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GENERAL FIXED POINT THEOREMS FOR PAIRS OF EXPANSIVE MAPPINGS WITH COMMON LIMIT RANGE PROPERTY IN G - METRIC SPACES

V. POPA[1] AND A.-M. PATRICIU[2]*

[1] \Vasile Alecsandri" University of Bacau, 600115, Romania

[2] Department of Mathematics and Computer Sciences, Faculty of Sciences and Environment, \Dunarea de Jos" University of Galati, 800201, Romania

ABSTRACT. In this paper two general fixed point theorems for pairs of extensive mappings with common limit range property in G - metric spaces are proved. In the last part of this paper, as applications, two general fixed point results for mappings satisfying extensive conditions of integral type are obtained.

KEYWORDS: Fixed point; G - metric space; Common limit range property; Implicit relation.
AMS Subject Classification: 54H25, 47H10.

1. INTRODUCTION

Let (X, d) be a metric space and S, T be two self mappings of X. In [11], Jungck defined S and T to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever (x_n) is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t,$$

for some $t \in X$.

This concept was frequently used to prove the existence theorems in fixed point theory.

Let f, g be self mappings of a nonempty set X. A point $x \in X$ is a coincidence point of f and g if $w = fx = gx$. The set of all coincidence points of f and g is denoted by $\mathcal{C}(f, g)$ and w is said a point of coincidence of f and g.

In 1994, Pant [28] introduced the notion of pointwise R - weakly commuting mappings.

* *Corresponding author.*
Email address : Alina.Patriciu@ugal.ro.
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It is proved in [29] that pointwise R – weakly commutativity is equivalent to commutativity in coincidence points.

In [12] Jungck introduced the notion of weakly compatible mappings.

Definition 1.1 ([12])**.** Let X be a nonempty set and f, g to be self mappings of X. f and g are weakly compatible if $fgu = gfu$ for $u \in \mathcal{C}(f, g)$.

Hence, f and g are weakly compatible if and only if f and g are pointwise R - weakly commuting.

The study of common fixed points for noncompatible mappings is also interesting, the work along this lines has been initiated by Pant in [25], [26], [27].

Aamri and El-Moutawakil [1] introduced a generalization of noncompatible mappings.

Definition 1.2 ([1])**.** Let S and T be two self mappings of a metric space (X, d). We say that S and T satisfy $(E.A)$ - property if there exists a sequence (x_n) in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t,$$

for some $t \in X$.

Remark 1.3. It is clear that two self mappings S and T of a metric space (X, d) will be noncompatible if there exists a sequence (x_n) in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$, but $\lim_{n\to\infty} d(TSx_n, STx_n)$ is nonzero or nonexistent. Therefore, two noncompatible self mappings of a metric space (X, d) satisfy $(E.A)$ - property.

It is proved in [30], [31] that the notions of weakly compatible mappings and mappings satisfying $(E.A)$ - property are independent.

There exists a vast literature concerning the study of fixed points for pairs of mappings satisfying $(E.A)$ - property.

In 2011, Sintunavarat and Kumam [55] introduced the idea of common limit range property.

Definition 1.4 ([55])**.** A pair (A, S) of self mappings of a metric space (X, d) is said to satisfy the limit range property with respect to S, denoted $CLR_{(S)}$, if there exists a sequence (x_n) in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t,$$

for some $t \in S(X)$.

Thus, one can infer that a pair (A, S) satisfying the $(E.A)$ - property along with the closedness of the subspace $S(X)$ always have the $CLR_{(S)}$ - property, with respect to S (see Examples 2.16, 2.17 [7]).

Some fixed point results for pairs of mappings with CLR - property are, also, obtained in [7], [8], [9], [10], [49], [54] and in other papers.

Wang et al. [58] proved some non unique fixed point theorems for expansive mappings which correspond some contractive mappings. Khan et al. [15] and Popa [32] generalized the results from [58].

Also, Rhoades [47], Taniguchi [56] generalized the results from [58] for pairs of mappings. In [33], Popa initiated the study of the unique fixed points for expansive mappings.

In [34], [35], [36] some unique fixed points theorems for two pairs of mappings are proved.

In [5], [6] Dhage introduced a new class of generalized metric space, named D – metric space. Mustafa and Sims [18], [19] proved that most of the claims concerning the fundamental topological structures on D – metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G – metric space.

In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G – metric spaces under certain conditions [20], [21], [22], [23], [53] and other papers.

Some fixed point theorems for expansive mappings in G - metric spaces are proved in [23], [17], [50], [53], [45], [46].

Some classical fixed point theorems and common fixed point theorems have recently unified considering a general condition by an implicit relation in [37], [38] and in other papers.

Recently, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi – metric spaces, ultra - metric spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two or three metric spaces, for single valued functions, hybrid pairs of mappings and set valued mappings.

Quite recently, the method is used in the study of fixed points for mappings satisfying a contractive/extensive condition of integral type, in fuzzy metric spaces, probabilistic metric spaces and G - metric spaces.

With this method, the proofs of some fixed points theorems are more simple. Also, the method allow the study of local and global properties of fixed point structures.

The study of fixed points for mappings satisfying implicit relations in G - metric spaces is initiated in [39], [42], [43], [44].

The study of fixed points for pairs of self mappings with common limit range property in metric spaces satisfying implicit relations is initiated in [9].

The study of fixed points for a pair of self mappings with common limit range property in G - metric spaces is initiated in [3].

Definition 1.5 ([14])**.** An altering distance is a function $\psi : [0, \infty) \to [0, \infty)$ satisfying:
(ψ_1) : ψ is increasing and continuous;
(ψ_2) : $\psi(t) = 0$ if and only if $t = 0$.

Fixed point theorems involving altering distances have been studied in [51], [52], [47].

Definition 1.6. An almost altering distance is a function $\psi : [0, \infty) \to [0, \infty)$ satisfying:
(ψ'_1) : ψ is continuous;
(ψ'_2) : $\psi(t) = 0$ if and only if $t = 0$.

In this paper, two general fixed point theorems for pairs of self extensive mappings with common limit range property in G - metric spaces are proved.

In the last part of this paper, as applications, two general fixed point theorems for mappings satisfying extensive conditions of integral type are proved.

2. PRELIMINARIES

Definition 2.1 ([19])**.** Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:

$(G_1) : G(x, y, z) = 0$ if $x = y = z$,
$(G_2) : 0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,
$(G_3) : G(x, y, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
$(G_4) : G(x, y, z) = G(y, z, x) = G(z, x, y) = \ldots$ (symmetry in all three variables),
$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

The function G is called a G - metric and the pair (X, G) is called a G - metric space.

Note that if $G(x, y, z) = 0$, then $x = y = z$.

Definition 2.2 ([19]). Let (X, G) be a G – metric space. A sequence (x_n) in X is said to be
a) G - convergent if for $\varepsilon > 0$, there is an $x \in X$ and $k \in \mathbb{N}$ such that for all $m, n \in \mathbb{N}$, $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$;
b) G - Cauchy if for $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for $m, n, p \in \mathbb{N}$, $m, n, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $n, m, p \to \infty$;
A G - metric space (X, G) is said to be G - complete if every G - Cauchy sequence in X is G - convergent.

Lemma 2.3 ([19]). *Let (X, G) be a G - metric space. Then, the following properties are equivalent:*
1) (x_n) is G - convergent to x;
2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$;
3) $G(x_n, x, x) \to 0$ as $n \to \infty$;
4) $G(x_n, x_m, x) \to 0$ as $n, m \to \infty$.

Lemma 2.4 ([19]). *Let (X, G) be a G - metric space. Then the following properties are equivalent:*
1) (x_n) is a G - Cauchy sequence;
2) for $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $m, n \in \mathbb{N}$, $m, n \geq k$.

Lemma 2.5 ([19]). *Let (X, G) be a G - metric space. The function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Let $\mathfrak{F}_{CL}$ be the set of all continuous functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ such that:
$(F_1) : F(t, 0, t, 0, 0, t) < 0, \forall t > 0$.
$(F_2) : F(t, t, 0, 0, t, t) < 0, \forall t > 0$.

The following functions are from $\mathfrak{F}_{CL}$.

Example 2.6. $F(t_1, ..., t_6) = t_1 - k \max\{t_2, t_3, ..., t_6\}$, where $k \in [1, \infty)$.

Example 2.7. $F(t_1, ..., t_6) = t_1 - at_2 - b \max\{t_3, t_4\} - c \max\{t_2, t_5, t_6\}$, where $a, b, c \geq 0$ and $c > 1$.

Example 2.8. $F(t_1, ..., t_6) = t_1 - k \max\{t_2, t_3, t_4, \frac{t_5 + t_6}{2}\}$, where $k \in [1, \infty)$.

Example 2.9. $F(t_1, ..., t_6) = t_1 - k \max\{t_2, \frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\}$, where $k \in [2, \infty)$.

Example 2.10. $F(t_1, ..., t_6) = t_1 - \alpha \max\{t_2, t_3, t_4\} - (1 - \alpha)(at_5 + bt_6)$, where $\alpha \in (0, 1)$ and $a, b \geq 0$, $b > 1$.

Example 2.11. $F(t_1, ..., t_6) = t_1 - at_2 - b(t_3 + t_4) - c \min\{t_5, t_6\}$, where $a, b \geq 0, b > 1$ and $a + c > 1$.

Example 2.12. $F(t_1, ..., t_6) = t_1 - at_2 - b\frac{t_5 + t_6}{1 + t_3 + t_4}$, where $a, b \geq 0$ and $b > 1$.

Example 2.13. $F(t_1,...,t_6) = t_1 - \max\{ct_2, ct_3, ct_4, at_5 + bt_6\}$, where $a, b, c \geq 0$ and $\max\{b, c\} > 1$.

Example 2.14. $F(t_1,...,t_6) = t_1 - at_2 - b\max\{t_3, t_4, \frac{2t_4+t_5}{3}, \frac{2t_4+t_6}{3}, \frac{t_5+t_6}{2}\}$, where $a, b \geq 0$ and $b > 1$.

Example 2.15. $F(t_1,...,t_6) = t_1 - at_2 - b\max\{2t_4 + t_5, 2t_4 + t_6, t_3 + t_5 + t_6\}$, where $a, b \geq 0$ and $b > \frac{1}{2}$.

Lemma 2.16 ([2]). *Let f and g be two weakly compatible self mappings on a nonempty set X. If f and g have an unique point of coincidence $w = fx = gx$, for some $x \in X$, then w is the unique common fixed point of f and g.*

3. MAIN RESULTS

Theorem 3.1. *Let T, S be self mappings of a G - metric space (X, G) such that*

$$\begin{aligned} &F(\psi(G(Tx,Tx,Ty)), \psi(G(Sx,Sx,Sy)), \psi(G(Tx,Tx,Sx)),\\ &\psi(G(Ty,Ty,Sy)), \psi(G(Sx,Sx,Ty)), \psi(G(Tx,Tx,Sy))) \geq 0 \end{aligned} \tag{3.1}$$

for all $x, y \in X$, where F satisfy property (F_2) and ψ is an almost altering distance. If there exists $u, v \in X$ such that $w = Su = Tu$ and $z = Sv = Tv$, then S and T have an unique point of coincidence.

Proof. First we prove that $Tu = Sv$. By (3.1) we obtain

$$\begin{aligned} &F(\psi(G(Tu,Tu,Tv)), \psi(G(Su,Su,Sv)), \psi(G(Tu,Tu,Su)),\\ &\psi(G(Tv,Tv,Sv)), \psi(G(Su,Su,Tv)), \psi(G(Tu,Tu,Sv))) \geq 0 \end{aligned}$$

which implies

$$F(\psi(G(w,w,z)), \psi(G(w,w,z)), 0, 0, \psi(G(w,w,z)), \psi(G(w,w,z))) \geq 0$$

a contradiction of (F_2) if $\psi(G(w,w,z)) \neq 0$. Hence $\psi(G(w,w,z)) = 0$ which implies $w = z$. Hence, $Tu = Sv = Su = Tv = w = z$. Therefore, z is a common point of coincidence of S and T.

Suppose that there exists two points of coincidence of T and S, $z_1 = Tu = Su$ and $z_2 = Tv = Sv$. By (3.1) we obtain

$$F(\psi(G(z_1,z_1,z_2)), \psi(G(z_1,z_1,z_2)), 0, 0, \psi(G(z_1,z_1,z_2)), \psi(G(z_1,z_1,z_2))) \geq 0,$$

a contradiction of (F_2) if $\psi(G(z_1,z_1,z_2)) \neq 0$. Hence $\psi(G(z_1,z_1,z_2)) = 0$ which implies $z_1 = z_2$. □

Theorem 3.2. *Let T, S be self mappings of a G - metric space (X, G) such that the inequality (3.1) holds for all $x, y \in X$, where $F \in \mathfrak{F}_{CL}$ and ψ is an almost altering distance. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S) \neq \varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.*

Proof. Since T and S satisfies $CLR_{(S)}$ - property, there exists a sequence (x_n) in X such that

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = Su,$$

for some $u \in X$.

By (3.1) we have

$$\begin{aligned} &F(\psi(G(Tu,Tu,Tx_n)), \psi(G(Su,Su,Sx_n)), \psi(G(Tu,Tu,Su)),\\ &\psi(G(Tx_n,Tx_n,Sx_n)), \psi(G(Su,Su,Tx_n)), \psi(G(Tu,Tu,Sx_n))) \geq 0. \end{aligned}$$

Letting n tend to infinity we obtain

$$F(\psi(G(Tu,Tu,Su)),0,\psi(G(Tu,Tu,Su)),0,0,\psi(G(Tu,Tu,Su)))\geq 0,$$

a contradiction of (F_2) if $\psi(G(Tu,Tu,Su))\neq 0$. Hence, $\psi(G(Tu,Tu,Su))=0$, which implies $Tu=Su=z$. Hence, $\mathcal{C}(T,S)\neq\varnothing$. By Theorem 3.1, z is the unique point of coincidence of T and S. Moreover, if T and S are weakly compatible, then by Lemma 2.16, z is the unique common fixed point of T and S. □

For $\psi(t)=t$, by Theorem 3.2 we obtain

Theorem 3.3. *Let T,S be self mappings of a G - metric space (X,G) such that:*

$$\begin{aligned}F(G(Tx,Tx,Ty),G(Sx,Sx,Sy),G(Tx,Tx,Sx),\\G(Ty,Ty,Sy),G(Sx,Sx,Ty),G(Tx,Tx,Sy))\geq 0\end{aligned} \tag{3.2}$$

for all $x,y\in X$ and $F\in\mathfrak{F}_{CL}$. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S)\neq\varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

Theorem 3.4. *Let T,S be self mappings of a G - metric space (X,G) such that:*

$$\begin{aligned}F(\psi(G(Tx,Ty,Ty)),\psi(G(Sx,Sy,Sy)),\psi(G(Tx,Sx,Sx)),\\\psi(G(Ty,Sy,Sy)),\psi(G(Sx,Ty,Ty)),\psi(G(Tx,Sy,Sy)))\geq 0\end{aligned} \tag{3.3}$$

for all $x,y\in X$, $F\in\mathfrak{F}_{CL}$ and ψ is an altering distance. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S)\neq\varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

Proof. The proof is similar to the proof of Theorem 3.2. □

If $\psi(t)=t$, by Theorem 3.4 we obtain

Theorem 3.5. *Let T,S be self mappings of a G - metric space (X,G) such that:*

$$\begin{aligned}F(G(Tx,Ty,Ty),G(Sx,Sy,Sy),G(Tx,Sx,Sx),\\G(Ty,Sy,Sy),G(Sx,Ty,Ty),G(Tx,Sy,Sy))\geq 0\end{aligned} \tag{3.4}$$

for all $x,y\in X$, where $F\in\mathfrak{F}_{CL}$. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S)\neq\varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

Remark 3.1. By Theorem 3.5 and Examples 2.6 - 2.15 we obtain particular results. For example, by 3.5 and Example 2.6 we obtain

Theorem 3.6. *Let T,S be self mappings of a G - metric space (X,G) such that*

$$\begin{aligned}G(Tx,Ty,Ty)\geq k\max\{G(Sx,Sy,Sy),G(Tx,Sx,Sx),\\G(Ty,Sy,Sy),G(Sx,Ty,Ty),G(Tx,Sy,Sy)\},\end{aligned} \tag{3.5}$$

for all $x,y\in X$, where $k>1$. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S)\neq\varnothing$. If T and S are weakly compatible, then T and S have an unique common fixed point.

4. APPLICATION: FIXED POINTS FOR MAPPINGS SATISFYING EXTENSIVE CONDITIONS OF INTEGRAL TYPE

In [4], Branciari established the following theorem which opened the way to the study of fixed points for mappings satisfying contractive/extensive conditions of integral type.

Theorem 4.1 ([4]). *Let (X, G) be a complete metric space, $c \in (0,1)$ and $f : (X,d) \to (X,d)$ such that for all $x, y \in X$*

$$\int_0^{d(fx,fy)} h(t)dt \leq c \int_0^{d(x,y)} h(t)dt,$$

whenever $h : [0,\infty) \to [0,\infty)$ is a Lebesgue measurable mapping which is summable (i.e. with finite integral) on each compact subset of $[0,\infty)$, such that $\int_0^{\varepsilon} h(t)dt > 0$, for each $\varepsilon > 0$. Then, f has an unique fixed point $z \in X$ such that for all $x \in X$, $z = \lim_{n\to\infty} f^n x$.

Some fixed point results for mappings satisfying contractive conditions of integral type are obtained in [16], [40], [41], [48] and in other papers.

The study of fixed points for pairs of mappings satisfying $CLR_{(S)}$ - property of integral type in G - metric spaces in initiated in [3].

Lemma 4.1. *Let $h : [0,\infty) \to [0,\infty)$ be as in Theorem 4.1. Then $\psi(t) = \int_0^t h(x)dx$ is an almost altering distance.*

Proof. The proof it follows from Lemma 2.5 [41]. □

Theorem 4.2. *Let T, S be self mappings of a G - metric space (X, G) such that*

$$F(\int_0^{G(Tx,Tx,Ty)} h(t)dt, \int_0^{G(Sx,Sx,Sy)} h(t)dt, \int_0^{G(Tx,Tx,Sx)} h(t)dt, \int_0^{G(Ty,Ty,Sy)} h(t)dt, \int_0^{G(Sx,Sx,Ty)} h(t)dt, \int_0^{G(Tx,Tx,Sy)} h(t)dt) \geq 0 \tag{4.1}$$

for all $x, y \in X$, where $F \in \mathfrak{F}_{LC}$ and $h(t)$ is as in Theorem 4.1. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S) \neq \varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

Proof. By Lemma 4.1, $\psi(t) = \int_0^t h(x)dx$ is an almost altering distance. By (4.1) we obtain

$$F(\psi(G(Tx,Tx,Ty)), \psi(G(Sx,Sx,Sy)), \psi(G(Tx,Tx,Sx)), \psi(G(Ty,Ty,Sy)), \psi(G(Sx,Sx,Ty)), \psi(G(Tx,Tx,Sy))) \geq 0,$$

which is the inequality (3.1). Hence, the conditions of Theorem 3.2 are satisfied and Theorem 4.3 it follows from Theorem 3.2. □

Similarly, from Theorem 3.4 we obtain

Theorem 4.3. *Let T and S be self mappings of a G - metric space (X, G) such that*

$$F(\int_0^{G(Tx,Ty,Ty)} h(t)dt, \int_0^{G(Sx,Sy,Sy)} h(t)dt, \int_0^{G(Tx,Sx,Sx)} h(t)dt, \int_0^{G(Ty,Sy,Sy)} h(t)dt, \int_0^{G(Sx,Ty,Ty)} h(t)dt, \int_0^{G(Tx,Sy,Sy)} h(t)dt) \geq 0 \tag{4.2}$$

for all $x, y \in X$, where $F \in \mathfrak{F}_{LC}$ and $h(t)$ is as in Theorem 4.1. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T,S) \neq \varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

By Theorems 4.2 and 4.3 and Examples 2.6 - 2.15 we obtain new particular results. For example, by Theorem 4.2 and Example 2.6 we obtain

Theorem 4.4. *Let T and S be self mappings of a G - metric space (X, G) such that*

$$\int_0^{G(Tx,Tx,Ty)} h(t)dt \geq k \max\{\int_0^{G(Sx,Sx,Sy)} h(t)dt, \int_0^{G(Tx,Tx,Sx)} h(t)dt, \int_0^{G(Ty,Ty,Sy)} h(t)dt, \int_0^{G(Sx,Sx,Ty)} h(t)dt, \int_0^{G(Tx,Tx,Sy)} h(t)dt\}, \quad (4.3)$$

for all $x, y \in X$, where $F \in \mathfrak{F}_{LC}$ and $h(t)$ as in Theorem 4.1. If T and S satisfies $CLR_{(S)}$ - property, then $\mathcal{C}(T, S) \neq \varnothing$. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

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