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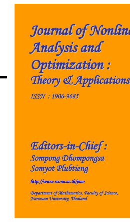

AN APPROACH FOR SIMULTANEOUSLY DETERMINING THE OPTIMAL TRAJECTORY AND CONTROL OF REDUCE THE SPREAD OF COMPUTER VIRUSES

H. R. SAHEBI*

Department of Mathematics, Ashtian Branch, Islamic Azad University, Ashtian, Iran.

ABSTRACT. In the recent decade, a considerable number of optimal control problems have been solved successfully based on the properties of the measures. Even the method, has many useful benefits, in general, it is not able to determine the optimal trajectory and control at the same time; moreover, it rarely uses the advantages of the classical solutions of the involved systems. In this article, for a Susceptible-Infected-Removed-Antidotal (SIRA) model for viruses in computer, we are going to present a new solution algorithm. First, by considering all necessary conditions, the problem is represented in a variational format in which the trajectory is shown by a trigonometric series with the unknown coefficients. Then the problem is converted into a new one that the unknowns are the mentioned coefficients and a positive Radon measure. It is proved that the optimal solution is exited and it is also explained how the optimal pair would be identified from the results deduced by a finite linear programming problem. A numerical examples is also given.

KEYWORDS : viruses Computer; Optimal Control; Measure Theory; radon Measure; SIRA Model.
AMS Subject Classification: 49QJ20, 49J45, 49M25, 76D33

1. INTRODUCTION

In recent, computer viruses are an important risk to computational systems endangering either corporation systems of all sizes or personal computers used for simple applications as accessing bank accounting or even consulting entertainment activities schedules. The viruses are being developed simultaneously with the computer systems and the use of internet facilities increases the number of damaging virus incidents. Since the first trials on studying how to combat viruses, biological analogies were established because biological organisms and computer networks share many characteristics as, for example, large number of connections among large number of simple components creating complex system [3]. Local systems

* *Corresponding author.*
Email address : sahebi@aiau.ac.ir.
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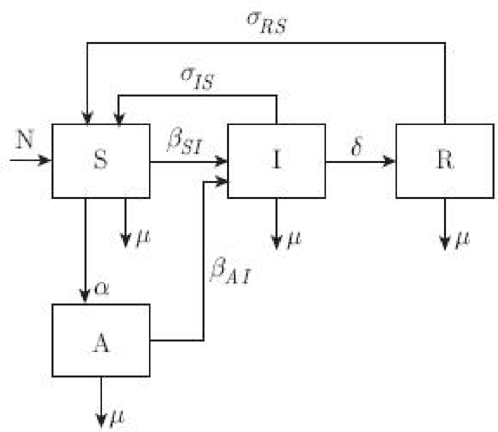
in a computer network can be attacked generating malfunctions that, spreading along the network, produce network-wide disorders following a similar qualitative model of disease spreading for a biological system. This is the main reason for designating attacks against networks by biological terms as worms and viruses. Using these ideas, it is important to consider that computer viruses have two different levels for being studied: microscopic and macroscopic [11].The microscopic level has been the subject of several studies. For instance, [1], [2] establishes theoretical principles about how to kill the new viruses created every day. Following the virus development, computer immunology is a new discipline capable of creating efficient anti-virus strategies as programs that are being sold all over the world guaranteeing protection to individual users of a global network [9]. However, the macroscopic approach has not been receiving the same attention in spite of epidemiology analogies being an important tool in order to establish the policies to preventing infections by giving figures about how to update the anti-virus programs. The interesting but simple model considering exponential variation in the number of computer viruses, proposed by [20], could not be considered realistic because the lack of limits for the growth, which is a natural phenomenon either in biological or in computer systems. There is vast catalog of Mathematical Biology models indicated for epidemiology [12]. One of them, called SIR (Susceptible-Infected-Removed) model, was originally proposed by [12]. Here, we employ a modified version of such a model in order to obtain parameter combinations representing situations with asymptotically stable disease-free solutions. The relations among network parameters can provide some hints about how to prevent infections in networks. An expression for the maximum infection rate of computers equipped with anti-virus to avoid the propagation of new infections is given. If this number is known, an updating plan for anti-virus programs in a computer network can be elaborated. According to an idea of L. C. Young, by transferring the problem in to a theoretical measure optimization, in 1986 Rubio introduced a powerful method for solving optimal control problems [17]. The important properties of the method (globality, automatic existence theorem a linear treatment even for extremely nonlinear problems, ...) caused it to be applied for the wide variety of problems. Even the method has been used frequency for solving several kinds of problems, like [4], [6] and [7] but at least two important points were not considered in applying the method yet. Generally the method can not be able to produce the acceptable optimal trajectory and control directly at the same time; and moreover, the classical format of the system solution, usually is not taken into account. Therefore, there is no any possibility to use this important fact and their related literatures in the analysis of the system. In this article, we try to bring attention these two facts; for these purposes, an optimal control problem governed by a classical epidemiological models for studying computer virus system (SIRA) with initial and boundary conditions and an integral criterion is considered as a sample. Regarding a general format of the classical solution, the problem is presented in a variational format and then by a doing deformation it is converted into a measure theoretical one with some positive coefficient. Next, extending the underlying space, using the density properties and applying some discretization scheme cause to approximate the optimal pair as a result of a finite linear programming. The approach would be improved if the number of constraints and nods are exceeded . In this manner, the optimal trajectory and control is determined at the same time.

2. The Dynamic System of SIRA

Due to the high similarity between computer virus and biological virus, some models for the spread of computer virus have been proposed . Piqueira and Navarro [15] suggested a modification of the SIRA epidemiological model for computer virus. In this section, we use the SIRA model for computer virus spread presented by Piqueira and Navarro [15] to set our control problem. In this model, they considered that the individuals in a computer system can switch between the Susceptible, Infected, Recovered and Antidotal states The system of differential equations with



time delay is defined by:

$$\begin{cases} \dot{S} = -\alpha SA - \beta_{SI}SI + \sigma_{IS}I + \sigma_{RS}R, \\ \dot{I} = \beta_{SI}SI + \beta_{AI}AI - \sigma_{IS}I - \delta I, \\ \dot{R} = \delta I - \sigma_{RS}R, \\ \dot{A} = \alpha SA - \beta_{AI}AI, \end{cases}$$

The parameters in the model are defined as follows:
δ: removing rate of infected computers;
β_{SI}: infection rate of susceptible computers;
β_{AI}: infection rate of antidotal computers due to the onset of new virus;
σ_{IS}: recovering rate of infected computers;
σ_{RS}: recovering rate of removed computers, with an operator intervention;
α: conversion of susceptible computers into antidotal ones, occurring when susceptible computers establish effective communication with antidotal ones and the antidotal install.
For $\dot{S}+\dot{I}+\dot{R}+\dot{A}=0$, then $S+A+I+R=T$, a constant for any time t. By using the optimal control theory developed by Pontryagin, we can set an optimal control problem in the $SIRA$ model to control the spread of computer virus. The main goal of this problem is to investigate an effective strategy to control the computer virus, which means that we can find an optimal strategy such that the infected nodes can meet the minimum within a specified time period. To set an optimal control problem, for given constants $\Lambda, T>0$, we choose the following as our control class:

$$U = \{u(t) \in L^2(0,T) : 0 \le u(t) \le \Lambda,\ 0 \le t \le T\}.$$

In this problem, the meaning of the control variable is that low levels of the number of infected nodes build contact to the susceptible nodes and better antivirus software. In case of high contact rate, the number of infected nodes increases while the number of susceptible, recovered and antidotal nodes decreases. With better antivirus software and lower contact rate, susceptible nodes begin to build again and more nodes are recovered from infection. Therefore, by an optimal control strategy $u(t)$, a fraction $u(t)I(t)$ of infected nodes moved from class I to class S, class R and class A. So, our optimal control problem is given by the following. The optimal control problem is formulated as:

$$\begin{array}{rl} min\ J(u) = & \int_0^T [I(t) + \frac{\epsilon u^2(t)}{2}]dt \\ \dot{S} = & -\alpha SA - \beta_{SI}SI + \sigma_{IS}I + \sigma_{RS}R + \omega u(t)I, \\ \dot{I} = & \beta_{SI}SI + \beta_{AI}AI - \sigma_{IS}I - \delta I - u(t)I, \\ \dot{R} = & \delta I - \sigma_{RS}R + (1-\omega)u(t)I, \\ \dot{A} = & \alpha SA - \beta_{AI}AI, \end{array} \tag{2.1}$$

with initial conditions $S(0) = S_0, I(0) = I_0, R(0) = R_0$ and $A(0) = A_0$ .Here $\epsilon \in [0,1]$ is a positive constant which represents the weight on the size of infected nodes and systemic cost. Note that for $\epsilon = 1$, the infected ones move to the susceptible class, while for $\epsilon = 0$, the infected nodes move to the recovered class at rate of control variable $u(t)$.

3. New Representation of the Problem

Setting $S = x_1, I = x_2, R = x_3$ and $A = x_4$. We define the function f_0 : $J \times S \times I \times R \times A \times U \to R$ as following where S, I, R, A and U are compact subsets of R.

$$f_0(t, S(t), I(t), R(t), A(t), u(t)) = f_0(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)) = x_2(t) + \frac{\epsilon u^2(t)}{2} \tag{3.1}$$

then we write the problem (2.1) in the following form:

$$\begin{array}{rl} min\ \Xi(x_1(.), x_2(.), x_3(.), x_4(.), u(.)) = & \int_0^T f_0(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t))dt \\ \dot{x_1} = & f_1(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)), \\ \dot{x_2} = & f_2(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)), \\ \dot{x_3} = & f_3(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)), \\ \dot{x_4} = & f_4(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)), \end{array} \tag{3.2}$$

where
$x_1(0) = x_{10}, x_2(0) = x_{20}, x_3(0) = x_{30}, x_4(0) = x_{40}$ and $x_1(T), x_2(T), x_3(T), x_4(T)$ are not specifed. Also,

$$f_1(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)) = -\alpha x_1 x_4 - \beta_{SI} x_1 x_2 + \sigma_{IS} x_2 + \sigma_{RS} x_3 + \omega u(t) x_2,$$
$$f_2(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)) = \beta_{SI} x_1 x_2 \beta_{AI} x_4 x_2 - \sigma_{IS} x_2 - \delta x_2 - u(t) x_2,$$
$$f_3(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)) = \delta x_2 - \sigma_{RS} x_4 + (1-\omega) u(t) x_2,$$
$$f_4(t, x_1(t), x_2(t), x_3(t), x_4(t), u(t)) = \alpha x_1 x_4 - \beta_{AI} x_4 x_2,$$

Let us consider $A = A_1 \times A_2 \times A_3 \times A_4$ and $\Omega = J \times A \times U$, where $J = [0,T], U = [0,1]$ and $A_i, i = 1, 2, 3, 4$ closed and bounded subset of R^n. Suppose that $X(t) = (x_1(t), x_2(t), x_3(t), x_4(t))$,consider the differential equation

$$\acute{X}(t) = f(t, X(t), u(t)), t \in J^0 = (0, 1) \tag{3.3}$$

where $g : \Omega \to R^n$ a continuous function, and the trajectory function $t \in J \to X(t) \in A$ is absolutely continuous and the control function $t \in J \to u(t) \in U$ is Lebesgue-measurable. We say that a trajectory control pair $w = [X(.), u(.)]$ is admissible if the following conditions hold :
(a) : $x(t) \in A, t \in J$
(b) : $u(t) \in U, t \in J$
(c) : The boundary conditions $X(0) = X_0$ is satisfied,
(d) : The pair w satisfies the differential equation (3.3) a.e. on J^0. We assume that the set of all admissible pairs is non-empty and denote it by W. Our control problem consists of finding the pair $w = [X(.), u(.)] \in W$, which minimizes the functional

$$I[X(.), u(.)] = \int_0^T f_0(t, X(t), u(t))dt \tag{3.4}$$

where $f_0 \in C(\Omega)$, the space of continuous functions on Ω, with topology of uniform convergence. This control problem may or may not have a solution in W.

4. Infinite-Dimensional Linear Programming

We may transform the above control problems to an infinite-dimensional linear programming problem. Let $w = [X(.), u(.)]$ be an admissible pair, and B an open ball in R^{n+1} containing $J \times A$ and $C(\acute{B})$ be the space of real valued continuously differentiable functions on B. Let $\phi \in C(\acute{B})$, and define function ϕ^i as follows:

$$\phi_{f_i}^{(i)}(t, X(t), u(t)) = \phi_X(t, X(t)).f_i(t, X(t), u(t)) + \phi_t(t, X(t)), i = 1, 2, 3, 4. \tag{4.1}$$

for all $(, X(t), u(t)) \in \Omega$, note that $\phi_X(t)$ is n-vector, and that the first term in the right-hand side of (4.1) is their inner product. The function ϕ^i is in the space $C(\Omega)$ the continuous functions on the compact set Ω. Since $w = [X(.), u(.)]$ is an admissible pair, we have

$$\begin{aligned} \int_0^T \phi_{f_i}^{(i)}(t, X(t), u(t))dt = & \\ &= \int_0^T \phi_X(t, X(t)).f_i(t, X(t), u(t)) + \phi_t(t, X(t)) \\ &= \int_0^T \acute{\phi}(t, X(t))dt \\ &= \phi(T, \acute{X}(T)) - \phi(0, \acute{X}(0)) = \Delta\phi_i \end{aligned} \tag{4.2}$$

for all $\phi \in \acute{C}(B)$.Let $D(J^0)$ be the space of infinitely differentiable real-valued functions with compact support in J^0. For $i = 1, 2, 3, 4$, we define

$$\psi_i(t, X(t), u(t)) = X(t)\acute{\psi}(t) + f_i(t, X(t), u(t)).\psi(t) \tag{4.3}$$

for all $\psi \in D(J^0)$, then for $\psi \in D(J^0)$ we have:

$$\begin{aligned} \int_0^T \psi_i(t, X(t), u(t))dt = & \quad \int_0^T X(t)\acute{\psi}(t)dt + \int_0^T f_i(t, X(t), u(t)).\psi(t)dt \\ &= X(t)\psi(t)|_J - \int_0^T (\acute{X}(t) - f_i(t, X(t), u(t)))\psi(t) = 0 \end{aligned}$$

since the trajectory and control function are an admissible pair satisfying (4.3) a.e. on J^0, and since the function ψ has compact support in J^0,$\psi(0) = \psi(T) = 0$, also by choosing a variable t, we have

$$\int_0^T f_i(t, X(t, u(t)))dt = a_f,\ f \in C_1(\Omega)$$

where $C_1(\Omega)$ is subspace of the space $C(\Omega)$ of all continuous function on Ω depending only on the variable t.
Now, The mapping

$$\Lambda_W : F \to \int_J F(t, X(t), u(t))dt,\ F \in C(\Omega)$$

defines a positive linear functional on $C(\Omega)$. By the Riesz representation theorem [19] there exist a unique positive Radon measure μ on Ω such that

$$\int_J F(t, X(t), u(t))dt = \int_\Omega F\ d\mu = \mu(F),\ F \in C(\Omega)$$

Thus, the minimization of the functional Ξ in (3.2) over Ω is equivalent to the minimization of

$$\Xi(\mu) = \int_\Omega f_0 d\mu = \mu(f_0) \in R \tag{4.4}$$

over the set of positive measures μ corresponding to admissible pairs w, which satisfy

$$\begin{aligned} \mu(\phi_f^{(i)}) &= \Delta\phi_i, \phi \in \acute{C}(B) \\ \mu(\psi_i) &= 0,\ \psi \in D(J^0) \\ \mu(g) &= a_g\ g \in C_1(\Omega). \end{aligned} \tag{4.5}$$

Define the set of all positive Radon measures on Ω satisfying (4.5) as Σ. Also we assume $M^+(\Omega)$ be the set of all positive Radon measures on Ω. Now if we topologize the space $M^+(\Omega)$ by the weak*- topology, it can be shown that $\Sigma$is compact [18]. In the sense of this topology,the functional $\Xi : \Sigma \to R$ define by (4.4) is a linear continuous functional on a compact set Σ, thus it attains its minimum on Σ, and so the measure theoretical problem, which consist of finding the minimum of the functional (4.4), over the subset of $M^+(\Omega)$, possesses a minimizing solution, μ^*, in Σ, [18].

5. Metamorphosis

We now estimate the optimal control by a nearly-optimal piecewise constant control. The problem (4.4) and (4.5) are an infinite dimensional linear programming problem, because all the functionals in (4.4) and (4.5) are linear in the variable μ, and furthermore μ is required to be positive. First we consider the minimization of (4.4) not only over the set Σ but over a subset of it defined by requiring that only a finite number of constraints in (4.5) be satisfied.

Theorem 5.1. *Let $\Sigma(M_1, M_2, L)$ be a subset of $M^+(\Omega)$consists of all measures which satisfy the*

$$\begin{aligned} \mu(\phi_f^{(i)}) &= \Delta\phi_i,\ i = 1, 2, ..., M_1\ \phi_i \in C_1(\Omega) \\ \mu(\psi_r) &= 0,\ r = 1, 2, ..., M_2\ \psi_r \in D(J^0) \\ \mu(g_s) &= a_{g_s},\ s = 1, 2, ..., L\ g_s \in C_1(\Omega) \end{aligned}$$

As M_1, M_2 and L tend to infinity, $\eta(M_1, M_2, L) = inf_{\Sigma(M_1,M_2,L)\mu(f_0)}$ tends to $\eta = inf_\Sigma \mu(f_0)$.

Proof. see [18].
The first stage of the approximation is completed successfully. As the second stage, it is possible to develop a finite-dimensional, linear program whose solution can be used to construct the solution of the infinite-dimensional linear program (4.4) and (4.5). From Theorem (A.5) of [18], we can characterize a measure, say μ^*, in the set $\Sigma(M_1, M_2, L)$ at which the functional $\mu \to \mu(f)$ attains its minimum, it follows from a result of [16].

Theorem 5.2. *The measure μ^* in the set $\Sigma(M_1, M_2, L)$ at which the function $\mu \to \mu(f)$ attains its minimum has the form*

$$\mu^* = \sum_{k=1}^{M_1+M_2+L} \alpha_k^* \delta(z_k^*),$$

where $z_k^* \in \Omega$; the coefficients $\alpha_k^* \geq 0,\ k = 1, 2, ..., M_1 + M_2 + L$.

Here $\delta(z)$ defines a unitary atomic measure, characterized by $\delta(z)(F) = F(z)$, where $F \in C(\Omega)$. Now the measure theoretical optimization problem is equivalent to a non-linear optimization problem, in which the unknowns are the coefficients α_k^* and supports $z_k^*,\ k = 1, 2, ..., M_1 + M_2 + L$. It would be convenient if we could minimize the functional $\mu \to \mu(f)$ only with respect to the coefficients $\alpha_k^*, k = 1, 2, ..., M_1 + M_2 + L$, this would be a linear programming problem. However, we do not know the support of the optimal measure. The answer lies in approximation of this support, by introducing a dense set in Ω.
Now, we construct a piecewise constant control function corresponding to the finite-dimensional problem. Therefore in the infinite-dimensional linear programming problem (4.4) with restriction defined by (4.5), we shall consider how one can choose total functions in the constraints (4.4) and (4.5). Consider first functions ϕ^i in $\acute{C}(B)$ as follows:

$$x_1, x_2, x_3, x_4, x_2^2, x_2^3, x_1x_2, x_3x_2, x_4x_2, tx_2 \tag{5.1}$$

Trivially the linear combinations of these functions are uniformly dense in the space $C_1(B)$ [19], we choose only M_1 number of them. Also, we choose M_2 functions with compact support in the following form:

$$\psi_r(t) = \begin{cases} sin[2\pi r\left(\frac{t-0}{T-0}\right)] & r = 1, 2, ..., M_{21}, \\ 1 - cos[2\pi r\left(\frac{t-0}{T-0}\right)] & r = M_{21} + 1, M_{21} + 2, ..., 2M_{21}. \end{cases} \tag{5.2}$$

where, $M_2 = 2M_{21}$.
Finally, it is necessary to choose L number of functions of time only, as follows:

$$g_s(t) = \begin{cases} 1 & t \in J_s, \\ 0 & otherwise, \end{cases} \tag{5.3}$$

where $J_s = \left(\frac{0+(s-1)(T-0)}{L}, \frac{0+s(T-0)}{L}\right),\ s = 1, 2, ..., L$

The set $\Omega = J \times A \times U$ will be covered with a grid, where the grid will be defined by taking all points in Ω as $z_j = (t_j, x_{1j}, x_{2j}, x_{3j}, x_{4j}, u_j)$; the points in the grid will be numbered sequentially from 1 to N, which can be estimated numerically. Instead of the infinite-dimensional linear programming problem (4.1), we consider the following .nite dimensional linear programming problem, where $z_i \in w$ (w is a

approximately dense subset of Ω).
$Minimize\ \sum_{j=1}^{N} \alpha_j f_0(z_j)$

$subject\ to:$

$$\begin{cases} \sum_{j=1}^{N} \alpha_j \phi_f^{(i)} = \Delta\phi^i & i = 1, 2, ..., M_1, \\ \sum_{j=1}^{N} \alpha_j \psi_r(z_j) = 0 & r = 1, 2, ..., M_2, \\ \sum_{j=1}^{N} \alpha_j g_s(z_j) = a_{g_s} & s = 1, 2, ..., L. \end{cases} \tag{5.4}$$

Now, by the solution of this problem, we can get the coefficients α_j, and from the analysis in Rubio [17] we can obtain the piecewise-constant control function $u(.)$ which approximate the action of the optimal measure.

6. Numerical Simulations

In this section, the optimality system is numerically solved by applying MATLAB . The values of the parameters are presented in the following Table 1.

$Parameters$	α	β_{AI}	β_{SI}	σ_{RS}	σ_{IS}	δ	ω	ϵ
$Values$	0.5	0.2	0.4	0.4	0.3	0.2	0.5	10

We consider the initial population contain susceptible nodes $S(0) = 7$, infected nodes $I(0) = 1$, recovered nodes $R(0) = 1$ and antidotal nodes $A(0) = 1$ for numerical simulation. Let us $t \in J = [0, 25]$, and $X(t) = [x_1(t), x_2(t), x_3(t), x_4(t)] \in A = A_1 \times A_2 \times A_3 \times A_4$, where $A1 = A2 = [0, 7]$, $A_3 = [0, 5], A_4 = [0, 2.5]$. Let the sets J and A_2 be divided into 10 equal subintervals, the set A_1, A_3 and A_2 are divided respectively into 4 equal subintervals, and also the set $U = [0, 1]$ divided into 4 equal subintervals, so that $\Omega = J \times A \times U$ is divided into 25600 equal subsets. We assume $Z_j = (t_j, x_{1j}, x_{2j}, x_{3j}, x_{4j}, u_j), j = 1, ..., 25600$ and

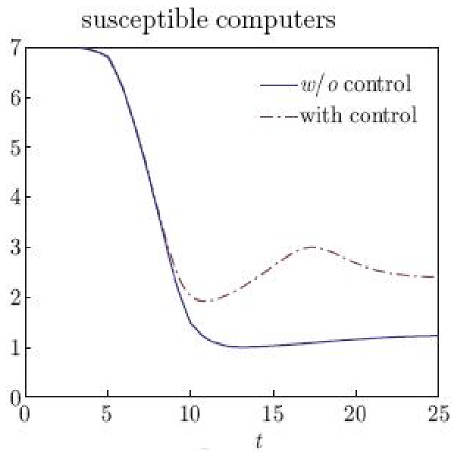
$$temp = k_1 + 4(i-1) + 16(j-1) + 64(k-1) + 640 * (f-1) + 2560(l-1)$$

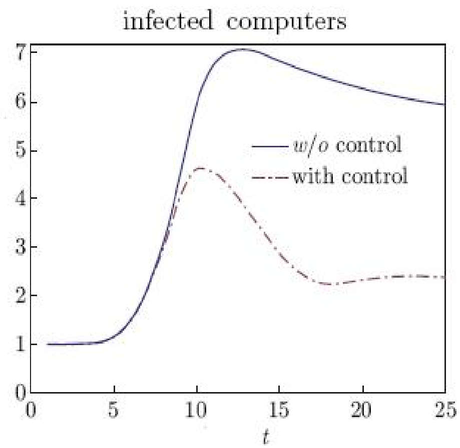
where

$$\begin{aligned} x_{1j}(temp) = -0.1 + \frac{7}{4}f;\ x_{2j}(temp) = -0.15 + \frac{7}{15}k; \\ x_{3j}(temp) = -0.25 + \frac{5}{4}j;\ x_{4j}(temp) = -0.35 + \frac{2.5}{4}i; \\ u_j(temp) = -0.45 + \frac{0.1}{4}k_1;\ t_j(temp) = 2.5l; \\ l = 1, \ldots 10; f = 1 \ldots 4; k = 1 \ldots 10; j = 1 \ldots 4; i = 1 \ldots 4; k_1 = 1 \ldots 4. \end{aligned}$$

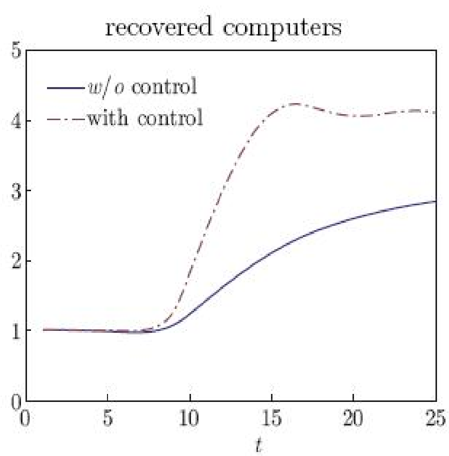
Also, we get $M_1 = 4, L = 4$ and $M_2 = 24$. And the functions $\phi_f^i\ i = 1 \ldots 4$, $h = 1 \ldots, 8$, will be chosen as the form (5.1). We have an linear programming (LP) with 25620 unknowns and 44 constrains which solved by the revised simplex code of the optimization toolbox in MATLAB. The total CPU time required on a laptop with CPU 5 GHz and 4 GB of RAM was 25.65 minutes.
In the following figures, the population size of each individual without control is marked by solid line,while the one with control is marked by dash-dotted line. The following figure represents the population size of susceptible nodes without control

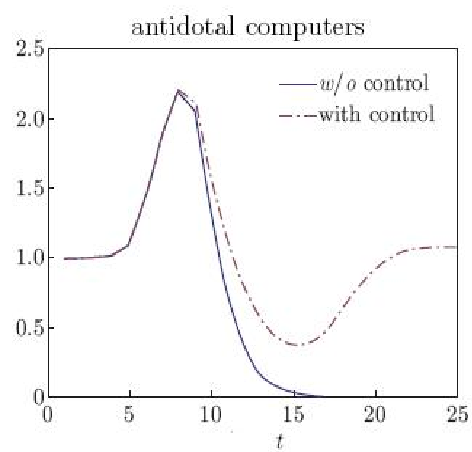
and with control. The result shows that the rate of infected susceptible nodes with optimal control strategy becomes slower and smaller number of nodes is infected from the computer virus. Also, represents the infected population in both cases. The population size of infected nodes without the optimal control strategy is larger than the nodes with control strategy.





In following figures, the recovered population becomes larger after control and the antidotal population also becomes larger after the control. Thus, after the optimal control strategy is introduced into this SIRA model, the infection rate of susceptible decreases and more infected nodes are recovered or become susceptible.

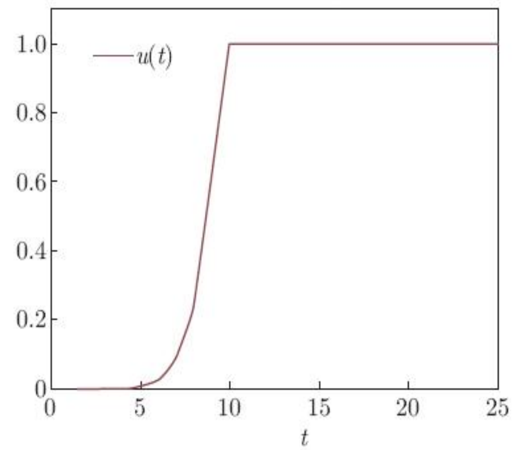




Finally, we need the optimal strategy to control the infected nodes, which is presented in following figure.

7. Conclusion

Our numerical results show that the number of infected nodes after the control is much smaller than that of infected nodes before the control. Therefore, it has



a realistic significance in the computer virus research by introducing an optimal control into an SIRA computer virus spread model. Viral attacks against computer networks are an important research area because the defense strategies need to be able to avoid infection propagation. In this work we presented the SIRA model based on epidemiological studies and conditions for the asymptotically stability of the disease free equilibrium were deduced. Some simulations were performed showing how a parameter, analogous to the epidemic basal reproduction rate, affects the dynamics of the infection propagation.

References

1. . Cohen, Computer Viruses, Theory and Experiments Therapies, Computer and Security, (19863) 22-35.
2. . Cohen, A Short Course of Computer Viruses, Computer and Security, (1990) 149-160.
3. . J. Denning, Computers under attack, Reading, Mass: Addison-Wesley, (1999).
4. . Fakharzadeh J., J. E. Rubio, Global Solution of Optimal Shape Designe Problems, J. Anal. Appl, **16**,No.1,(1999) 143-155.
5. . Fakharzadeh J, Determining the Best Domain for a Non-linear Wave System, J. Appl. Math, **13**,No.1-2,(2003)183-194.
6. . H. Farahi, The Boundary Control of the Wave Equation, PhD thesis, Dept. of Applied Mathematical Studies, Leeds University, April (1996).
7. . H. Farahi, H. H. Mehneh, A. H. Borzabadi, Wing Drag Minimization by Using Measure Theory, J. Optimization Method and Software,(2005)1-9.
8. . Friedman, Foundation of Modern Analysis, Rinehart and Winston, New York, (1970).
9. . Forrest, S. A. Hofmayer , A. Somayaji, Computer Immunology, Communications of the ACM,(1997) 88-96.
10. . V. Kamyad, J. E. Rubio , D. A. Wilson, The Optimal Control of Multidimensional Diffusion Equation, JOTA, **1(70)**, (1991)191-209.
11. . O. Kephart, S.R. White, D.M. Chess, Computers and Epidemiology. IEEE Spectrum, (1993)20-26.
12. . D. Murray, Mathematical Biology, New York: Springer-Verlag, 3rd Edn (2002).
13. . G. McKendrick, Application of Mathematics to Medical Problems Proc.Ed.Math. Soc, (1926) 98-130.
14. . Nachenberg, Computer Virus-Anti virus convolution, Communications of the ACM, (1997)40-51.
15. . R. C. Piqueira, B. F. Navarro, Epidemiological models applied to viruses in computer networks, J. Comp. Sci, **1(1)**, (2005)31-34.
16. . C. Rosenbloom, Qudques Classes de Problems Exteremaux, Bulltein de Societe Mathematique de France, **80**, (1952)183-216.
17. . E. Rubio, Control and Optimization: the Linear Treatment of Non-linear Problems, Manchester University Press, Manchester, (1986).

18. . E. Rubio, The Global Control of Non-linear Elliptic Equation, J. Franklin Institute, **330(1)**, (1993)29-35.
19. . Rudin, Real and Complex Analysis, McGraw-Hill (3Ed),(1987).
20. . S. Tippet, Computer Virus Replication, Comput. Syst. Eur,(1990)33-36.