



***I*-CONVERGENT TRIPLE DIFFERENCE SEQUENCE SPACES DEFINED BY A SEQUENCE OF MODULUS FUNCTION**

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ABSTRACT. The main objective of this paper is to introduce classes of *I*-convergent triple difference sequence spaces, $c_{0I}^3(\Delta, F)$, $c_I^3(\Delta, F)$, $\ell_{\infty I}^3(\Delta, F)$, $M_I^3(\Delta, F)$ and $M_{0I}^3(\Delta, F)$, by using sequence of moduli function $F = (f_{pqr})$. We also study some algebraic and topological properties of these new sequence spaces.

KEYWORDS: Triple sequence spaces, Difference sequence space, *I*-convergence, Modulus functions, Ideal, Statistical convergence.

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1. INTRODUCTION

A triple sequence (real or complex) is a function $x : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}(\mathbb{C})$, where \mathbb{N} , \mathbb{R} and \mathbb{C} are the set of natural numbers, real numbers, and complex numbers respectively. We denote by ω''' the class of all complex triple sequence (x_{pqr}) , where $p, q, r \in \mathbb{N}$. Then under the coordinate wise addition and scalar multiplication ω''' is a linear space. A triple sequence can be represented by a matrix, in case of double sequences we write in the form of a square. In case of triple sequence it will be in the form of a box in three dimensions.

The different types of notions of triple sequences and their statistical convergence were introduced and investigated initially by Sahiner et. al [19]. Later Debnath et.al [1, 2], Esi et.al [3, 4, 5], Jalal and Malik [11, 12, 13] and many others authors have studied it further and obtained various results. Kizmaz [14] introduced the notion of difference sequence spaces, he defined the difference sequence spaces $\ell_{\infty}(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ as follows.

$$Z(\Delta) = \{x = (x_k) \in \omega : (\Delta x_k) \in Z\}$$

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for $Z = \ell_\infty$, c and c_0

Where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$ and $\Delta^0 x_k = x_k$ for all $k \in \mathbb{N}$

The difference operator on triple sequence is defined as

$$\begin{aligned} \Delta x_{mnk} &= x_{mnk} - x_{(m+1)nk} - x_{m(n+1)k} - x_{mn(k+1)} + x_{(m+1)(n+1)k} \\ &\quad + x_{(m+1)n(k+1)} + x_{m(n+1)(k+1)} - x_{(m+1)(n+1)(k+1)} \end{aligned}$$

and $\Delta_{mnk}^0 = (x_{mnk})$.

Statistical convergence was introduced by Fast [6] and later on it was studied by Fridy [7, 8] from the sequence space point of view and linked it with summability theory. The notion of statistical convergent double sequence was introduced by Mursaleen and Edely [17].

I -convergence is a generalization of the statistical convergence. Kostyrko et. al. [15] introduced the notion of I -convergence of real sequence and studied its several properties. Later Jalal [9, 10], Salat et.al [18] and many other researchers contributed in its study. Tripathy and Goswami [22] extended this concept in probabilistic normed space using triple difference sequences of real numbers. Sahiner and Tripathy [20] studied I -related properties in triple sequence spaces and showed some interesting results. Tripathy [21] extended the concept in I -convergent double sequence and later Kumar [16] obtained some results on I -convergent double sequence. In this paper we have defined I -convergent triple difference sequence spaces, $c_{0I}^3(\Delta, F)$, $c_I^3(\Delta, F)$, $\ell_{\infty I}^3(\Delta, F)$, $M_I^3(\Delta, F)$ and $M_{0I}^3(\Delta, F)$, by using sequence of moduli function $F = (f_{pqr})$ and also studied some algebraic and topological properties of these new sequence spaces.

2. DEFINITIONS AND PRELIMINARIES

Definition 2.1. Let $X \neq \phi$. A class $I \subset 2^X$ (Power set of X) is said to be an ideal in X if the following conditions holds good:

- (i) I is additive that is if $A, B \in I$ then $A \cup B \in I$;
- (ii) I is hereditary that is if $A \in I$, and $B \subset A$ then $B \in I$.

I is called non-trivial ideal if $X \notin I$

Definition 2.2. [19] A triple sequence (x_{pqr}) is said to be convergent to L in Pringsheim's sense if for every $\epsilon > 0$, there exists $\mathbf{N} \in \mathbb{N}$ such that

$$|x_{pqr} - L| < \epsilon \text{ whenever } p \geq \mathbf{N}, q \geq \mathbf{N}, r \geq \mathbf{N}$$

and write as $\lim_{p,p,r \rightarrow \infty} x_{pqr} = L$.

Note: A triple sequence is convergent in Pringsheim's sense may not be bounded [19].

Example Consider the sequence (x_{pqr}) defined by

$$x_{pqr} = \begin{cases} p + q & \text{for all } p = q \text{ and } r = 1 \\ \frac{1}{p^2qr} & \text{otherwise} \end{cases}$$

Then $x_{pqr} \rightarrow 0$ in Pringsheim's sense but is unbounded.

Definition 2.3. A triple sequence (x_{pqr}) is said to be I -convergence to a number L if for every $\epsilon > 0$,

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{pqr} - L| \geq \epsilon\} \in I.$$

In this case we write $I - \lim x_{pqr} = L$.

Definition 2.4. A triple sequence (x_{pqr}) is said to be I -null if $L = 0$. In this case we write $I - \lim x_{pqr} = 0$.

Definition 2.5. [19] A triple sequence (x_{pqr}) is said to be Cauchy sequence if for every $\epsilon > 0$, there exists $\mathbf{N} \in \mathbb{N}$ such that

$$|x_{pqr} - x_{lmn}| < \epsilon \text{ whenever } p \geq l \geq \mathbf{N}, q \geq m \geq \mathbf{N}, r \geq n \geq \mathbf{N}$$

Definition 2.6. A triple sequence (x_{pqr}) is said to be I -Cauchy sequence if for every $\epsilon > 0$, there exists $\mathbf{N} \in \mathbb{N}$ such that

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{pqr} - a_{lmn}| \geq \epsilon\} \in I$$

whenever $p \geq l \geq \mathbf{N}, q \geq m \geq \mathbf{N}, r \geq n \geq \mathbf{N}$

Definition 2.7. [19] A triple sequence (x_{pqr}) is said to be bounded if there exists $M > 0$, such that $|x_{pqr}| < M$ for all $p, q, r \in \mathbb{N}$.

Definition 2.8. A triple sequence (x_{pqr}) is said to be I -bounded if there exists $M > 0$, such that $\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{pqr}| \geq M\} \in I$ for all $p, q, r \in \mathbb{N}$.

Definition 2.9. A triple sequence space E is said to be solid if $(\alpha_{pqr}x_{pqr}) \in E$ whenever $(x_{pqr}) \in E$ and for all sequences (α_{pqr}) of scalars with $|\alpha_{pqr}| \leq 1$, for all $p, q, r \in \mathbb{N}$.

Definition 2.10. Let E be a triple sequence space and $x = (x_{pqr}) \in E$. Define the set $S(x)$ as

$$S(x) = \{(x_{\pi(pqr)}) : \pi \text{ is a permutations of } \mathbb{N}\}$$

If $S(x) \subseteq E$ for all $x \in E$, then E is said to be symmetric.

Definition 2.11. A triple sequence space E is said to be convergence free if $(y_{pqr}) \in E$ whenever $(x_{pqr}) \in E$ and $x_{pqr} = 0$ implies $y_{pqr} = 0$ for all $p, q, r \in \mathbb{N}$.

Definition 2.12. A triple sequence space E is said to be sequence algebra if $x \cdot y \in E$, whenever $x = (x_{pqr}) \in E$ and $y = (y_{pqr}) \in E$, that is product of any two sequences is also in the space.

Definition 2.13. A function $f : [0, \infty) \rightarrow [0, \infty)$ is called a modulus function if it satisfies the following conditions

- (i) $f(x) = 0$ if and only if $x = 0$.
- (ii) $f(x + y) \leq f(x) + f(y)$ for all $x \geq 0$ and $y \geq 0$.
- (iii) f is increasing.
- (iv) f is continuous from the right at 0.

Since $|f(x) - f(y)| \leq f(|x - y|)$, it follows from condition (4) that f is continuous on $[0, \infty)$. Furthermore, from condition (2) we have $f(nx) \leq nf(x)$, for all $n \in \mathbb{N}$, and so

$$f(x) = f(nx(\frac{1}{n})) \leq nf(\frac{x}{n}).$$

Hence $\frac{1}{n}f(x) \leq f(\frac{x}{n})$ for all $n \in \mathbb{N}$

We now define the following sequence spaces

$$\begin{aligned} c_{0I}^3(\Delta, F) &= \left\{ x \in \omega''' : I - \lim f_{pqr}(|\Delta x_{pqr}|) = 0 \right\} \\ c_I^3(\Delta, F) &= \left\{ x \in \omega''' : I - \lim f_{pqr}(|\Delta x_{pqr} - b|) = 0, \text{ for some } b \right\} \\ \ell_{\infty I}^3(\Delta, F) &= \left\{ x \in \omega''' : \sup_{p, q, r \in \mathbb{N}} f_{pqr}(|\Delta x_{pqr}|) = 0 \right\} \\ M_I^3(\Delta, F) &= c_I^3(\Delta, F) \cap \ell_{\infty I}^3(\Delta, F) \\ M_{0I}^3(\Delta, F) &= c_{0I}^3(\Delta, F) \cap \ell_{\infty I}^3(\Delta, F) \end{aligned}$$

3. ALGEBRAIC AND TOPOLOGICAL PROPERTIES OF THE NEW SEQUENCE SPACES

Theorem 3.1. *The triple difference sequence spaces $c_{0I}^3(\Delta, F)$, $c_I^3(\Delta, F)$, $\ell_{\infty I}^3(\Delta, F)$, $M_I^3(\Delta, F)$ and $M_{0I}^3(\Delta, F)$ all are linear for the sequence of moduli $F = (f_{pqr})$.*

Proof. We shall prove it for the sequence space $c_I^3(\Delta, F)$, for the other spaces, it can be established similarly.

Let $x = (x_{pqr}), y = (y_{pqr}) \in c_I^3(\Delta, F)$ and $\alpha, \beta \in \mathbb{R}$ such that $|\alpha| \leq 1$ and $|\beta| \leq 1$, then

$$I - \lim f_{pqr}(|\Delta x_{pqr} - b_1|) = 0, \text{ for some } b_1 \in \mathbb{C}$$

$$I - \lim f_{pqr}(|\Delta y_{pqr} - b_2|) = 0, \text{ for some } b_2 \in \mathbb{C}$$

Now for a given $\epsilon > 0$ we set

$$X_1 = \left\{ (p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - b_1|) > \frac{\epsilon}{2} \right\} \in I \quad (2.1)$$

$$X_2 = \left\{ (p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta y_{pqr} - b_2|) > \frac{\epsilon}{2} \right\} \in I \quad (2.2)$$

Since f_{pqr} is a modulus function, so it is non-decreasing and convex, hence we get

$$\begin{aligned} f_{pqr}(|(\alpha \Delta x_{pqr} + \beta \Delta y_{pqr}) - (\alpha b_1 + \beta b_2)|) &= f_{pqr}(|(\alpha \Delta x_{pqr} - \alpha b_1) + (\beta \Delta y_{pqr} - \beta b_2)|) \\ &\leq f_{pqr}(|\alpha| |\Delta x_{pqr} - b_1|) + f_{pqr}(|\beta| |\Delta y_{pqr} - b_2|) \\ &= |\alpha| f_{pqr}(|\Delta x_{pqr} - b_1|) + |\beta| f_{pqr}(|\Delta y_{pqr} - b_2|) \\ &\leq f_{pqr}(|\Delta x_{pqr} - b_1|) + f_{pqr}(|\Delta y_{pqr} - b_2|) \end{aligned}$$

From (2.1) and (2.2) we can write

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|(\alpha \Delta x_{pqr} + \beta \Delta y_{pqr}) - (\alpha b_1 + \beta b_2)|) > \epsilon\} \subset X_1 \cup X_2$$

Thus $\alpha x + \beta y \in c_I^3(\Delta, F)$

This completes the proof. \square

Theorem 3.2. *The triple difference sequence $x = (x_{pqr}) \in M_I^3(\Delta, F)$ is I -convergent if and only if for every $\epsilon > 0$ there exists $I_\epsilon, J_\epsilon, K_\epsilon \in \mathbb{N}$ such that*

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - \Delta x_{I_\epsilon J_\epsilon K_\epsilon}|) \leq \epsilon\} \in M_I^3(\Delta, F)$$

Proof. Let $b = I - \lim \Delta x$. Then we have

$$A_\epsilon = \left\{ (p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - b|) \leq \frac{\epsilon}{2} \right\} \in M_I^3(\Delta, F) \quad \text{for all, } \epsilon > 0.$$

Next fix $I_\epsilon, J_\epsilon, K_\epsilon \in A_\epsilon$ then we have

$$|\Delta x_{pqr} - \Delta x_{I_\epsilon J_\epsilon K_\epsilon}| \leq |\Delta x_{pqr} - b| + |b - \Delta x_{I_\epsilon J_\epsilon K_\epsilon}| \leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon \text{ for all, } p, q, r \in A_\epsilon$$

Thus

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - \Delta x_{I_\epsilon J_\epsilon K_\epsilon}|) \leq \epsilon\} \in M_I^3(\Delta, F)$$

Conversely suppose that

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - \Delta x_{I_\epsilon J_\epsilon K_\epsilon}|) \leq \epsilon\} \in M_I^3(\Delta, F)$$

we get $\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - \Delta x_{I_\epsilon J_\epsilon K_\epsilon}|) \leq \epsilon\} \in M_I^3(\Delta, F)$, for all $\epsilon > 0$.

Then given $\epsilon > 0$ we can find the set

$$B_\epsilon = \{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \Delta x_{pqr} \in [\Delta x_{I_\epsilon J_\epsilon K_\epsilon} - \epsilon, \Delta x_{I_\epsilon J_\epsilon K_\epsilon} + \epsilon]\} \in M_I^3(\Delta, F)$$

Let $J_\epsilon = [I_\epsilon - \epsilon, I_\epsilon + \epsilon]$ if $\epsilon > 0$ is fixed then $B_\epsilon \in M_I^3(\Delta, F)$ as well as $B_{\frac{\epsilon}{2}} \in M_I^3(\Delta, F)$.

Hence $B_\epsilon \cap B_{\frac{\epsilon}{2}} \in M_I^3(\Delta, F)$

Which gives $J = J_\epsilon \cap J_{\frac{\epsilon}{2}} \neq \emptyset$ that is $\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \Delta x_{pqr} \in \mathbb{N}\} \in M_I^3(\Delta, F)$

Which implies $\text{diam } J \leq \text{diam } J_\epsilon$

where the diam of J denotes the length of interval J .

Now by the principal of induction a sequence of closed interval can be found

$$J_\epsilon = I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots \supseteq I_s \supseteq \cdots$$

with the help of the property that $\text{diam } I_s \leq \frac{1}{2} \text{diam } I_{s-1}$, for $s = 1, 2, \cdots$ and $\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \Delta x_{pqr} \in I_{pqr}\}$ for $(p, q, r = 1, 2, 3 \cdots)$

Then there exists a $\xi \in \cap I_s$ where $s \in \mathbb{N}$ such that $\xi = I - \lim \Delta x$

So that $f_{pqr}(\xi) = I - \lim f_{pqr}(\Delta x)$ therefore $b = I - \lim f_{pqr}(\Delta x)$.

Hence the proof is complete. \square

Theorem 3.3. *The $F = (f_{pqr})$ be a sequence of modulus functions then the inclusions $c_{0I}^3(\Delta, F) \subset c_I^3(\Delta, F) \subset \ell_{\infty I}^3(\Delta, F)$ holds .*

Proof. The inclusion $c_{0I}^3(\Delta, F) \subset c_I^3(\Delta, F)$ is obvious.

We prove $c_I^3(\Delta, F) \subset \ell_{\infty I}^3(\Delta, F)$, let $x = (x_{pqr}) \in c_I^3(\Delta, F)$ then there exists $b \in \mathbb{C}$ such that $I - \lim f_{pqr}(|\Delta x_{pqr} - b|) = 0$,

Which gives $f_{pqr}(|\Delta x_{pqr}|) \leq f_{pqr}(|\Delta x_{pqr} - b|) + f_{pqr}(|b|)$

On taking supremum over p, q and r on both sides gives

$x = (x_{pqr}) \in \ell_{\infty I}^3(\Delta, F)$

Hence the inclusion $c_{0I}^3(\Delta, F) \subset c_I^3(\Delta, F) \subset \ell_{\infty I}^3(\Delta, F)$ holds. \square

Theorem 3.4. *The triple difference sequence $c_{0I}^3(\Delta, F)$ and $M_{0I}^3(\Delta, F)$ are solid.*

Proof. We prove the result for $c_{0I}^3(\Delta, F)$.

Consider $x = (x_{pqr}) \in c_{0I}^3(\Delta, F)$, then $I - \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|) = 0$

Consider a sequence of scalar (α_{pqr}) such that $|\alpha_{pqr}| \leq 1$ for all $p, q, r \in \mathbb{N}$.

Then we have

$$\begin{aligned} I - \lim_{p,q,r} f_{pqr}(|\Delta \alpha_{pqr}(x_{pqr})|) &\leq I - |\alpha_{pqr}| \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|) \\ &\leq I - \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|) \\ &= 0 \end{aligned}$$

Hence $I - \lim_{p,q,r} f_{pqr}(|\Delta \alpha_{pqr} x_{pqr}|) = 0$ for all $p, q, r \in \mathbb{N}$

Which gives $(\alpha_{pqr} x_{pqr}) \in c_{0I}^3(\Delta, F)$

Hence the sequence space $c_{0I}^3(\Delta, F)$ is solid.

The result for $M_{0I}^3(\Delta, F)$ can be similarly proved. \square

Theorem 3.5. *The triple difference sequence spaces $c_{0I}^3(\Delta, F)$, $c_I^3(\Delta, F)$, $\ell_{\infty I}^3(\Delta, F)$, $M_I^3(\Delta, F)$ and $M_{0I}^3(\Delta, F)$ are sequence algebras.*

Proof. We prove the result for $c_{0I}^3(\Delta, F)$.

Let $x = (x_{pqr}), y = (y_{pqr}) \in c_{0I}^3(\Delta, F)$

Then we have $I - \lim f_{pqr}(|\Delta x_{pqr}|) = 0$ and $I - \lim f_{pqr}(|\Delta y_{pqr}|) = 0$

and $I - \lim f_{pqr}(|\Delta(x_{pqr} \cdot y_{pqr})|) = 0$ as

$$\begin{aligned} \Delta(x_{pqr} \cdot y_{pqr}) &= x_{pqr} \cdot y_{pqr} - x_{(p+1)qr} \cdot y_{(p+1)qr} - x_{p(q+1)r} \cdot y_{p(q+1)r} - x_{pq(r+1)} \cdot y_{pq(r+1)} + \\ &\quad x_{(p+1)(q+1)r} \cdot y_{(p+1)(q+1)r} + x_{(p+1)q(r+1)} \cdot y_{(p+1)q(r+1)} + x_{p(q+1)(r+1)} \cdot \\ &\quad y_{p(q+1)(r+1)} - x_{(p+1)(q+1)(r+1)} \cdot y_{(p+1)(q+1)(r+1)} \end{aligned}$$

It implies that $x \cdot y \in c_{0I}^3(\Delta, F)$

Hence the proof.

The result can be proved for the spaces $c_I^3(\Delta, F)$, $\ell_{\infty I}^3(\Delta, F)$, $M_I^3(\Delta, F)$ and $M_{0I}^3(\Delta, F)$ in the same way. \square

Theorem 3.6. *In general the sequence spaces $c_{0I}^3(\Delta, F)$, $c_I^3(\Delta, F)$ and $\ell_{\infty I}^3(\Delta, F)$ are not convergence free.*

Proof. We prove the result for the sequence space $c_I^3(\Delta, F)$ using an example. Example. Let $I = I_f$ define the triple sequence $x = (x_{pqr})$ as

$$x_{pqr} = \begin{cases} 0 & \text{if } p = q = r \\ 1 & \text{otherwise} \end{cases}$$

Then if $f_{pqr}(x) = x_{pqr} \forall p, q, r \in \mathbb{N}$, we have $x = (x_{pqr}) \in c_I^3(\Delta, F)$.

Now define the sequence $y = y_{pqr}$ as

$$y_{pqr} = \begin{cases} 0 & \text{if } r \text{ is odd, and } p, q \in \mathbb{N} \\ lmn & \text{otherwise} \end{cases}$$

Then for $f_{pqr}(x) = x_{pqr} \forall p, q, r \in \mathbb{N}$, it is clear that $y = (y_{pqr}) \notin c_I^3(\Delta, F)$

Hence the sequence spaces $c_I^3(\Delta, F)$ is not convergence free.

The space $c_I^3(\Delta, F)$ and $\ell_{\infty I}^3(\Delta, F)$ are not convergence free in general can be proved in the same fashion. \square

Theorem 3.7. *In general the triple difference sequences $c_{0I}^3(\Delta, F)$ and $c_I^3(\Delta, F)$ are not symmetric if I is neither maximal nor $I = I_f$.*

Proof. We prove the result for the sequence space $c_{0I}^3(\Delta, F)$ using an example. Example. Define the triple sequence $x = (x_{pqr})$ as

$$x_{pqr} = \begin{cases} 0 & \text{if } r = 1, \text{ for all } p, q \in \mathbb{N} \\ 1 & \text{otherwise} \end{cases}$$

Then if $f_{pqr}(x) = x_{pqr} \forall p, q, r \in \mathbb{N}$, we have $x = (x_{pqr}) \in c_{0I}^3(\Delta, F)$.

Now if $x_{\pi(pqr)}$ be a rearrangement of $x = (x_{pqr})$ defined as

$$x_{\pi(pqr)} = \begin{cases} 1 & \text{for } p, q, r \text{ even} \in K \\ 0 & \text{otherwise} \end{cases}$$

Then $\{x_{\pi(p,q,r)}\} \notin c_{0I}^3(\Delta, F)$ as $\Delta x_{\pi(pqr)} = 1$

Hence the sequence spaces $c_{0I}^3(\Delta, F)$ is not symmetric in general.

The space $c_I^3(\Delta, F)$ is not symmetric in general can be proved in the same fashion. \square

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