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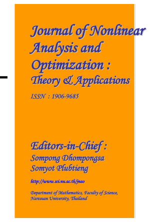

MODELLING HUMAN PERIPHERAL TISSUE TEMPERATURE BASED ON NUMERICAL COMPUTATION OF NON-LINEAR BOUNDARY VALUE PROBLEM

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ABSTRACT. A mathematical model has been established to analyse the changes in the tissue temperature on the peripheral regions of human body with respect to varying physiological conditions. The formulation is based on a parabolic heat equation with appropriate physiological parameters. The solution has been established using finite difference scheme and the simulation was done with the help of MATLAB software. It has been observed that the results have shown a considerable effect on temperature profiles due to variable physiological parameters.

KEYWORDS : Heat regulation; finite element method; bio-heat model; boundary value problem.
AMS Subject Classification(2010): 92BXX; 92CXX; 92C35; 92C50; 46N60.

1. INTRODUCTION

The heat and mass transport in human body are two important physiological processes. Any change in the homostasis disturbs the overall function of the well coordinated system. The skin is the largest organ of the human body and is responsible for all kinds of processes including temperature regulation, protecting the body from external invasion and for the manufacture of vitamin D. The body loses heat mainly through skin to its surroundings by radiation, conduction and evaporation and thus the body temperature maintains constant level under normal physiological and atmospheric conditions. Under the normal physiological and atmospheric conditions, the skin and subcutaneous tissue (SST) region of body maintains its core temperature at 37^0C. The skin is divided into two main layers epidermis and dermis. The subcutaneous or bottom layer contains muscles and fatty tissue that help keep the skin toned and firm. The dermis or main layer contains sensory nerve endings, blood and lymph vessels, hair follicles and the

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sebaceous and sweat glands. This is also where new living cells are manufactured before emerging on the surface. The epidermis, or top layer, is the visible surface of the skin, which is composed of flat, essentially dead skin cells. The unstable environmental temperature plays a great role for the disturbance in human thermoregulatory system. The effect of surrounding temperature makes its way via dermal layers and leads to hyperthermia and hypothermia in body core and other various thermal stresses to the body peripherals. From the last few years various researchers studied the distribution of temperature in the human body in relation to several environment temperatures. If T represents the temperature of the tissue at any point in the dermal layers, then the mathematical model for the distribution of temperature in the human dermal layers can be represented as:

$$k\frac{d^2T}{dr^2} + \frac{2k}{r} + \frac{dT}{dr} + Q = 0 \tag{1.1}$$

and the associated boundary conditions are

$$\lim_{r\to 0^+} \frac{\partial T}{\partial r} = 0, \quad -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = E\left(T_H - T_a\right) \tag{1.2}$$

where r is the radial distance from the origin, R is the radius of the domain, E is the ambient cooling constant, T_a ambient temperature, T_H periphery temperature, Q - the heat production per unit volume and k is the thermal conductivity inside the dermal region.

To study the effect of environmental temperatures on human dermal regions various mathematical models were formulated. Earlier experimental investigations were made by Patterson[11] to obtain temperature profiles in the SST region. Some theoretical investigations have been carried out during the last few decades by Cooper and Trezek [2], Chao et. al. [1] discussed temperature distribution in SST region under normal environmental and physiological conditions. Song et. al [16] established models describing the macro and micro vascular level heat transfer in limbs. and Jas [6] studied the thermal behaviour of human organs in malignancies. Our group had also developed numerous models in this direction [7], but the thermal conductivity was mostly assumed as constant function. In the present study, we assumed the thermal conductivity as a function of temperature in the estimation of heat regulation in human peripheral tissues.

Thron [17] studied the above model to estimate the temperature distribution in human head and suggested that if there is no singularity in the differential equation(1.1), then the solution is given by

$$T(r) = T_a + \frac{QR^2}{6k}\left[1 + \frac{2k}{ER} - \left(\frac{r}{R}\right)^2\right] \tag{1.3}$$

In addition, he calculated the temperature distribution by assuming additional heat sources while the cooling of blood at peripheral regions is given by the equation (1.3) with

$$Q = Q_0 + Q_b \tag{1.4}$$

where $Q_b = Vs(T_1 - T), Q_0$ is the heat production of tissue, V is volume of the flow of blood in unit time, T_1 is the deep temperature of the human head and $s = 0.9cal/^0Ccm^3$.

Richardson and Whitelaw [14] predicted the temperature profiles and the heat conduction and skin surface as functions of surface temperature. Flesch [4] estimated the temperature distribution using the heat equation (1.3) by assuming a heat generation rate as an explicit function of the radial distance and an implicit function of the environment temperature. Khanday and Saxena [8],[9] calculated the mass and temperature distribution at multilayered skin and sub-dermal tissues by using variational finite element method with respect to various environmental temperatures. Also, they studied the conditions under which the brain maintains thermostat and also estimated the cold effect with respect to ambient temperatures.

The present paper is an attempt to study the temperature distribution at deep dermal layers of the human body with heterogeneous thermal conductivity being a function of temperature.

2. MATHEMATICAL FORMULATION

Estimation of temperature distribution in human body using mathematical techniques has gained interest among many researchers. The heat transfer in biological tissues was studied initially by Pennes' [12] and later on Perl [13]. The existing models for heat transfer in dermal regions mostly assumed thermal conductivity term k either as constant or function of displacement. The thermal conductivity of the material may also depend on the temperature, thus it is meaningful to assume thermal conductivity of the material k as temperature dependent.

Assume k as a function of temperature as $k(T) = k_0(T - T_H)^n$. Hence, a mathematical model of the heat transfer in the human tissue follows the differential equation of heat conduction as

$$\begin{aligned}\rho c\frac{\partial T}{\partial t} =& k_0(T-T_H)^n\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)\\ &+k_0(T-T_H)^{(n-1)}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}+\frac{\partial T}{\partial z}\right)+Q \end{aligned} \tag{2.1}$$

where ρ, c and k_0 represent the density, specific heat of the tissue and thermal conductivity respectively.
In case of steady state, the above equation reduces to

$$k_0\frac{d^2T}{dx^2}+\frac{2k_0}{r}+\frac{nk_0}{(T-T_H)}\left(\frac{dT}{dr}\right)^2+\frac{Q}{(T-T_H)^n}=0 \tag{2.2}$$

Assuming the heat generation term Q as a function of temperature T and from equation (1.4) we can take $Q = q_1(37 - T)$ for some positive constant q_1, thus we have

$$k_0\frac{d^2T}{dx^2}+\frac{2k_0}{r}+\frac{nk_0}{(T-T_H)}\left(\frac{dT}{dr}\right)^2+\frac{q_1(37-T)}{(T-T_H)^n}=0 \tag{2.3}$$

The boundary conditions associated with the system are given as

$$\left(\frac{dT}{dr}\right)_{r=0}=0,\quad T(r)=T_H \tag{2.4}$$

The singular boundary value problem determining the conduction of heat in human dermal layers.

$$k_0\frac{d^2T}{dx^2}+\frac{2k_0}{r}+\frac{nk_0}{(T-T_H)}\left(\frac{dT}{dr}\right)^2+\frac{q_1(37-T)}{(T-T_H)^n}=0,\quad 0<r<R \tag{2.5}$$

$$\left(\frac{dT}{dr}\right)_{r=0}=0,\quad T(r)=T_H \tag{2.6}$$

Using transformations

$$y=T-T_H,\quad t=r/R, \tag{2.7}$$

the boundary value problem reduces to the following system of equations

$$k_0 \frac{d^2y}{dt^2} + \frac{2k_0}{y}\frac{dy}{dt} + \left(\frac{dy}{dt}\right)^2 + \frac{q_1(37 - y - T_H)R^2}{(y)^n} = 0, \quad 0 < t < 1 \tag{2.8}$$

$$\left(\frac{dT}{dt}\right)_{t=0} = 0, \quad y = T_H \tag{2.9}$$

By using the following substitution

$c(t) = t^2$ and $f(t, y, cy') = \frac{nk_0}{y}\left(\frac{dy}{dt}\right)^2 + \frac{q_1(37 - y - T_H)R^2}{(y)^n}$

(2.8) and (2.9) reduces to

$$\frac{1}{c(t)}\left[c(t)y'(t)\right]' + f(t, y, cy') = 0 \tag{2.10}$$

$$y'(0) = 0, \qquad y(1) = 0$$

The solution of this singular non-linear boundary value problem exists and is unique. To compute the approximate solution by using finite difference method has been used.

3. SOLUTION AND INTERPRETATION OF THE MODEL

The temperature distribution in human dermal regions can be obtained by solving the above boundary value problem numerically discussed as

$$k_0 \frac{d^2y}{dt^2} + \frac{2k_0}{y}\frac{dy}{dt} + \frac{nk_0}{y}\left(\frac{dy}{dt}\right)^2 + \frac{q_1(37 - y - T_H)R^2}{(y)^n} = 0, \quad 0 < t < 1 \tag{3.1}$$

$$y'(0) = 0, \qquad y(1) = 0$$

Partitioning the interval $(0, 1)$ into p subintervals with the length of each subinterval as $\frac{1}{p}$, then by the central differences, the equation (3.1) above for $i = 0$ changes into the following form

$$2k_0 y_1 + \frac{q_1(37 - y - T_H)R^2h^2}{(y_0)^n} - 2k_0 y_0 = 0, \quad 0 < t < 1 \tag{3.2}$$

and for $i = 1, 2, 3, ...(p - 1)$, we have

$$\left(1 + \frac{1}{i}\right) y_{i+1} + nk_0 \frac{(y_{i+1} - y_{i-1})^2}{4y_i} + \frac{q_1(37 - y - T_H)R^2}{(y)^n} - 2k_0 y_1 + \left(1 - \frac{1}{i}\right) y_{i-1} \tag{3.3}$$

where $p = \frac{1}{3}, q_1 = 0.000002, T_H^2 = 33.03 + 0.14(T_a - 10)$,
$k_0 = 0.00009T_H(37 - T_H)^{(1/3)}$ and T_a is an ambient temperature.

Making use of numerical technique to solve the resulting non-linear system of equations, the temperature distribution in human dermal regions at various environmental temperatures can be computed.

The heat generation calculated at the body core (brain and heart) is given in Figure-(3). It has been observed from Figure-(3) that the heat generation in these regions increases when the environment temperature decreases from 10^0C to 0^0C. The present solution is useful and realistic due to the fact that it maintains constant temperature at these regions.

4. DISCUSSION AND CONCLUSION

A Heat distribution model to understand the peripheral tissues temperature of human body has been formulated. The formulation is based on bio-heat equation with variable physiological parameters and appropriate boundary conditions. The boundary value problem determining the distribution of heat in the biological tissue has been solved using numerical method. The computation and simulation has been carried out by MATLAB software. The outcome of this study reflects some innovations in the existing models by means of temperature dependent. It is important to mention that the experiments have shown a great role of thermal conductivity on the thermal behaviour of the biological tissue. The numerical solution of the boundary value problem was carried out and the results were compared with the existing solutions as discussed by Thron [17]. The variation of temperature at various dermal layers of the underlying tissue is demonstrated in Figure (1). The curves (1) and (2) are due to thron[17] at two atmospheric temperatures $T_a = 0^0C, 10^0C$ while as the curves (3) and (4) are our results at the same ambient temperatures respectively. The figure reveals that there are gradual changes of temperature with radial distances. The study in this paper shows same temperature variation of tissues with that of Thron [17] with few significant differences in some results. The main reason for such differences is that Thron [17] treated the thermal conductivity as constant whereas it is temperature dependent in our case. Therefore, it may be said that the present study is comparatively more realistic. Similarly Figure-(2) is described at ambient temperatures $T_a = 21^0C, 25^0C$. It is evident from the Figures-$(1, 2)$ that the temperature variation in radial distances shows the same tendency with existing study of temperature variations on the human periphery. For temperature dependent thermal conductivity k, the present study shows some realistic values for the estimation of thermoregulation in human dermal layers. The value of thermal conductivity gradually increases from outer regions towards core with increase in temperature. Also a nonlinear singular differential equation with suitable boundary conditions was developed according to the equation-(14). The estimation of temperature distribution in human head for various ambient temperatures was done by various researchers including Khanday and Saxena[10], Thron [17]. They have realized that sub-lingual temperature of the head is not affected from the environmental temperature. Some realistic results were observed in this study while comparing these results with some experimental work carried out by Hodgson [5]. It is worthwhile to mention that the model can be used extensively in medical sciences and biomedical engineering.

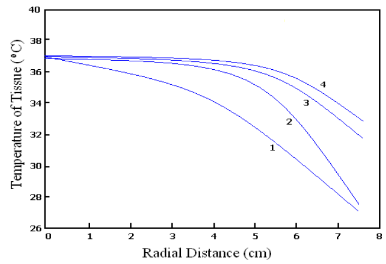

FIGURE 1. Temperature distribution in the tissue of human head versus radial distance for various values of environmental temperatures.

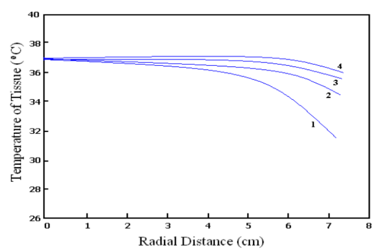

FIGURE 2. Temperature distribution in the tissue of human head versus radial distance for various values of environmental temperatures.

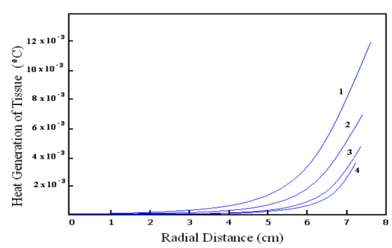

FIGURE 3. Heat generation of tissue of the human head versus radial distance for several environmental temperatures.

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