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SOME FIXED POINT RESULTS FOR GENERALIZED CONTRACTIONS IN PARTIALLY ORDERED CONE METRIC SPACES

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ABSTRACT. The aim of this paper is to present some fixed point theorems for generalized contractions by altering distance functions in a complete cone metric spaces endowed with a partial order. We also generalize fixed point theorems of J. Harjani, K. Sadarangani [J. Harjani, K. Sadarangani, Generalized contractions in partially ordered metric spaces and applications to ordinary differential equations, Nonlinear Analysis 72 (2010) 1188-1197] from metric spaces to cone metric spaces.

KEYWORDS : Cone metric space; Fixed point; Partially ordered set
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1. INTRODUCTION AND PRELIMINARIES

Let E be a real Banach space. A nonempty convex closed subset $P \subset E$ is called a cone in E if it satisfies:

(i) P is closed, nonempty and $P \neq \{0\}$,
(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$ and $x, y \in P$ imply that $ax + by \in P$,
(iii) $x \in P$ and $-x \in P$ imply that $x = 0$.

The space E can be partially ordered by the cone $P \subset E$; that is, $x \leq y$ if and only if $y - x \in P$. Also we write $x \ll y$ if $y - x \in P^o$, where P^o denotes the interior of P. A cone P is called normal if there exists a constant $K > 0$ such that $0 \leq x \leq y$ implies $\|x\| \leq K\|y\|$.
In the sequel we always suppose that E is a real Banach space, P is a cone in E with nonempty interior i.e. $P^o \neq \emptyset$ and $\leq$ is the partial ordering with respect to P.

Definition 1.1. ([1]) *Let X be a nonempty set. Assume that the mapping $d : X \times X \to E$ satisfies*

(i) $0 \leq d(x, y)$ *for all* $x, y \in X$ *and* $d(x, y) = 0$ *iff* $x = y$
(ii) $d(x, y) = d(y, x)$ *for all* $x, y \in X$

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(iii) $d(x, y) \le d(x, z) + d(z, y)$ *for all* $x, y, z \in X$.

Then d *is called a cone metric on* X, *and* (X, d) *is called a cone metric space.*

Definition 1.2. *Let* (X, d) *be a cone metric space,* $x \in X$ *and* $\{x_n\}$ *a sequence in* X. *Then*

(i) $\{x_n\}$ *is said to be convergent to* $x \in X$ *whenever for every* $c \in E$ *with* $0 \ll c$ *there is* N *such that for all* $n > N$, $d(x_n, x) \ll c$, *that is,* $\lim_{n\to\infty} x_n = x$.

(ii) $\{x_n\}$ *is called a Cauchy sequence in* X *whenever for every* $c \in E$ *with* $0 \ll c$ *there is* N *such that for all* $n, m > N$, $d(x_n, x_m) \ll c$.

(iii) (X, d) *is a complete cone metric space if every Cauchy sequence is convergent.*

The following remark will be useful in the sequel.

Remark 1.3. *([2])*

(1) *If* $u \le v$ *and* $v \ll w$, *then* $u \ll w$.

(2) *If* $0 \le u \ll c$ *for each* $c \in P^o$, *then* $u = 0$.

(3) *If* $a \le b + c$ *for each* $c \in P^o$ *then* $a \le b$.

(4) *If* $0 \le x \le y$, *and* $0 \le a$, *then* $0 \le ax \le ay$.

(5) *If* $0 \le x_n \le y_n$ *for each* $n \in \mathbb{N}$, *and* $\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y$, *then* $0 \le x \le y$.

(6) *If* $0 \le d(x_n, x) \le b_n$ *and* $b_n \to 0$, *then* $d(x_n, x) \ll c$ *where* x_n, x *are, respectively, a sequence and a given point in* X.

(7) *If* E *is a real Banach space with a cone* P *and if* $a \le \lambda a$ *where* $a \in P$ *and* $0 < \lambda < 1$, *then* $a = 0$.

(8) *If* $c \in P^o$, $0 \le a_n$ *and* $a_n \to 0$, *then there exists* N *such that for all* $n > N$ *we have* $a_n \ll c$.

The altering distance functions were introduced by Khan et al. in [3] and now we define this functions on a cone. If $P := \mathbb{R}^+$ then we have the definition 1.1 in [4].

Definition 1.4. *An altering distance function is a function* $\psi : P \to P$ *which satisfies*

(a) ψ *is continuous and nondecreasing.*

(b) $\psi(x) = 0$ *if and only if* $x = 0$.

Definition 1.5. *If* $(X, \sqsubseteq)$ *is a partially ordered set and* $f : X \to X$, *we say that* f *is monotone nondecreasing if* $x, y \in X$, $x \sqsubseteq y \Rightarrow fx \sqsubseteq fy$.

Definition 1.6. *The cone* P *is called regular if every increasing sequence which is bounded from above is convergent. That is, if* $\{x_n\}$ *is a sequence such that* $x_1 \le x_2 \le \cdots \le y$ *for some* $y \in E$, *then there is* $x \in E$ *such that* $\lim_{n\to\infty} \|x_n - x\| = 0$. *Equivalently the cone* P *is regular if and only if every decreasing sequence which is bounded from below is convergent. It has been mentioned that every regular cone is normal* [5].

Definition 1.7. P *is called minihedral cone if* $\sup\{x, y\}$ *exists for all* $x, y \in E$, *and strongly minihedral if every subset of* E *which is bounded above has a supremum* [6]. *So if cone* P *is strongly minihedral then, every subset of* P *has infimum.*

For more details and some examples about definition 1.7 and some applications on cone metric spaces refer to [7, 8].

The purpose of this paper is to present some fixed point theorems for generalized contractions involving altering distance functions that generalize the theorems of the paper [4] by Harjani and Sadarangani in the context of ordered cone metric spaces with arbitrary cones.

Existence of fixed point in partially ordered sets has been considered recently in [9]-[16].

2. MAIN RESULTS

Let $(X, \sqsubseteq)$ be a partially ordered set and suppose there exists a cone metric d in X. We define (ID) property as follows,

for all $x, y \in X$ if there exists $z \in X$ such that, $x \sqsubseteq y \sqsubseteq z$ then $d(x, y)$ and $d(y, z)$ are comparable.

Theorem 2.1. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose there exists a cone metric d in X such that (X, d) is a complete cone metric space which the (ID) property holds and if there exists a bounded decreasing sequence in P, then it converges to an element in P. Let $f : X \to X$ be a continuous and nondecreasing mapping such that*

$$\psi(d(fx, fy)) \leq \psi(d(x, y)) - \varphi(d(x, y)), \quad \text{for } x \sqsubseteq y, \tag{2.1}$$

where ψ and φ are altering distance functions. If there exists $x_0 \in X$ with $x_0 \sqsubseteq fx_0$ then f has a fixed point. Further if fixed points of f are comparable, then f has a unique fixed point.

Proof. If $x_0 = fx_0$ then the proof is finished. Suppose that $x_0 \neq fx_0$. Since $x_0 \sqsubseteq fx_0$ and f is a nondecreasing function, so

$$x_0 \sqsubseteq fx_0 \sqsubseteq f^2x_0 \sqsubseteq f^3x_0 \sqsubseteq \cdots .$$

Put $x_{n+1} := fx_n = f^n x_0$ and $a_n := d(x_{n+1}, x_n)$. Then for $n \geq 1$ we have

$$\psi(d(x_{n+1}, x_n)) = \psi(d(fx_n, fx_{n-1})) \leq \psi(d(x_n, x_{n-1})) - \varphi(d(x_n, x_{n-1})),$$

therefore

$$0 \leq \psi(a_n) \leq \psi(a_{n-1}) - \varphi(a_{n-1}) \leq \psi(a_{n-1}). \tag{2.2}$$

Since $x_n \sqsubseteq x_{n+1} \sqsubseteq x_{n+2}$ by the (ID) property we have

$$a_n \leq a_{n+1} \tag{2.3}$$

or

$$a_{n+1} \leq a_n. \tag{2.4}$$

If (2.3) holds, since ψ is nondecreasing by (2.2) we have

$$0 \leq \psi(a_n) \leq \psi(a_{n-1}) - \varphi(a_{n-1}) \leq \psi(a_n) - \varphi(a_{n-1}) \leq \psi(a_n). \tag{2.5}$$

This implies that $\varphi(a_{n-1}) = 0$ and so $a_{n-1} = 0$ for $n \geq 1$ hence

$$x_n = x_{n-1} = fx_{n-1}$$

for $n \geq 1$ are fixed points of f. If (2.4) holds, since ψ and φ are nondecreasing by relation (2.2) and induction we have

$$\begin{aligned}
\varphi(a_{n+1}) &\leq \varphi(a_n) \leq \psi(a_n) \leq \psi(a_{n-1}) - \varphi(a_{n-1}) \\
&\leq \psi(a_{n-1}) - \varphi(a_n) \\
&\leq \psi(a_{n-2}) - \varphi(a_{n-2}) - \varphi(a_n) \\
&\leq \psi(a_{n-2}) - 2\varphi(a_n) \leq \cdots \\
&\leq \psi(a_0) - n\varphi(a_n),
\end{aligned}$$

so

$$0 \leq \varphi(a_n) \leq \frac{1}{n+1}\psi(a_0) \tag{2.6}$$

for all n. By Remark 1.3-5 and since $\lim_{n\to\infty} a_n$ exists by (2.4), so

$$0 \le \varphi(a_n) \le \frac{1}{n+1}\psi(a_0) \Rightarrow 0 \le \varphi(\lim_{n\to\infty} a_n) \le \lim_{n\to\infty} \frac{1}{n+1}\psi(a_0) = 0,$$

thus $\varphi(\lim_{n\to\infty} a_n) \in P \cap -P$ and we obtain $\varphi(\lim_{n\to\infty} a_n) = 0$ and since φ is altering distance function, hence $\lim_{n\to\infty} a_n = 0$ so

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = 0. \tag{2.7}$$

Now, we will show that $\{x_n\}$ is a Cauchy sequence. Suppose that $\{x_n\}$ is not a Cauchy sequence. Then, there exists $c \gg 0$ for which we can find subsequences $\{x_{m_k}\}$ and $\{x_{n_k}\}$ of $\{x_n\}$ with $n_k > m_k > k$ such that

$$d(x_{n_k}, x_{m_k}) \ge c. \tag{2.8}$$

Further, corresponding to m_k we can choose n_k in such a way that it is the smallest integer with $n_k > m_k$ and satisfying (2.8). Then

$$d(x_{n_k-1}, x_{m_k}) \ll c. \tag{2.9}$$

Using (2.8), (2.9) and the triangular inequality, we have

$$\begin{aligned} c &\le d(x_{n_k}, x_{m_k}) \\ &\le d(x_{n_k}, x_{n_k-1}) + d(x_{n_k-1}, x_{m_k}) \\ &\ll d(x_{n_k}, x_{n_k-1}) + c. \end{aligned}$$

Letting $k \to \infty$ and using (2.7)

$$\lim_{k\to\infty} d(x_{n_k}, x_{m_k}) = c. \tag{2.10}$$

Again, the triangular inequality gives us

$$\begin{aligned} d(x_{n_k}, x_{m_k}) &\le d(x_{n_k}, x_{n_k-1}) + d(x_{n_k-1}, x_{m_k-1}) + d(x_{m_k-1}, x_{m_k}), \\ d(x_{n_k-1}, x_{m_k-1}) &\le d(x_{n_k-1}, x_{n_k}) + d(x_{n_k}, x_{m_k}) + d(x_{m_k}, x_{m_k-1}), \end{aligned}$$

Letting $k \to \infty$ in the above two inequalities and using (2.7) and (2.10), we have

$$\lim_{k\to\infty} d(x_{n_k-1}, x_{m_k-1}) = c. \tag{2.11}$$

As $n_k > m_k$ and x_{n_k} and x_{m_k} are comparable (in fact, $x_{m_k-1} \sqsubseteq x_{n_k-1}$, setting $x := x_{n_k-1}$ and $y := x_{m_k-1}$ in (2.1), we obtain

$$\psi(d(x_{n_k}, x_{m_k})) \le \psi(d(x_{n_k-1}, x_{m_k-1})) - \varphi(d(x_{n_k-1}, x_{m_k-1})).$$

Letting $k \to \infty$ and taking into account (2.10) and (2.11), we have

$$\psi(c) \le \psi(c) - \varphi(c).$$

As ψ is an altering distance function, the last inequality gives us $\varphi(c) = 0$ and, consequently, $c = 0$ which is a contradiction. This implies that the sequence $\{x_n\}$ is Cauchy and since (X, d) is complete, thus there exists $x^* \in X$ such that $x_n \to x^*$ and on the other hand f is continuous and $x_{n+1} = fx_n$ so we obtain $x^* = fx^*$.

For uniqueness let $x, y \in X$ be fixed points and x is comparable to y. Hence $fx = x$ is comparable to $fy = y$ and

$$\psi(d(x, y)) = \psi(d(fx, fy)) \le \psi(d(x, y)) - \varphi(d(x, y)).$$

The last inequality gives us $\varphi(d(x, y)) = 0$ and by altering distance functions properties this implies $d(x, y) = 0$ therefore $x = y$.

□

Example 2.1. *Let* $E = (C^1([0,1], \mathbb{R}^+), \|.\|)$*, with* $\|f\| = \|f\|_\infty + \|f'\|_\infty$*,* $X = \{f, g, h\} \subseteq E$*, and*

$$\sqsubseteq = \{(f,f), (g,g), (h,h), (g,h), (h,f), (g,f)\}$$

where $f(t) = 0, g(t) = e^t = 2h(t)$*, for all* $t \in [0,1]$*, so* $\sqsubseteq$ *is a partial order on* X*. Define* $d : X \times X \to E$ *by* $d(f,g) = f + g$ *and* $f \neq g$ *and* $d(f,f) = 0$*. It is easy to see that every Cauchy sequence on* X *is convergent, i.e.,* (X,d) *is a complete cone metric space, and if we put* $P = \{f \in E : f(t) \geq 0\}$*, then* P *is a non-normal cone while is not minihedral by* [7]*. Further, let* $T : X \to X$ *be* $Tf = f, Tg = h, Th = f$*,* $\psi(f) = f$ *and* $\varphi(f) = \frac{f}{2}$*, for all* $f \in P$*. We notice that* $g \sqsubseteq Tg$*, ID property and all conditions of Theorem 2.1 hold. Therefore* T *has a unique fixed point, i.e.,* $Tf = f$*.*

Example 2.2. *With hypothesis of Example 2.1, define* $X = \{f, g, h, k\} \subseteq E$*, and*

$$\sqsubseteq = \{(f,f), (g,g), (h,h), (k,k), (g,h), (h,f), (g,f)\}$$

where $f(t) = 0, g(t) = e^t = 2h(t) = 3k(t)$*, for all* $t \in [0,1]$*, so* $\sqsubseteq$ *is a partial order on* X*. Let* $T : X \to X$ *be* $Tf = f, Tg = h, Th = f, Tk = k, \psi(f) = f$ *and* $\varphi(f) = \frac{f}{2}$*, for all* $f \in P$*. Therefore* T *have two fixed points, i.e.,* $Tf = f$ *and* $Tk = k$*, where* f *and* k *aren't comparable.*

In the next theorem, we replace the (ID) property by strongly minihedrallity of the cone.

Theorem 2.2. *Let* $(X, \sqsubseteq)$ *be a partially ordered set and suppose that there exists a cone metric* d *in* X *with strongly minihedral cone* P*, such that* (X,d) *is a complete cone metric space. Let* $f : X \to X$ *be a continuous and nondecreasing mapping such that*

$$\psi(d(fx, fy)) \leq \psi(d(x,y)) - \varphi(d(x,y)),$$

for $x \sqsubseteq y$*, where* ψ *and* φ *are altering distance functions. If there exists* $x_0 \in X$ *with* $x_0 \sqsubseteq fx_0$ *then* f *has a fixed point.*

Proof. By the proof of the Theorem 2.1 the sequence $\{\psi(a_n)\}$ has infimum. Put $b = \inf_n \psi(a_n)$. So there exists $\{\psi(a_{n_k})\}_k$ such that $\psi(a_{n_k}) \to b$ as $k \to \infty$. Now by (2.2)

$$0 \leq \psi(a_{n_k}) \leq \psi(a_{n_k-1}) - \varphi(a_{n_k-1}) \leq \psi(a_{n_k-1}), \tag{2.12}$$

letting $k \to \infty$

$$b \leq b - \varphi(\lim_{k\to\infty} a_{n_k-1}) \leq b,$$

this implies that $\varphi(\lim_{k\to\infty} a_{n_k-1}) \in P \bigcap -P$ so $\varphi(\lim_{k\to\infty} a_{n_k-1}) = 0$. □

In the next corollary, we replace the (ID) property and strongly minihedrality of the cone by regularity.

Corollary 2.3. *Let* $(X, \sqsubseteq)$ *be a partially ordered set and suppose that there exists a cone metric* d *in* X *with regular cone* P *such that* (X,d) *is a complete cone metric space. Let* $f : X \to X$ *be a continuous and nondecreasing mapping such that*

$$\psi(d(fx, fy)) \leq \psi(d(x,y)) - \varphi(d(x,y)),$$

for $x \sqsubseteq y$*, where* ψ *and* φ *are altering distance functions. If there exists* $x_0 \in X$ *with* $x_0 \sqsubseteq fx_0$ *then* f *has a fixed point.*

Proof. By proofing of the Theorem 2.1 and relation (2.2) the sequence $\{\psi(a_n)\}$ is decreasing and bounded below and P is regular cone so

$$\varphi(\lim_{n\to\infty} a_n) = 0.$$

Now similar as the proof of the previous theorem the proof is completed. □

In the sequel, we prove that Theorems 2.1, 2.2 and corollary 2.3 are still valid where f is not necessarily continuous, but the following hypothesis holds in X, "if $\{x_n\}$ is a nondecreasing sequence in X such that $x_n \to x$ then $x_n \sqsubseteq x$ for all $n \in \mathbb{N}$".

Theorem 2.3. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exists a cone metric d in X such that (X, d) is a complete cone metric space which the (ID) property holds. Let $f : X \to X$ be a nondecreasing mapping such that*

$$\psi(d(fx, fy)) \leq \psi(d(x, y)) - \varphi(d(x, y)),$$

for $x \sqsubseteq y$, where ψ and φ are altering distance functions. If there exists $x_0 \in X$ with $x_0 \sqsubseteq fx_0$ and X satisfies in following condition
if $\{x_n\}$ is a nondecreasing sequence in X such that $x_n \to x$ then $x_n \sqsubseteq x$ for all $n \in \mathbb{N}$, then f has a fixed point.

Proof. Following the proof of Theorem 2.1 it is enough to prove that $fx^* = x^*$. Since $\{x_n\} \subset X$ is a nondecreasing sequence and $x_n \to x^*$ as $n \to \infty$. Now by hypothesis we conclude that $x_n \sqsubseteq x^*$ for all $n \in \mathbb{N}$ and for all $c \gg 0$ there exists N such that $d(x_n, x^*) \ll c$ and

$$\psi(d(x_{n+1}, fx^*)) = \psi(d(fx_n, fx^*)) \leq \psi(d(x_n, x^*)) - \varphi(d(x_n, x^*)) \leq \psi(c),$$

for all $n \geq N$. Since ψ and φ are altering distance function if $n \to \infty$ we have,

$$0 \leq \psi(\lim_{n\to\infty} d(x_{n+1}, fx^*)) \leq \psi(c),$$

for all $c \gg 0$. Thus $0 \leq \psi(\lim_{n\to\infty} d(x_{n+1}, fx^*)) \leq \psi(\frac{c}{m})$, for all $c \gg 0$ and every $m \in \mathbb{N}$, hence

$$\psi(\lim_{n\to\infty} d(x_{n+1}, fx^*)) = 0$$

so

$$\lim_{n\to\infty} d(x_{n+1}, fx^*) = 0.$$

Let $c \in E$ and $c \gg 0$ so there exists N such that $d(x_{n+1}, fx^*) \ll c$ for every $n \geq N$. Thus for some N we have

$$d(x^*, fx^*) \leq d(x^*, x_{n+1}) + d(x_{n+1}, fx^*) \ll c,$$

for every $n \geq N$. This implies that $0 \leq d(x^*, fx^*) \ll c$ for all $c \gg 0$. Then $d(x^*, fx^*) = 0$ and consequently $x^* = fx^*$. □

In what follows, we give a sufficient condition for the uniqueness of the fixed point in Theorem 2.2 and corollary 2.3. This condition is:

$$\text{"for } x, y \in X \text{ there exists } z \in X \text{ which is comparable to } x \text{ and } y." \tag{2.13}$$

Theorem 2.4. *Adding condition (2.13) to the hypothesis of Theorem 2.2 (resp. corollary 2.3) we obtain uniqueness of the fixed point of f.*

Proof. Let $x, y \in X$ are fixed points. We distinguish two cases:
Case 1. If x is comparable to y then $fx = x$ is comparable to $fy = y$ and

$$\psi(d(x, y)) = \psi(d(fx, fy)) \leq \psi(d(x, y)) - \varphi(d(x, y)).$$

The last inequality gives us $\varphi(d(x, y)) = 0$ and by altering distance functions properties this implies $d(x, y) = 0$ therefore $x = y$.
Case 2. If x is not comparable to y then there exists $z \in X$ comparable to x and y.

Monotonicity of f implies that $f^n z$ is comparable to $f^n x = x$ and to $f^n y = y$, for $n = 0, 1, 2, \cdots$. Moreover,

$$\begin{aligned}\psi(d(x, f^n z)) &= \psi(d(f^n x, f^n z)) \\ &\leq \psi(d(f^{n-1}x, f^{n-1}z)) - \varphi(d(f^{n-1}x, f^{n-1}z)) \\ &= \psi(d(x, f^{n-1}z)) - \varphi(x, f^{n-1}z)) \leq \psi(d(x, f^{n-1}z)).\end{aligned} \quad (2.14)$$

according to regularity or strongly minihedrality of the cone P, there exists $b \in E$ such that $\psi(d(x, f^n z)) \to b$ as $n \to \infty$. Now by (2.14) and altering distance functions properties ψ and φ we have

$$\psi(d(x, f^n z)) \leq \psi(d(x, f^{n-1}z)) - \varphi(d(x, f^{n-1}z)) \leq \psi(d(x, f^{n-1}z)),$$

letting $n \to \infty$

$$b \leq b - \varphi(\lim_{n\to\infty} d(x, f^{n-1}z)) \leq b,$$

this implies that

$$\varphi(\lim_{n\to\infty} d(x, f^{n-1}z)) \in P \cap -P$$

so $\varphi(\lim_{n\to\infty} d(x, f^{n-1}z)) = 0$ thus $\lim_{n\to\infty} d(x, f^{n-1}z) = 0$. And similarly $d(y, f^n z) \to 0$. Let $c \gg 0$ and $c \in E$, so there exists N such that $d(x, f^n z) \ll c$ and $d(y, f^n z) \ll c$ for all $n \geq N$. Now by triangle inequality

$$d(x, y) \leq d(x, f^n z) + d(f^n z, y) \ll 2c,$$

for all $n \geq N$. Namely $0 \leq d(x, y) \ll c$ for all $c \gg 0$. Then $d(x, y) = 0$ so $x = y$. □

Our Theorems 2.1, 2.2 with non-normal cone and Corollary 2.3 with normal cone generalize Theorems 2.1, 2.2 [4] and also Theorem 2.4 extend Theorem 2.3 [4] to cone metric version.

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