarrow X^\star$ *be* α*-strongly* η*-monotone with respect to* A*,* β*-relaxed* η*-monotone with respect to* B *and* h_1*-Lipschitz continuous and* h_2*-Lipschitz continuous with respect to* A *and* B*, respectively. Let* $M : X \times X \longrightarrow 2^{X^\star}$ *be set-valued mapping such that for fixed* $z \in G(X), M(., z) : X \times X \longrightarrow 2^{X^\star}$ *be* $(H(., .), \eta)$*-monotone operator with respect to* A *and* B *and* $Range(g - p) \cap dom(M(., z)) \neq \emptyset$*. In addition, suppose there exists a constant* $\sigma > 0$ *such that*

$$\left\| J^{H(.,.),\eta}_{M(.,z_{n+1}),\rho}(u) - J^{H(.,.),\eta}_{M(.,z),\rho}(u) \right\| \leq \sigma \parallel z_{n+1} - z_n \parallel . \tag{3.1}$$

Furthermore, suppose the following condition is satisfied

$$0 < Q < 1,$$

where Q *is given by,*

$$\begin{aligned} Q &= \frac{1}{\sqrt{1+2\lambda}} \Big\{ k \Big[\sqrt{s^2 h_1^2 - 2\rho l_1 r_1 \alpha_1 (s h_1 + \rho l_1 r_1 \alpha_1)} \\ &\quad + \sqrt{s^2 h_2^2 - 2\rho l_2 r_2 \alpha_2 (s h_2 + \rho l_2 r_2 \alpha_2)} + \rho(\beta_1 + \beta_2 + \beta_3 \alpha_3) \Big] + \sigma \alpha_3 \Big\}, \end{aligned} \tag{3.2}$$

then the sequences $\{u_n\}, \{x_n\}, \{y_n\}$ *and* $\{z_n\}$*, generated by the Iterative Algorithm* 3.2 *converge strongly to the unique solution* (u, x, y, z)*, respectively, where* $u \in X,\ x \in S(u), y \in T(u)$ *and* $z \in G(u)$ *is the solution of GVLIP (2.1).*

Proof. From Iterative Algorithm 3.2 and Lemma 2.7, we have

$$\begin{aligned} &\|(g-p)u_{n+2} - (g-p)u_{n+1}\| \\ &= \Big\| J^{H(.,.),\eta}_{M(.,z_{n+1}),\rho} \Big[H\Big(A((g-p)(u_{n+1})), B((g-p)(u_{n+1})) \Big) \\ &\quad - \rho \Big\{ N\Big(g(x_{n+1}), A(y_{n+1}) \Big) + F(u_{n+1}, u_{n+1}, z_{n+1}) + f \Big\} \Big] \\ &\quad - J^{H(.,.),\eta}_{M(.,z_n),\rho} \Big[H\Big(A((g-p)(u_n)), B((g-p)(u_n)) \Big) \\ &\quad - \rho \Big\{ N\Big(g(x_n), A(y_n) \Big) + F(u_n, u_n, z_n) + f \Big\} \Big] \Big\| \\ &\leq \Big\| J^{H(.,.),\eta}_{M(.,z_{n+1}),\rho} \Big[H\Big(A((g-p)(u_{n+1})), B((g-p)(u_{n+1})) \Big) \\ &\quad - \rho \Big\{ N\Big(g(x_{n+1}), A(y_{n+1}) \Big) + F(u_{n+1}, u_{n+1}, z_{n+1}) + f \Big\} \Big] \\ &\quad - J^{H(.,.),\eta}_{M(.,z_{n+1}),\rho} \Big[H\Big(A((g-p)(u_n)), B((g-p)(u_n)) \Big) \\ &\quad - \rho \Big\{ N\Big(g(x_n), A(y_n) \Big) + F(u_n, u_n, z_n) + f \Big\} \Big] \Big\| \\ &\quad + \Big\| J^{H(.,.),\eta}_{M(.,z_{n+1}),\rho} \Big[H\Big(A((g-p)(u_n)), B((g-p)(u_n)) \Big) \end{aligned}$$

$$
\begin{aligned}
& -\rho\Big\{N\Big(g(x_n),A(y_n)\Big)+F(u_n,u_n,z_n)+f\Big\}\Big] \\
& -J^{H(.,.),\eta}_{M(.,z_n),\rho}\Big[H\Big(A((g-p)(u_n)),B((g-p)(u_n))\Big) \\
& -\rho\Big\{N\Big(g(x_n),A(y_n)\Big)+F(u_n,u_n,z_n)+f\Big\}\Big]\Big\| \\
\leq\ & k\Big\|H\Big(A((g-p)(u_{n+1})),B((g-p)(u_{n+1}))\Big) \\
& -\rho\Big\{N\Big(g(x_{n+1}),A(y_{n+1})\Big)+F(u_{n+1},u_{n+1},z_{n+1})\Big\} \\
& -\Big[H\Big(A((g-p)(u_n)),B((g-p)(u_n))\Big) \\
& -\rho\Big\{N\Big(g(x_n),A(y_n)\Big)+F(u_n,u_n,z_n)\Big\}\Big]\Big\|+\sigma\Big\|z_{n+1}-z_n\Big\| \\
\leq\ & k\Big[\Big\|H\Big(A((g-p)(u_{n+1})),B((g-p)(u_{n+1}))\Big) \\
& -H\Big(A((g-p)(u_n)),B((g-p)(u_{n+1}))\Big)-\rho\Big\{N\Big(g(x_{n+1}),A(y_{n+1})\Big) \\
& -N\Big(g(x_n),A(y_{n+1})\Big)\Big\}\Big\|+\Big\|H\Big(A((g-p)(u_n)),B((g-p)(u_{n+1}))\Big) \\
& -H\Big(A((g-p)(u_n)),B((g-p)(u_n))\Big)-\rho\Big\{N\Big(g(x_n),A(y_{n+1})\Big) \\
& -N\Big(g(x_n),A(y_n)\Big)\Big\}\Big\|+\rho\Big\|F(u_{n+1},u_{n+1},z_{n+1})-F(u_n,u_n,z_n)\Big\|\Big] \\
& +\sigma\Big\|z_{n+1}-z_n\Big\|. \qquad (3.3)
\end{aligned}
$$

Since $(g-p)$ is s-Lipschitz continuous, $H(.,.)$ is h_1-Lipschitz continuous with respect to A, $N(.,.)$ is l_1-Lipschitz continuous with respect to first argument and from Lemma 1.6, we have

$$
\begin{aligned}
& \Big\|H\Big(A((g-p)(u_{n+1})),B((g-p)(u_{n+1}))\Big)-H\Big(A((g-p)(u_n)),B((g-p)(u_{n+1}))\Big) \\
& \qquad -\rho\left\{N\Big(g(x_{n+1}),A(y_{n+1})\Big)-N\Big(g(x_n),A(y_{n+1})\Big)\right\}\Big\|^2 \\
\leq\ & \Big\|H\Big(A((g-p)(u_{n+1})),B((g-p)(u_{n+1}))\Big) \\
& -H\Big(A((g-p)(u_n)),B((g-p)(u_{n+1}))\Big)\Big\|^2-2\rho\Big\langle N\Big(g(x_{n+1}),A(y_{n+1})\Big) \\
& -N\Big(g(x_n),A(y_{n+1})\Big),j^\star\Big(H\Big(A((g-p)(u_{n+1})),B((g-p)(u_{n+1}))\Big) \\
& -H\Big(A((g-p)(u_n)),B((g-p)(u_{n+1}))\Big)-\rho\Big\{N\Big(g(x_{n+1}),A(y_{n+1})\Big)
\end{aligned}
$$

$$
\begin{aligned}
& -N\Big(g(x_n), A(y_{n+1})\Big)\Big\}\Big)\Big\rangle \\
\leq\ & \Big\|H\Big(A((g-p)(u_{n+1})), B((g-p)(u_{n+1}))\Big) \\
& -H\Big(A((g-p)(u_n)), B((g-p)(u_{n+1}))\Big)\Big\|^2 \\
& -2\rho\Big\|N\Big(g(x_{n+1}), A(y_{n+1})\Big) - N\Big(g(x_n), A(y_{n+1})\Big)\Big\| \\
& \times\Big[\Big\|H\Big(A((g-p)(u_{n+1})), B((g-p)(u_{n+1}))\Big) \\
& -H\Big(A((g-p)(u_n)), B((g-p)(u_{n+1}))\Big)\Big\| \\
& +\rho\Big\|N\Big(g(x_{n+1}), A(y_{n+1})\Big) - N\Big(g(x_n), A(y_{n+1})\Big)\Big\|\Big] \\
\leq\ & s^2h_1^2\|u_{n+1}-u_n\|^2 - 2\rho l_1\|g(x_{n+1}) - g(x_n)\| \\
& \times\Big[sh_1\|u_{n+1}-u_n\| + \rho l_1\|g(x_{n+1}) - g(x_n)\|\Big]. \qquad (3.4)
\end{aligned}
$$

Since g is r_1-Lipschitz continuous and S is α_1-D-Lipschitz continuous, we have

$$
\begin{aligned}
& \Big\|N\Big(g(x_{n+1}), A(y_{n+1})\Big) - N\Big(g(x_n), A(y_{n+1})\Big)\Big\| \\
& \leq\ l_1\|g(x_{n+1}) - g(x_n)\| \\
& \leq\ l_1r_1\|x_{n+1} - x_n\| \\
& \leq\ l_1r_1\alpha_1\Big(1 + (1+n)^{-1}\Big)\|u_{n+1} - u_n\|. \qquad (3.5)
\end{aligned}
$$

Using (3.5) in (3.4), we have

$$
\begin{aligned}
& \Big\|H\Big(A((g-p)(u_{n+1})), B((g-p)(u_{n+1}))\Big) - H\Big(A((g-p)(u_n)), B((g-p)(u_{n+1}))\Big) \\
& \qquad -\rho\Big\{N\Big(g(x_{n+1}), A(y_{n+1})\Big) - N\Big(g(x_n), A(y_{n+1})\Big)\Big\}\Big\| \\
\leq\ & \sqrt{s^2h_1^2 - 2\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big) \times \Big[sh_1 + \rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\Big]} \\
& \times\|u_{n+1} - u_n\|. \qquad (3.6)
\end{aligned}
$$

Similarly, using h_2-Lipschitz continuity of $H(.,.)$ with respect to B, l_2-Lipschitz continuity of $N(\cdot,\cdot)$ with respect to second argument, s-Lipschitz continuity of $(g-p)$ and Lemma 1.6, we have the following estimate:

$$
\begin{aligned}
& \Big\|H\Big(A((g-p)(u_n)), B((g-p)(u_{n+1}))\Big) - H\Big(A((g-p)(u_n)), B((g-p)(u_n))\Big) \\
& \qquad -\rho\Big\{N\Big(g(x_n), A(y_{n+1})\Big) - N\Big(g(x_n), A(y_n)\Big)\Big\}\Big\|^2 \\
& \leq\ s^2h_2^2\|u_{n+1} - u_n\|^2 - 2\rho l_2\|A(y_{n+1}) - A(y_n)\|
\end{aligned}
$$

$$\times\Big[sh_2\|u_{n+1}-u_n\|+\rho l_2\|A(y_{n+1})-A(y_n)\|\Big]. \tag{3.7}$$

Since A is r_2-Lipschitz continuous and T is α_2-D-Lipschitz continuous, we have

$$\begin{aligned}\Big\|A(y_{n+1})-A(y_n)\Big\| &\leq r_2\Big\|y_{n+1}-y_n\Big\| \\ &\leq r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\|u_{n+1}-u_n\|.\end{aligned} \tag{3.8}$$

Using (3.8) in (3.7), we have

$$\begin{aligned}&\Big\|H\Big(A((g-p)(u_n)),B((g-p)(u_{n+1}))\Big)-H\Big(A((g-p)(u_n)),B((g-p)(u_n))\Big) \\ &\qquad -\rho\left\{N\Big(g(x_n),A(y_{n+1})\Big)-N\Big(g(x_n),A(y_n)\Big)\right\}\Big\| \\ &\leq \sqrt{s^2h_2^2-2\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\times\Big[sh_2+\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\Big]} \\ &\quad\times\|u_{n+1}-u_n\|.\end{aligned} \tag{3.9}$$

Since $F(.,.,.)$ is β_j-Lipschitz continuous in the jth argument, for $j=1,2,3$, G is α_3-D-Lipschitz continuous and using Iterative Algorithm 3.2, we have

$$\begin{aligned}&\Big\|F(u_{n+1},u_{n+1},z_{n+1})-F(u_n,u_n,z_n)\Big\| \\ &\leq \Big\|F(u_{n+1},u_{n+1},z_{n+1})-F(u_n,u_{n+1},z_{n+1})\Big\| \\ &\quad+\Big\|F(u_n,u_{n+1},z_{n+1})-F(u_n,u_n,z_{n+1})\Big\| \\ &\quad+\Big\|F(u_n,u_n,z_{n+1})-F(u_n,u_n,z_n)\Big\| \\ &\leq \beta_1\Big\|u_{n+1}-u_n\Big\|+\beta_2\Big\|u_{n+1}-u_n\Big\|+\beta_3\Big\|z_{n+1}-z_n\Big\| \\ &\leq \Big[\beta_1+\beta_2+\beta_3\alpha_3\Big(1+(1+n)^{-1}\Big)\Big]\Big\|u_{n+1}-u_n\Big\|.\end{aligned} \tag{3.10}$$

Using (3.6),(3.9) and (3.10) in (3.3), we have

$$\begin{aligned}&\|(g-p)u_{n+2}-(g-p)u_{n+1}\| \\ &\leq \Bigg\{k\Bigg[\sqrt{s^2h_1^2-2\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\times\Big[sh_1+\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\Big]} \\ &\quad+\sqrt{s^2h_2^2-2\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\times\Big[sh_2+\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\Big]} \\ &\quad+\rho\Big\{\beta_1+\beta_2+\beta_3\alpha_3\Big(1+(1+n)^{-1}\Big)\Big\}\Bigg] \\ &\quad+\sigma\alpha_3\Big(1+(1+n)^{-1}\Big)\Bigg\}\ \Big\|u_{n+1}-u_n\Big\|.\end{aligned} \tag{3.11}$$

Since $(g-p-I)$ is λ-strongly accretive, by Lemma 1.6 and (3.11), we have the following estimate:

$$\begin{aligned}\|u_{n+2}-u_{n+1}\|^2 &\le \Big\|(g-p)u_{n+2}-(g-p)u_{n+1}+u_{n+2}-u_{n+1}\\ &\quad -\Big((g-p)u_{n+2}-(g-p)u_{n+1}\Big)\Big\|^2\\ &\le \|(g-p)u_{n+2}-(g-p)u_{n+1}\|^2\\ &\quad -2\Big\langle (g-p-I)u_{n+2}-(g-p-I)u_{n+1}, j(u_{n+2}-u_{n+1})\Big\rangle\\ &\le \|(g-p)u_{n+2}-(g-p)u_{n+1}\|^2-2\lambda\|u_{n+2}-u_{n+1})\|^2.\end{aligned}$$

Hence,

$$\begin{aligned}&\|u_{n+2}-u_{n+1}\|\\ &\le \frac{1}{\sqrt{1+2\lambda}}\Bigg\{k\Bigg[\sqrt{s^2h_1^2-2\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\times\Big[sh_1+\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\Big]}\\ &\quad+\sqrt{s^2h_2^2-2\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\times\Big[sh_2+\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\Big]}\\ &\quad+\rho\Big\{\beta_1+\beta_2+\beta_3\alpha_3\Big(1+(1+n)^{-1}\Big)\Big\}\Bigg]+\sigma\alpha_3\Big(1+(1+n)^{-1}\Big)\Bigg\}\times\ \|u_{n+1}-u_n\|\\ &\le \phi_{n+1}\ \|u_{n+1}-u_n\|, \qquad (3.12)\end{aligned}$$

where

$$\begin{aligned}&\phi_{n+1}\\ &= \frac{1}{\sqrt{1+2\lambda}}\Bigg\{k\Bigg[\sqrt{s^2h_1^2-2\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\times\Big[sh_1+\rho l_1r_1\alpha_1\Big(1+(1+n)^{-1}\Big)\Big]}\\ &\quad+\sqrt{s^2h_2^2-2\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\times\Big[sh_2+\rho l_2r_2\alpha_2\Big(1+(1+n)^{-1}\Big)\Big]}\\ &\quad+\rho\Big\{\beta_1+\beta_2+\beta_3\alpha_3\Big(1+(1+n)^{-1}\Big)\Big\}\Bigg]+\sigma\alpha_3\Big(1+(1+n)^{-1}\Big)\Bigg\}.\end{aligned}$$

Let

$$\begin{aligned}\phi &= \frac{1}{\sqrt{1+2\lambda}}\Bigg\{k\Bigg[\sqrt{s^2h_1^2-2\rho l_1r_1\alpha_1\Big(sh_1+\rho l_1r_1\alpha_1\Big)}\\ &\quad+\sqrt{s^2h_2^2-2\rho l_2r_2\alpha_2\Big(sh_2+\rho l_2r_2\alpha_2\Big)}+\rho\Big(\beta_1+\beta_2+\beta_3\alpha_3\Big)\Bigg]+\sigma\alpha_3\Bigg\}.\end{aligned}$$

Then we know that $\phi_n \longrightarrow \phi$ as $n \longrightarrow \infty$.
By condition (3.2), we know that $\phi \in (0,1)$ and hence there exist $n_0 > 0$ and $\phi_0 \in (0,1)$ such that $\phi_{n+1} \le \phi_0$ for all $n \ge n_0$. Therefore by (3.12), we have

$$\|u_{n+2}-u_{n+1}\| \le \phi_0\|u_{n+1}-u_n\|,\ \forall\, n \ge n_0.$$

This implies

$$\|u_{n+1}-u_n\| \le \phi_0^{n-n_0}\|u_{n_0+1}-u_{n_0}\|.$$

Hence, for any $m \geq n > n_0$, we have

$$\begin{aligned}\|u_m - u_n\| &\leq \sum_{t=n}^{m-1} \|u_{t+1} - u_t\| \\ &\leq \sum_{t=n}^{m-1} \phi_0^{t-n_0} \|u_{n_0+1} - u_{n_0}\|.\end{aligned}$$

It follows $\|u_m - u_n\| \longrightarrow 0$ as $n \longrightarrow \infty$ so that $\{u_n\}$ is a Cauchy sequence in X. Then there exists $u \in X$ such that $u_n \longrightarrow u$ as $n \longrightarrow \infty$.

Now from α_1-D-Lipschitz continuity of S and Iterative Algorithm 3.2, we have

$$\begin{aligned}\|x_{n+1} - x_n\| &\leq \Big(1 + (1+n)^{-1}\Big) D\Big(S(u_{n+1}), S(u_n)\Big) \\ &\leq \Big(1 + (1+n)^{-1}\Big) \alpha_1 \|u_{n+1} - u_n\|. \end{aligned} \tag{3.13}$$

Since $\{u_n\}$ being Cauchy in X, (3.13) implies that $\{x_n\}$ is a Cauchy sequence in X. Thus, in general, there exist x, y, z in X such that $x_n \longrightarrow x$, $y_n \longrightarrow y$, $z_n \longrightarrow z$ as $n \longrightarrow \infty$.

Now, we show that $x \in S(u)$. Since $x_n \in S(u_n)$, we have

$$\begin{aligned}d(x, S(u)) &\leq \|x - x_n\| + d(x_n, S(u)) \\ &\leq \|x - x_n\| + D\Big(S(u_n), S(u)\Big) \\ &\leq \|x - x_n\| + \alpha_1 \|u_n - u\| \longrightarrow 0 \text{ as } n \longrightarrow \infty.\end{aligned}$$

Since $S(u)$ is closed, it implies that $x \in S(u)$. Similarly, we can show that $y \in T(u)$, $z \in G(u)$. By assumption (3.1), Lipschitz continuity of proximal mapping $J^{H(.,.),\eta}_{M(.,z),\rho}$, continuity of the respective mappings and Iterative Algorithm 3.2, it follows that $u \in X$, $x \in S(u)$, $y \in T(u)$, $z \in G(u)$, where $J^{H(.,.),\eta}_{M(.,z),\rho}(u) = \Big(H(A,B) + \rho M(.,z)\Big)^{-1}(u)$ and ρ are constants. By Lemma 3.1, (u, x, y, z) is the solution of GVLIP 2.1. This completes the proof. □

Finally, we give the following result which gives the convergence of the sequences generated by the Iterative Algorithm 3.3 to the solution of Problem 2.2.

Theorem 3.5. *Let X be a real Banach space. Let $S, T, G : X \longrightarrow CB(X)$ be α_1-D-Lipschitz, α_2-D-Lipschitz, α_3-D-Lipschitz continuous mappings, respectively. Let $N : X \times X \longrightarrow X^\star$ be l_1-Lipschitz continuous and l_2-Lipschitz continuous with respect to first and second arguments, respectively. $\eta : X \times X \longrightarrow X$ be τ-Lipschitz continuous, $F : X \times X \times X \longrightarrow X^\star$ be β_j-Lipschitz continuous with respect to jth argument, for $j = 1, 2, 3$ and $A, B, p, g : X \longrightarrow X$ be single-valued mappings such that g is r_1-Lipschitz continuous, A is r_2-Lipschitz continuous. Let $H : X \times X \longrightarrow X^\star$ be α-strongly η-monotone with respect to A, β-relaxed η-monotone with respect to B and h_1-Lipschitz continuous and h_2-Lipschitz continuous with respect to A and B, respectively. Let $M : X \times X \longrightarrow 2^{X^\star}$ be set-valued mapping such that for fixed $z \in G(X)$, $M(., z) : X \times X \longrightarrow 2^{X^\star}$ be $(H(.,.), \eta)$-monotone operator with respect to A and B. In addition, suppose there exists a constant $\sigma > 0$ such that*

$$\left\| J^{H(.,.),\eta}_{M(.,z_{n+1})}(u) - J^{H(.,.),\eta}_{M(.,z)}(u) \right\| \leq \sigma \parallel z_{n+1} - z_n \parallel .$$

Furthermore, suppose the following condition is satisfied

$$0 < P < 1,$$

where P is given by,

$$P = k\Big\{\sqrt{h_1^2 - 2l_1r_1\alpha_1(h_1 + l_1r_1\alpha_1)} + \sqrt{h_2^2 - 2l_2r_2\alpha_2(h_2 + l_2r_2\alpha_2)} + (\beta_1 + \beta_2 + \beta_3\alpha_3)\Big\} + \sigma\alpha_3,$$

then the sequences $\{u_n\}, \{x_n\}, \{y_n\}$ and $\{z_n\}$, generated by the Iterative Algorithm 3.3 converge strongly to the unique solution (u, x, y, z), respectively, where $u \in X,\ x \in S(u), y \in T(u)$ and $z \in G(u)$ is the solution of the problem (2.2).

Remark 3.6. Using the technique developed in this paper we can extend the results of Bhat and Zahoor [1], Chang *et. al* [2], Kazmi and Bhat [3-6], Mitrovic [9], Verma [14] and the related results cited therein for the system of variational inclusions.

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