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STABILITY AND RATE OF CONVERGENCE OF SOME ITERATION METHODS FOR BERINDE CONTRACTIONS

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ABSTRACT. In this paper, we first prove strong convergence theorems of a new iteration method finding a common fixed point of three Berinde nonexpansive mappings and introduce a new iteration method and study stability of the proposed method and Noor iteration for a class of Berinde contraction mappings in complete metric space. We also compare the rate of convergence between our iteration method and Noor iteration under some suitable control conditions.

KEYWORDS: stability, rate of convergence, Noor iteration, Berinde contractions.
AMS Subject Classification: 47H09, 47H10.

1. Introduction and preliminaries

Let C be a nonempty convex subset of a Banach space X, and $T : C \to C$ be a mapping. A point $x \in C$ is a fixed point of T if $Tx = x$. We denote $F(T)$ the set of all fixed points of T. There are two important problems in fixed point theory. The first one is the existence problem. Many mathematicians are interested in finding sufficient conditions to guarantee the existence of fixed point and common fixed point of mappings. The second problem is to study how to approximate a fixed point and a common fixed point of mappings. Many iteration methods were introduced and studied. Some conditions for convergence of those methods were given, see for instance [11, 9, 13, 18, 8].

In 2003, Berinde [3, 4] introduced and studied a weak contraction mapping. It is very interesting to study this mapping because it generalizes many well-known mappings such as contraction and Zamfirescu mappings.

In 2004, Berinde [5] provided the concept of how to compare the rate of convergence of the iterative methods and proved that Picard iteration converges faster than Mann iteration for a class of Zamfirescu operators and a class of quasi-contractive

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operator in arbitrary Banach spaces. After that there are many works concerning comparision of the rate of convergence, see [2, 14, 19, 16, 13] for examples.

A mapping $T : X \to X$ is said to be

(1) a *contraction* if there exists $k \in [0, 1)$ such that for $x, y \in X$,

$$\|Tx - Ty\| \ \leq \ k\|x - y\|, \tag{1.1}$$

(2) *nonexpansive* if for $x, y \in X$,

$$\|Tx - Ty\| \ \leq \ \|x - y\|, \tag{1.2}$$

In 1968, Kannan extended a contraction mapping to mapping that need not be continuous. A mapping $T : X \to X$ is called a *Kannan mapping* if for $x, y \in X$, there is a constant $0 < b < \dfrac{1}{2}$ such that

$$\|Tx - Ty\| \ \leq \ b[\ \|x - Tx\| + \|y - Ty\|\ ]. \tag{1.3}$$

In 1972, Chatterjea introduced a mapping that is dual of Kannan mapping.

A mapping $T : X \to X$ is called a *Chatterjea mapping* if for $x, y \in X$, there exists $0 < c < \dfrac{1}{2}$ such that

$$\|Tx - Ty\| \ \leq \ c[\ \|x - Ty\| + \|y - Tx\|\ ]. \tag{1.4}$$

In 1972, Zamfirescu obtained a very interesting nonlinear mapping by combining (1.1), (1.3) and (1.4). A mapping $T : X \to X$ is said to be a *Zamfirescu operator* if there exist $a \in [0, 1), b, c \in (0, \dfrac{1}{2})$ such that for $x, y \in X$, satisfy at least one of the following :

(a) $\|Tx - Ty\| \ \leq \ a\|x - y\|$;
(b) $\|Tx - Ty\| \ \leq \ b[\ \|x - Tx\| + \|y - Ty\|\ ]$;
(c) $\|Tx - Ty\| \ \leq \ c[\ \|x - Ty\| + \|y - Tx\|\ ]$.

In 1974, Ciric introduced a mapping that is one of the most general contraction condition. A mapping $T : X \to X$ is called a *quasi-contraction mapping* if for $x, y \in X$, there exists $0 < h < 1$ such that

$$\|Tx - Ty\| \ \leq \ h \max\{\ \|x - y\|, \|x - Tx\|, \|y - Ty\|, \|x - Ty\|, \|y - Tx\|\ \}. \tag{1.5}$$

It is obvious any mapping that satisfies (1.1), (1.3), (1.4) and Zamfirescu mapping is a quasi-contraction mapping.

Definition 1.1. (condition $(*)$) Let X be a Banach space. A mapping $T : X \to X$ is said to satisfy condition $(*)$ if there exists a constant $\delta' \in (0, 1)$ and $L' \geq 0$ such that for all $x, y \in C$,

$$\|Tx - Ty\| \ \leq \ \delta'\|x - y\| + L'\|y - Ty\|. \tag{1.6}$$

Definition 1.2. Let X be a Banach space. A mapping $T : X \to X$ is called a F-*contraction* if $F(T) \neq \emptyset$ and there exists $0 \leq k < 1$ such that for $x \in X$, $p \in F(T)$,

$$\|Tx - p\| \ \leq \ k\|x - p\|. \tag{1.7}$$

We can show that a F-contraction mapping with $F(T) \neq \emptyset$ has a unique fixed point and it easy to see that any mapping which satisfies condition $(*)$ (1.6) with $F(T) \neq \emptyset$ is F-contraction.

Definition 1.3. Let X be a Banach space. A mapping $T : X \to X$ with $F(T) \neq \emptyset$ is called a *quasi-nonexpansive* if for $x \in X$, $p \in F(T)$,

$$\|Tx - p\| \ \leq \ \|x - p\|. \tag{1.8}$$

It is clearly that any F-contraction mapping is quasi-nonexpansive.

Example 1.4. [7] Let $X = l^\infty$ and $C = \{\{x_n\} : -1 \le x_1 \le 3,\ -1 \le x_2 \le 1,\ x_n = 0,\ \forall n \ge 3\}$. Define $T : C \to C$ by

$$T(x_1, x_2, 0, ...) = (x_1, -x_2, 0, ...),\ \ \forall x_2 \neq 0,$$

$$T(x_1, 0, ...) = \begin{cases} (x_1, |x_1|, 0, ...), & \text{if } -1 \le x_1 \le 1, \\ (x_1, |x_1 - 2|, 0, ...), & \text{if } 1 \le x_1 \le 3. \end{cases}$$

Then T is a quasi-nonexpansive mapping with $F(T) = \{(0, 0, 0, ...), (2, 0, 0, ...)\}$.

In 2003, Berinde [3] introduced a weak contraction mapping and proved the following existence fixed point theorem in Banach spaces.

Definition 1.5. Let C be a nonempty subset of a Banach space X. A mapping $T : C \to C$ is called *weak contraction or Berinde contraction* if there exists a constant $\delta \in (0, 1)$ and $L \ge 0$ such that for all $x, y \in C$,

$$\|Tx - Ty\| \ \le\ \delta\|x - y\| + L\|y - Tx\|. \tag{1.9}$$

The class of Berinde contraction mappings includes classes of contraction, Kannan, Zamfirescu, Chatterjea and quasi-contraction mappings.

Proposition 1.6. [3] Let C be a nonempty closed subset of a Banach space X and $T : C \to C$ be a weak contraction, Then $F(T) \neq \emptyset$. Moreover, the Picard iteration $\{x_n\}$ defined by $x_1 \in C$ and $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$, converges to a fixed point of T.

Let C be a nonempty convex subset of a Banach space X, and $T : C \to C$ be a mapping. The *Mann iteration* is defined by $s_0 \in C$,

$$s_{n+1} = (1 - \alpha_n)s_n + \alpha_n T s_n,\ \text{ for all }\ n \ge 0, \tag{1.10}$$

where $\{\alpha_n\}$ is a sequence in $[0, 1]$.
For $\alpha_n = \lambda$ (constant), the iteration (1.10) reduces to the so-called Krasnoselskij iteration. For $\alpha_n = 1$, we obtain the Picard iteration.
The *Ishikawa iteration* is defined by $s_0 \in C$,

$$\begin{aligned} w_n &= (1 - \beta_n)s_n + \beta_n T s_n, \\ s_{n+1} &= (1 - \alpha_n)s_n + \alpha_n T w_n, \text{ for all } n \ge 0, \end{aligned} \tag{1.11}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0, 1]$.
The *Noor iteration* is defined by $s_0 \in C$,

$$\begin{aligned} h_n &= (1 - \gamma_n)s_n + \gamma_n T s_n, \\ w_n &= (1 - \beta_n)s_n + \beta_n T h_n, \\ s_{n+1} &= (1 - \alpha_n)s_n + \alpha_n T w_n,\ \text{ for all }\ n \ge 0, \end{aligned} \tag{1.12}$$

where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are sequences in $[0, 1]$.
It is easy to see that Mann and Ishikawa iterations are special cases of Noor iterations.

In this paper, we introduce an iterative method as follows.
Let $\{x_n\}$ be a sequence defined by $x_0 \in C$,

$$\begin{aligned} z_n &= (1 - \gamma_n)x_n + \gamma_n T x_n, \\ y_n &= (1 - \beta_n)z_n + \beta_n T z_n, \\ x_{n+1} &= (1 - \alpha_n)T z_n + \alpha_n T y_n,\ \text{ for all }\ n \ge 0, \end{aligned} \tag{1.13}$$

where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are sequences in $[0,1]$. The main purpose of the paper is to study stability of the proposed method and Noor iteration for a class of Berinde contraction mappings in a complete metric space. We also compare rate of convergence between our iteration method and Noor iteration under some suitable control conditions.

2. Convergence Theorem and Stability

We first recall the definition of Berinde nonexpansive mappings introduced by Kosol [10] as follow:

Definition 2.1. Let C be a nonempty subset of a Banach space X. A mapping $T : C \to C$ is called *Berinde nonexpansive* if there exists a constant $L \geq 0$ such that for all $x, y \in C$,

$$\|Tx - Ty\| \leq \|x - y\| + L\|y - Tx\|. \tag{2.1}$$

It is easy to see that all nonexpansive mappings and weak contraction mappings are Berinde nonexpansive.

Example 2.2. Let $X = \mathbb{R}$ and $C = [0,1]$. Define $T : C \to C$ by

$$T(x) = \begin{cases} x^2, & \text{if } x \in [0, \frac{1}{2}), \\ 1, & \text{if } x \in [\frac{1}{2}, 1]. \end{cases}$$

Then T is a Berinde nonexpansive with $L = 4$, but is not a nonexpansive mapping.

Proof. (i). If $x, y \in [0, \frac{1}{2})$, we have

$$\begin{aligned} |Tx - Ty| = |x^2 - y^2| &= |(x+y)(x-y)| \\ &= |x+y||x-y| \\ &\leq |x - y|. \end{aligned}$$

(ii). If $x \in [0, \frac{1}{2}), y \in [\frac{1}{2}, 1]$,

$$\begin{aligned} |Tx - Ty| = |x^2 - 1| &= 1 - x^2 \\ &\leq 4 \cdot \frac{1}{4} \\ &\leq 4|y - Tx|, \end{aligned}$$

and $|Tx - Ty| \leq |x - y| + 4|x - Ty|$.

(iii). If $x, y \in [\frac{1}{2}, 1]$, then $|Tx - Ty| = 0 \leq |x - y|$.

So, we have $|Tx-Ty| \leq |x-y|+4|x-Ty|$ and $|Tx-Tx| \leq |x-y|+4|y-Tx|$, for all $x, y \in [0,1]$. So T is a Berinde nonexpansive mapping, but it is not a nonexpansive mapping because $|T(\frac{1}{3}) - T(1)| \geq |\frac{1}{3} - 1|$. □

Let $T_i : C \longrightarrow C, i = 1,2,3$ be mappings. In order to approximate a common fixed point of Berinde nonexpansive mappings, we introduce the following iterative method. Let $\{x_n\}$ be a sequence defined by $x_0 \in C$,

$$\begin{aligned} z_n &= (1-\gamma_n)x_n + \gamma_n T_1 x_n, \\ y_n &= (1-\beta_n)z_n + \beta_n T_2 z_n, \\ x_{n+1} &= (1-\alpha_n)T_3 z_n + \alpha_n T_3 y_n, \text{ for all } n \geq 0, \end{aligned} \tag{2.2}$$

where $\{\alpha_n\},\{\beta_n\}$ and $\{\gamma_n\}$ are sequences in $[0,1]$.
The Noor iteration is defined by $s_0 \in C$ and

$$\begin{aligned} h_n &= (1-\gamma_n)s_n + \gamma_n T_1 s_n, \\ w_n &= (1-\beta_n)s_n + \beta_n T_2 h_n, \\ s_{n+1} &= (1-\alpha_n)s_n + \alpha_n T_3 w_n, \text{ for all } n \in \mathbb{N}, \end{aligned} \tag{2.3}$$

where $\{\alpha_n\},\{\beta_n\}$ and $\{\gamma_n\}$ are sequences in $[0,1]$.

First, we will prove a convergence theorem of the proposed iteration method (2.2) for finding a common fixed point of Berinde nonexpansive mappings.

Let $T_i : C \to C,\ i = 1,2,3$, be Berinde nonexpansive mappings. Through out this thesis, we let $L_i, i = 1,2,3$, be nonnegative real numbers such that for $x, y \in C$,

$$\|T_i x - T_i y\| \le \|x-y\| + L_i\|y - T_i x\|.$$

Lemma 2.3. [6] Let X be a uniformly convex Banach space and $B_r = \{x \in X : \|x\| \le r\},\ r > 0$. Then there exists a continuous, strictly increasing, and convex function $g : [0,\infty) \to [0,\infty),\ g(0) = 0$ such that

$$\|\lambda x + \beta y + \gamma z\|^2 \le \lambda\|x\|^2 + \beta\|y\|^2 + \gamma\|z\|^2 - \lambda\beta \cdot g(\|x-y\|),$$

for all $x, y, z \in B_r$ and all $\lambda, \beta, \gamma \in [0,1]$ with $\lambda + \beta + \gamma = 1$.

Lemma 2.4. Let X be a Banach space and C be a nonempty closed convex subset of X. For each $i = 1,2,3.$, let $T_i : C \to C$ be a quasi-nonexpansive mapping. Assume that $\bigcap_{i=1}^{3} F(T_i) \neq \emptyset$ and $\{x_n\}$ is a sequence generated by (2.2) and $\{\alpha_n\},\{\beta_n\},\{\gamma_n\}$ are sequences in $[0,1]$. Then,

(i) $\|x_{n+1} - p\| \le \|x_n - p\|$, $\forall n \in \mathbb{N}$ and $\forall p \in \bigcap_{i=1}^{3} F(T_i)$.

(ii) $\lim_{n\to\infty} \|x_n - p\|$ exists.

Proof. Let $p \in \bigcap_{i=1}^{3} F(T_i)$. By using (2.2), we have

$$\begin{aligned} \|z_n - p\| &\le (1-\gamma_n)\|x_n - p\| + \gamma_n\|T_1 x_n - p\| \\ &\le (1-\gamma_n)\|x_n - p\| + \gamma_n\|x_n - p\| \\ &= \|x_n - p\|, \end{aligned}$$

$$\begin{aligned} \|y_n - p\| &\le (1-\beta_n)\|z_n - p\| + \beta_n\|T_2 z_n - p\| \\ &\le (1-\beta_n)\|z_n - p\| + \beta_n\|z_n - p\| \\ &\le \|x_n - p\|. \end{aligned}$$

From above inequalities, we get

$$\begin{aligned} \|x_{n+1} - p\| &\le (1-\alpha_n)\|T_3 z_n - p\| + \alpha_n\|T_3 y_n - p\| \\ &\le (1-\alpha_n)\|z_n - p\| + \alpha_n\|y_n - p\| \\ &\le \|x_n - p\|, \end{aligned}$$

so $\|x_{n+1} - p\| \le \|x_n - p\|$. Since $\{\|x_n - p\|\}$ is a non-increasing sequence and bounded below by 0, $\lim_{n\to\infty} \|x_n - p\|$ exists. □

Theorem 2.5. Let X be a uniformly convex Banach space and C be a nonempty closed convex subset of X. For each $i = 1, 2, 3$, let $T_i : C \to C$ be a Berinde nonexpansive and quasi-nonexpansive mapping. Assume that $\bigcap_{i=1}^{3} F(T_i) \neq \emptyset$ and T_i is demicompact, for some $i \in \{1, 2\}$. Let $\{x_n\}$ be a sequence generated by (2.2) where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[0, 1]$ which satisfy the following conditions :

(i) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$;
(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$;
(iii) $0 < \liminf_{n\to\infty} \gamma_n \leq \limsup_{n\to\infty} \gamma_n < 1$.

Then $\{x_n\}$ converges strongly to a common fixed point of $\{T_i\}_{i=1}^{3}$.

Proof. Let $p \in \bigcap_{i=1}^{3} F(T_i)$. From Lemma 2.4, $\lim_{n\to\infty} \|x_n - p\|$ exists, then we have $\{\|x_n - p\|\}$ is bounded, that is, $\exists M > 0$ such that for each $n \in \mathbb{N}$, $\|x_n - p\| \leq M$. By quasi-nonexpansiveness of T_i, $\{x_n - p\}, \{T_1x_n - p\}, \{z_n - p\}, \{T_2z_n - p\}, \{T_3z_n - p\}, \{T_3y_n - p\} \subset B_M$. By Lemma 2.3, there exists a continuous, strictly increasing, and convex function $g : [0, \infty) \to [0, \infty)$, with $g(0) = 0$ such that

$$
\begin{aligned}
\|z_n - p\|^2 &= \|(1 - \gamma_n)(x_n - p) + \gamma_n(T_1x_n - p)\|^2 \\
&\leq (1 - \gamma_n)\|x_n - p\|^2 + \gamma_n\|T_1x_n - p\|^2 - (1 - \gamma_n)\gamma_n g(\|x_n - T_1x_n\|) \\
&\leq (1 - \gamma_n)\|x_n - p\|^2 + \gamma_n\|x_n - p\|^2 - (1 - \gamma_n)\gamma_n g(\|x_n - T_1x_n\|) \\
&= \|x_n - p\|^2 - (1 - \gamma_n)\gamma_n g(\|x_n - T_1x_n\|),
\end{aligned}
$$

$$
\begin{aligned}
\|y_n - p\|^2 &= \|(1 - \beta_n)(z_n - p) + \beta_n(T_2z_n - p)\|^2 \\
&\leq (1 - \beta_n)\|z_n - p\|^2 + \beta_n\|T_2z_n - p\|^2 - (1 - \beta_n)\beta_n g(\|z_n - T_2z_n\|) \\
&\leq (1 - \beta_n)\|z_n - p\|^2 + \beta_n\|z_n - p\|^2 - (1 - \beta_n)\beta_n g(\|z_n - T_2z_n\|) \\
&= \|z_n - p\|^2 - (1 - \beta_n)\beta_n g(\|z_n - T_2z_n\|).
\end{aligned}
$$

From above inequalities, we have

$$
\begin{aligned}
\|x_{n+1} - p\|^2 &= \|(1 - \alpha_n)(T_3z_n - p) + \alpha_n(T_3y_n - p)\|^2 \\
&\leq (1 - \alpha_n)\|z_n - p\|^2 + \alpha_n\|y_n - p\|^2 - (1 - \alpha_n)\alpha_n g(\|T_3z_n - T_3y_n\|) \\
&\leq (1 - \alpha_n)\|z_n - p\|^2 + \alpha_n\|z_n - p\|^2 - (1 - \beta_n)\beta_n\alpha_n g(\|z_n - T_2z_n\|) \\
&\quad - (1 - \alpha_n)\alpha_n g(\|T_3z_n - T_3y_n\|) \\
&= \|z_n - p\|^2 - (1 - \beta_n)\beta_n\alpha_n g(\|z_n - T_2z_n\|) \\
&\quad - (1 - \alpha_n)\alpha_n g(\|T_3z_n - T_3y_n\|) \\
&\leq \|x_n - p\|^2 - (1 - \gamma_n)\gamma_n g(\|x_n - T_1x_n\|) \\
&\quad - (1 - \beta_n)\beta_n\alpha_n g(\|z_n - T_2z_n\|) - (1 - \alpha_n)\alpha_n g(\|T_3z_n - T_3y_n\|).
\end{aligned}
$$

By assumptions on the control sequences, there exist $n_0 \in \mathbb{N}$ and $\eta_1, \eta_2 \in (0, 1)$ such that $0 < \eta_1 < \min\{\alpha_n, \beta_n, \gamma_n\}$ and $\max\{\alpha_n, \beta_n, \gamma_n\} < \eta_2 < 1$, for all $n \geq n_0$. Then,

$$
\begin{aligned}
\eta_1(1 - \eta_2)g(\|x_n - T_1x_n\|) &\leq (1 - \gamma_n)\gamma_n g(\|x_n - T_1x_n\|) \\
&\leq \|x_n - p\|^2 - \|x_{n+1} - p\|^2,
\end{aligned}
$$

$$\begin{aligned}
\eta_1^2(1-\eta_2)g(\|z_n - T_2z_n\|) &\leq (1-\beta_n)\beta_n\alpha_n g(\|z_n - T_2z_n\|) \\
&\leq \|x_n - p\|^2 - \|x_{n+1} - p\|^2, \\
\eta_1(1-\eta_2)g(\|T_3z_n - T_3y_n\|) &\leq (1-\alpha_n)\alpha_n g(\|T_3z_n - T_3y_n\|) \\
&\leq \|x_n - p\|^2 - \|x_{n+1} - p\|^2.
\end{aligned}$$

Since $\lim_{n\to\infty}\|x_n - p\|$ exists, it implies that $\lim_{n\to\infty} g(\|x_n - T_1x_n\|) = \lim_{n\to\infty} g(\|z_n - T_2z_n\|) = \lim_{n\to\infty} g(\|T_3z_n - T_3y_n\|) = 0$. Since g is continuous and $g(0) = 0$, we have $\lim_{n\to\infty}\|x_n - T_1x_n\| = \lim_{n\to\infty}\|z_n - T_2z_n\| = \lim_{n\to\infty}\|T_3z_n - T_3y_n\| = 0$. It follows that

$$\begin{aligned}
\|z_n - x_n\| &\leq \gamma_n\|x_n - T_1x_n\| \\
&\leq \|x_n - T_1x_n\| \quad \to 0, \\
\|y_n - z_n\| &\leq \beta_n\|z_n - T_2z_n\| \\
&\leq \|z_n - T_2z_n\| \quad \to 0, \\
\|x_{n+1} - T_3z_n\| &\leq \alpha_n\|T_3z_n - T_3y_n\| \\
&\leq \|T_3z_n - T_3y_n\| \quad \to 0.
\end{aligned}$$

By Berinde nonexpansiveness of T_2, we have

$$\begin{aligned}
\|x_n - T_2x_n\| &\leq \|x_n - z_n\| + \|z_n - T_2z_n\| + \|T_2z_n - T_2x_n\| \\
&\leq \|x_n - z_n\| + \|z_n - T_2z_n\| + (\ \|z_n - x_n\| + L_2\|x_n - T_2z_n\|\ ) \\
&\leq \|x_n - z_n\| + \|z_n - T_2z_n\| + \|z_n - x_n\| \\
&\qquad + L_2(\ \|x_n - z_n\| + \|z_n - T_2z_n\|\ ) \to 0.
\end{aligned}$$

It implies that $\lim_{n\to\infty}\|x_n - T_2x_n\| = 0$. Now, suppose that T_{i_0} is demicompact, for some $i_0 \in \{1, 2\}$. Then $\exists\{x_{n_k}\} \subset \{x_n\}$ such that $x_{n_k} \to q$, $\exists q$. From above inequalities, we have

$$\begin{aligned}
\|T_1x_{n_k} - T_1q\| &\leq \|x_{n_k} - q\| + L_1\|q - T_1x_{n_k}\| \\
&\leq \|x_{n_k} - q\| + L_1(\ \|q - x_{n_k}\| + \|x_{n_k} - T_1x_{n_k}\|\ ) \to 0,
\end{aligned}$$

$$\begin{aligned}
\|T_2z_{n_k} - T_2q\| &\leq \|z_{n_k} - q\| + L_2\|q - T_2z_{n_k}\| \\
&\leq \|z_{n_k} - x_{n_k}\| + \|x_{n_k} - q\| \\
&\qquad + L_2(\ \|q - x_{n_k}\| + \|x_{n_k} - z_{n_k}\| + \|z_{n_k} - T_2z_{n_k}\|\ ) \to 0,
\end{aligned}$$

and

$$\begin{aligned}
\|T_3y_{n_k} - T_3q\| &\leq \|y_{n_k} - q\| + L_3\|q - T_3y_{n_k}\| \\
&\leq \|y_{n_k} - z_{n_k}\| + \|z_{n_k} - x_{n_k}\| + \|x_{n_k} - q\| \\
&\qquad + L_3(\ \|q - x_{n_k+1}\| + \|x_{n_k+1} - T_3z_{n_k}\| + \|T_3z_{n_k} - T_3y_{n_k}\|\ ) \to 0.
\end{aligned}$$

It follows that

$$\begin{aligned}
\|q - T_1q\| &\leq \|q - x_{n_k}\| + \|x_{n_k} - T_1x_{n_k}\| + \|T_1x_{n_k} - T_1q\| \to 0, \\
\|q - T_2q\| &\leq \|q - x_{n_k}\| + \|x_{n_k} - z_{n_k}\| + \|z_{n_k} - T_2z_{n_k}\| \\
&\qquad + \|T_2z_{n_k} - T_2q\| \to 0, \\
\|q - T_3q\| &\leq \|q - x_{n_k+1}\| + \|x_{n_k+1} - T_3z_{n_k}\| + \|T_3z_{n_k} - T_3y_{n_k}\| \\
&\qquad + \|T_3y_{n_k} - T_3q\| \to 0.
\end{aligned}$$

So $q \in \bigcap_{i=1}^{3} F(T_i)$. By Theorem 2.4, $\lim_{n\to\infty} \|x_n - q\|$ exists. Since $\|x_{n_k} - q\| \to 0$, it implies that $\lim_{n\to\infty} x_n = q$. ☐

Theorem 2.6. [20] Let X be a uniformly convex Banach space and C be a nonempty closed convex subset of X. For each $i = 1, 2, 3.$, let $T_i : C \to C$ be a Berinde nonexpansive and quasi-nonexpansive mapping. Assume that $\bigcap_{i=1}^{3} F(T_i) \neq \emptyset$ and T_i is a demicompact, for some $i \in \{1, 2, 3\}$. Suppose $\{s_n\}$ is a sequence generated by Noor iteration (2.3) and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[0, 1]$ which satisfy the following conditions :

(i) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$;
(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$;
(iii) $0 < \liminf_{n\to\infty} \gamma_n \leq \limsup_{n\to\infty} \gamma_n < 1$.

Then $\lim_{n\to\infty} \|s_n - T_i s_n\| = 0$, for all $i = 1, 2, 3$ and $\{s_n\}$ converges strongly to a common fixed point of $\{T_i\}_{i=1}^{3}$.

In concrete applications, when calculating $\{x_n\}$, we usually follow the step :

(i) We choose the initail approximation $x_0 \in X$.
(ii) We compute $x_1 = f(T, x_0)$. Because of the various error, we do not get the exact value of x_1, but a different one, say y_1, which is however closed enough to x_1, i.e., $y_1 \approx x_1$.
(iii) Consequently, when computing $x_2 = f(T, x_1)$ we will actually compute x_2 as $x_2 = f(T, y_1)$, and error again from the computations, we will obtain in fact another valued, say y_2, closed enough to x_2, i.e., $y_2 \approx x_2$, and so on.

In this way, instead of the theoretical sequence $\{x_n\}$ defined by the given iterative method, we will practically obtain an *approximate sequence* $\{y_n\}$. We shall consider the given fixed point iteration method to be numerically **stable** if and only if for $\{y_n\}$ closed enough to $\{x_n\}$ at each stage, the approximate sequence $\{y_n\}$ still converges to the fixed point of T.

Definition 2.7. Let $\{x_n\}$ be a sequence in above procedure and converge to a fixed point p of T. Let $\{y_n\}$ be an arbitary sequence in X and set

$$\varepsilon_n = \|y_{n+1} - f(T, y_n)\|.$$

We shall say that the fixed point iteration procedure $\{x_n\}$ is T-stable or stable with respect to T if

$$\lim_{n\to\infty} \varepsilon_n = 0 \quad \text{if and only if} \quad \lim_{n\to\infty} y_n = p.$$

Theorem 2.8. Let C be a nonempty closed convex subset of a uniformly convex Banach space X and let $T : C \to C$ be a weak contraction and F-contraction mapping. Suppose $\{x_n\}$ is a sequence generated by (1.13) and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[0, 1]$ which satisfy the following conditions :

(i) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$;
(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$;
(iii) $0 < \liminf_{n\to\infty} \gamma_n \leq \limsup_{n\to\infty} \gamma_n < 1$.

Let $\{y_n\}$ be an arbitary sequence in C and define

$$\begin{aligned} s_n &= (1-\gamma_n)y_n + \gamma_n T y_n, \\ h_n &= (1-\beta_n)s_n + \beta_n T s_n, \\ \epsilon_n &= \|y_{n+1} - ((1-\alpha_n)Ts_n + \alpha_n T h_n)\|. \end{aligned}$$

Then $\{x_n\}$ is T-stable.

Proof. By Proposition 1.6 and Theorem 2.5, $\{x_n\}$ converges strongly to a unique fixed point of T, say x^*. Since T is a weak contraction and F-contraction, we have

$$\begin{aligned} \|(1-\alpha_n)Ts_n + \alpha_n T h_n - x^*\| &\le (1-\alpha_n)\|Ts_n - x^*\| + \alpha_n \|Th_n - x^*\| \\ &\le (1-\alpha_n)\delta\|s_n - x^*\| + \alpha_n\delta\|h_n - x^*\| \\ &\le (1-\alpha_n)\delta\|s_n - x^*\| \\ &\qquad + \alpha_n\delta\,[\,(1-\beta_n)\|s_n - x^*\| + \beta_n\delta\|s_n - x^*\|\,] \\ &= [\,\delta(1-\alpha_n + \alpha_n(1-\beta_n) + \alpha_n\beta_n\delta)\,]\|s_n - x^*\| \\ &= \delta(1-\alpha_n\beta_n(1-\delta))\|s_n - x^*\| \\ &\le \delta(1-\alpha_n\beta_n(1-\delta))\,[\,(1-\gamma_n)\|y_n - x^*\| + \gamma_n\delta\|y_n - x^*\|\,] \\ &= \delta(1-\alpha_n\beta_n(1-\delta))(1-\gamma_n(1-\delta))\|y_n - x^*\| \\ &\le (1-\alpha_n\beta_n(1-\delta))(1-\gamma_n(1-\delta))\|y_n - x^*\|. \end{aligned}$$

Next, assume that $\lim_{n\to\infty} \varepsilon_n = 0$. By above inequality, we have

$$\begin{aligned} \|y_{n+1} - x^*\| &\le \|y_{n+1} - [(1-\alpha_n)Ts_n + \alpha_n Th_n]\| + \|[(1-\alpha_n)Ts_n + \alpha_n Th_n] - x^*\| \\ &\le \varepsilon_n + (1-\alpha_n\beta_n(1-\delta))(1-\gamma_n(1-\delta))\|y_n - x^*\|. \end{aligned}$$

Since $\lim_{n\to\infty} \varepsilon_n = 0$, by assumption of control sequences and Lemma 2.4, we conclude that $\lim_{n\to\infty} y_n = x^*$. Conversely, suppose that $\lim_{n\to\infty} y_n = x^*$, then

$$\begin{aligned} \varepsilon_n &= \|y_{n+1} - ((1-\alpha_n)Ts_n + \alpha_n Th_n)\| \\ &\le \|y_{n+1} - x^*\| + \|x^* - [(1-\alpha_n)Ts_n + \alpha_n Th_n]\| \\ &\le \|y_{n+1} - x^*\| + (1-\alpha_n\beta_n(1-\delta))(1-\gamma_n(1-\delta))\|y_n - x^*\| \\ &\le \|y_{n+1} - x^*\| + \|y_n - x^*\|. \end{aligned}$$

Since $y_n \to x^*$, we obtain that $\lim_{n\to\infty} \varepsilon_n = 0$.
Hence $\{x_n\}$ is T-stable. □

Theorem 2.9. [20] Let C be a nonempty closed convex subset of a uniformly convex Banach space X and $T : C \to C$ be a weak contraction and F-contraction mapping. Suppose that $\{s_n\}$ is a sequence generated by Noor iteration (1.12) where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences in $[0, 1]$ which satisfy the following conditions :

(i) $0 < \liminf_{n\to\infty} \alpha_n \le \limsup_{n\to\infty} \alpha_n < 1$;
(ii) $0 < \liminf_{n\to\infty} \beta_n \le \limsup_{n\to\infty} \beta_n < 1$;
(iii) $0 < \liminf_{n\to\infty} \gamma_n \le \limsup_{n\to\infty} \gamma_n < 1$.

Let $\{y_n\}$ be an arbitary sequence in C and define

$$\begin{aligned} u_n &= (1-\gamma_n)y_n + \gamma_n T y_n, \\ z_n &= (1-\beta_n)y_n + \beta_n T u_n, \\ \epsilon_n &= \|y_{n+1} - [\,(1-\alpha_n)y_n + \alpha_n T z_n\,]\|. \end{aligned}$$

Then $\{s_n\}$ is T-stable.

3. The Rate of Convergence Theorem

There are a few papers concerning comparison of the rate of convergence of iteration methods. In 1976, Rhoades [15] introduced the concept to compare the rate of convergence of iterative methods as follows :

Definition 3.1. Let $\{x_n\}$ and $\{z_n\}$ be two iteration methods which converge to the same fixed point p, we shall say that $\{x_n\}$ *converges faster* than $\{z_n\}$ if

$$\|x_n - p\| \ \leq \ \|z_n - p\|, \ \ \text{for all } n \in \mathbb{N}.$$

In 2004, Berinde [5] provided the following concept to compare the rate of convergence of the iterative methods.

Definition 3.2. Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers that converge to a and b, respectively, and assume that there exists

$$l \ = \ \lim_{n\to\infty} \frac{|a_n - a|}{|b_n - b|}.$$

(i) If $l = 0$, then it can be said that $\{a_n\}$ converges *faster* to a than $\{b_n\}$ to b.
(ii) If $0 < l < \infty$, then it can be said that $\{a_n\}$ and $\{b_n\}$ have the same rate of convergence.

Remark 3.3. (i) In the case 1. we use the notation $a_n - a = o(b_n - b)$.
(ii) If $l = \infty$, then the sequence $\{b_n\}$ converges faster than $\{a_n\}$, that is $b_n - b = o(a_n - a)$.

Suppose that for two fixed point iteration methods $\{x_n\}$ and $\{y_n\}$, both converging to the same fixed point p, the error estimates

$$\|x_n - p\| \ \leq \ a_n, \quad n = 0, 1, 2, 3, ...$$
$$\|y_n - p\| \ \leq \ b_n, \quad n = 0, 1, 2, 3, ...$$

are available, where $\{a_n\}$ and $\{b_n\}$ are two sequences of positive numbers (converging to zero). Then, in view of Definition 3.2, the following concept appears to be very natural.

Definition 3.4. Let $\{x_n\}$ and $\{y_n\}$ be two fixed point iteration procedures that converge to the same fixed point p and satisfy above inequalities. If $\{a_n\}$ converges faster than $\{b_n\}$, then it can be said that $\{x_n\}$ *converges faster* than $\{y_n\}$ to p.

To comparision the rate of convergence in above definition depends on the error estimate sequences. So, in 2013, Phuengrattana and Suantai [13] modified above definition to compare the rate of convergence as follows :

Definition 3.5. Let $\{x_n\}$ and $\{y_n\}$ be two iterative methods converging to the same fixed point z of a mapping T. We say that $\{x_n\}$ converges faster than $\{y_n\}$ to z if

$$\lim_{n\to\infty} \frac{\|x_n - z\|}{\|y_n - z\|} \ = \ 0.$$

Theorem 3.6. Let C be a nonempty closed convex subset of Banach space X and let $T : C \to C$ be a weak contraction and F-contraction mapping. Suppose $\{x_n\}, \{p_n\}$ and $\{s_n\}$ are sequences generated by (1.13) and Noor iteration (1.12), respectively, which converge to a fixed point of T where $x_0 = s_0$, and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$,

are sequences in $[0,1]$. Then,
if $0<\alpha_n<\dfrac{1}{1+\delta}$, $\dfrac{\alpha_n(1+\delta)}{(1-\delta)}\leq\gamma_n<1$ and $\displaystyle\sum_{n=0}^{\infty}\alpha_n\beta_n=\infty$,
then $\{x_n\}$ converges faster than $\{s_n\}$.

Proof. By Proposition 1.6, $F(T)$ is nonempty. Since T is a F-contraction mapping, we obtain that a fixed point of map T is unique, say p. and by assumption, $\{x_n\}$ and $\{s_n\}$ converge to p.
First, from iteration (1.13), we have

$$\begin{aligned}
\|[x_{n+1}-p\| &\leq (1-\alpha_n)\|Tz_n-p\|+\alpha_n\|Ty_n-p\| \\
&\leq (1-\alpha_n)\delta\|z_n-p\|+\alpha_n\delta\|y_n-p\| \\
&\leq (1-\alpha_n)\delta\|z_n-p\|+\alpha_n\delta\,[\,(1-\beta_n)\|z_n-p\|+\beta_n\delta\|z_n-p\|\,] \\
&= [\,\delta(1-\alpha_n+\alpha_n(1-\beta_n)+\alpha_n\beta_n\delta)\,]\|z_n-p\| \\
&= \delta(1-\alpha_n\beta_n(1-\delta))\|z_n-p\| \\
&\leq \delta(1-\alpha_n\beta_n(1-\delta))\,[\,(1-\gamma_n)\|x_n-p\|+\gamma_n\delta\|x_n-p\|\,] \\
&= \delta(1-\alpha_n\beta_n(1-\delta))(1-\gamma_n(1-\delta))\|x_n-p\| \\
&\vdots \\
&\leq \delta^{n+1}\prod_{k=1}^{n+1}(1-\alpha_k\beta_k(1-\delta))(1-\gamma_k(1-\delta))\|x_0-p\|. \qquad (3.1)
\end{aligned}$$

Next, by iteration (1.12), we have

$$\begin{aligned}
\|s_{n+1}-p\| &= \|(1-\alpha_n)(s_n-p)+\alpha_n(Tw_n-p)\| \\
&\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\|Tw_n-p\| \\
&\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\delta\|w_n-p\| \\
&\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\delta(\,(1-\beta_n)\|s_n-p\|+\beta_n\|Th_n-p\|\,) \\
&\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\delta(\,(1-\beta_n)\|s_n-p\|+\beta_n\delta\|h_n-p\|\,) \\
&\geq (1-\alpha_n-\alpha_n\delta(1-\beta_n))\|s_n-p\| \\
&\quad -\alpha_n\beta_n\delta^2(1-\gamma_n(1-\delta))\|s_n-p\| \\
&= (1-\alpha_n(1+\delta(1-\beta_n(1-\delta(1-\gamma_n(1-\delta))))))\|s_n-p\| \\
&\geq (1-\alpha_n(1+\delta))\|s_n-p\| \\
&\vdots \\
&\geq \prod_{k=1}^{n+1}(1-\alpha_k(1+\delta))\|s_0-p\|.
\end{aligned}$$

Then

$$\frac{1}{\|s_{n+1}-p\|} \leq \frac{1}{\prod_{k=1}^{n+1}(1-\alpha_k(1+\delta))\|s_0-p\|}. \qquad (3.2)$$

It follows by (3.1) and (3.2) that

$$\frac{\|x_{n+1}-p\|}{\|s_{n+1}-p\|} \leq \frac{\prod_{k=1}^{n+1}(1-\alpha_k\beta_k(1-\delta))(1-\gamma_k(1-\delta))}{\prod_{k=1}^{n+1}(1-\alpha_k(1+\delta))}$$

$$\leq \prod_{k=1}^{n+1}(1-\alpha_k\beta_k(1-\delta)) \quad \to 0.$$

Then $\{x_n\}$ converges faster than $\{s_n\}$. □

Example 3.7. [13] Let $T:[0,1]\to[0,1]$ be defined by

$$Tx = \begin{cases} \frac{x}{3}, & \text{if } x\in [0,\frac{2}{5}), \\ \frac{2x}{5}, & \text{if } x\in [\frac{2}{5},1]. \end{cases}$$

Then T is a weak contraction and F-contraction mapping.

Proof. Let $x,y\in[0,1]$.
If $x,y\in[0,\frac{2}{5})$,

$$|Tx-Ty| = \left|\frac{x}{3}-\frac{y}{3}\right| = \frac{1}{3}|x-y|.$$

If $x,y\in[\frac{2}{5},1]$,

$$|Tx-Ty| = \left|\frac{2x}{5}-\frac{2y}{5}\right| = \frac{2}{5}|x-y|.$$

If $x\in[0,\frac{2}{5})$ and $y\in[\frac{2}{5},1]$,

$$\begin{aligned} |Tx-Ty| &= \left|\frac{x}{3}-\frac{2y}{5}\right| \leq \frac{1}{3}|x-y| + \left|\frac{y}{3}-\frac{2y}{5}\right| \\ &\leq \frac{1}{3}|x-y| + \frac{1}{15}. \\ &\leq \frac{1}{3}|x-y| + |Tx-y|. \end{aligned}$$

Choose $\delta=\frac{2}{5}$ and $L=1$, so T is a weak contraction. With the same argument as above, we can show that T satisfies condition (1.1) with $\delta'=\frac{2}{5}$, $L'=\frac{1}{4}$. So T is a F-contraction. □

Let $\{x_n\}$ and $\{s_n\}$ be sequences generated by iteration (1.13) and Noor iteration (1.12), respectively. The comparison of the convergence, we assume that the initial point $x_0=s_0=1$ and the control conditions $\alpha_n=\beta_n=\lambda_n=\frac{1}{3(n^{0.2}+1)}$ and $\gamma_n=\frac{1}{n^{0.2}+1}$. Then these control conditions satisfy Theorem 3.6.

Proof. We know that $\{n^{0.2}+1\}$ is a strictly increasing sequence in $[2,\infty)$ and by above example, we have $1+\delta=\frac{7}{5}$ and $1-\delta=\frac{3}{5}$. Then

$$\frac{1}{3(n^{0.2}+1)} \leq \frac{1}{n^{0.2}+1} < \frac{5}{7} = \frac{1}{1+\delta},$$

that is $\alpha_n+\beta_n+\lambda_n < \frac{1}{1+\delta}$. It is clearly that $\lim_{n\to\infty}\frac{1}{n^{0.2}+1}=0$. So we obtain that $\alpha_n+\beta_n+\lambda_n\to 0$. Next,

$$\alpha_n(1+\delta) = \frac{7}{5}\alpha_n \leq \frac{9}{5}\alpha_n = \gamma_n(1-\delta).$$

Then $\dfrac{\alpha_n(1+\delta)}{(1-\delta)} \leq \gamma_n < 1$. Next, we will show that $\sum_{n=1}^{\infty} \alpha_n\beta_n = \infty$.
Since $9(n^{0.2}+1)^2 \leq 9(n^{0.2}+n^{0.2})^2 = 9(2n^{0.2})^2 = 36n^{0.4}$, then we get $\dfrac{1}{36n^{0.4}} \leq \dfrac{1}{9(n^{0.2}+1)^2}$. By the p-series ($p \leq 1$), implies that $\sum_{n=1}^{\infty} \alpha_n\beta_n = \infty$.
Moreover, by the same argument as above we can show that $\sum_{n=1}^{\infty} \alpha_n = \sum_{n=1}^{\infty} \gamma_n = \infty$.
It make all sequences, $\{x_n\}$ and $\{s_n\}$, converge to unique fixed point of T, that is 0. □

n	Iteration (1.13) $\{x_n\}$	Noor iteration $\{s_n\}$
1	0.2753333333333333	0.892
2	0.0622866840398323	0.8028618015300142
3	0.0143843416773601	0.7263593546244763
4	0.0033696365323228	0.6595078099795465
5	7.9797524777686E-4	0.6004531818917709
⋮	⋮	⋮
22	3.9117640537377E-14	0.1322059479152645
23	9.9227426469139E-15	0.1215354334180748
24	2.5215123540173E-15	0.1117829732321454
25	6.4183893837157E-16	0.1028630625344521
26	1.6364198777878E-16	0.0946989830319073
27	4.1786674254824E-17	0.0872217855864026
28	1.0686318847965E-17	0.0803694081047037
29	2.7367843754926E-18	0.0740859078126916
30	7.0186215448254E-19	0.0683207907741172

Table 1. Comparison of the rate of convergence of the iterative methods (1.13) and Noor iterations for the mapping given in Example 3.7

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