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ON GENERALIZED VECTOR QUASI EQUILIBRIUM INEQUALITY PROBLEM VIA SCALARIZATION

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ABSTRACT. In this note, we consider the nonlinear scalarization function in topological vector spaces and present some properties of it and by using the nonlinear scalarization function we establish an existence theorem for a solution of a generalized quasi-vector equilibrium problems. Moreover, we show that under suitable conditions the solution set of the generalized quasi-vector equilibrium problem is compact.

KEYWORDS : Scalarization function; Topological vector space; Generalized quasi-vector equilibrium.
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1. INTRODUCTION AND PRELIMINARIES

Equilibrium problems have been extensively studied in recent years, the origin of which can be traced back to Blum and Oettli [1]. It is well known that vector equilibrium problems provide a unified model for several classes of problems, for examples, vector variational inequality problems, vector complementarity problems, vector optimization problems, and vector saddle point problems, see [1, 10] and the references therein. It is notable that vector variational inequality was first introduced and studied by Giannessi [8] in 1980.

Later on, vector variational inequality and its various extensions have been studied by Chen and Cheng [3], Chen, Huang and Yang [4] and other authors.

Recently, Chen et al. [5] introduced a nonlinear scalarization function for a variable domain structure and obtained several important properties , such as, the subadditivity and the continuity of it in the setting of a locally convex space.

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Further, they applied their nonlinear scalarization function in order to study the existence of solutions for a generalized vector quasi-equilibrium problem (GVQEP, for short).

Inspired and motivated by the works mentioned above, we first consider nonlinear scalarization function for a variable domain structure and present some properties of it in the setting of topological vector spaces and then using the function we deal with an existence theorem for a solution of (GVQEP) without any continuity on the maps and show that the solution set of (GVQEP) is compact under our assumptions. It worth noting that the author of [2] introduced the weakest notion of continuity for a (scalar) equilibrium problem. The results presented in this paper generalized some corresponding results in the literature.

In the rest of this section we recall some definitions and preliminaries results which we need in the sequel.

Definition 1.1. Let X and Y be two topological spaces. A multi-valued map $T : X \longrightarrow 2^Y$ is :

(i) **upper semi-continuous** (u.s.c.) at $x \in X$ if for each open set V containing $T(x)$, there is an open set U containing x such that for each $t \in U$, $T(t) \subseteq V$; T is said to be u.s.c. on X if it is u.s.c. at all $x \in X$.

(ii) **lower semi-continuous** (l.s.c.) at $x \in X$ if for each open set V with $T(x) \cap V \neq \emptyset$, there is an open set U containing x such that for each $t \in U$, $T(t) \cap V \neq \emptyset$; T is said to be l.s.c. on X if it is l.s.c. at all $x \in X$.

(iii) **continuous** on X if it is at the same time u.s.c. and l.s.c. on X.

(iv) **closed** if the graph $G_r(T)$ of T, i.e., $\{(x, y) : x \in X, y \in T(x)\}$, is a closed set in $X \times Y$.

(v) **compact** if the closure of range T, i.e., $\overline{T(X)}$, is compact, where $T(X) = \bigcup_{x \in X} T(x)$.

Lemma 1.2. ([11]) Let X and Y be topological spaces and $T : X \longrightarrow 2^Y$ be a multi-valued map. The following assertions are valid.

(i) T is l.s.c. at $x \in X$ if and only if for any $y \in T(x)$, and any net $\{x_\alpha\}$, $x_\alpha \longrightarrow x$, there is a net $\{y_\alpha\}$ such that $y_\alpha \in T(x_\alpha)$ and $y_\alpha \longrightarrow y$.

(ii) If for any $x \in X, T(x)$ is compact, then T is u.s.c. on X if and only if for any net $\{x_\alpha\}$ in X such that $x_\alpha \longrightarrow x$ ($x \in X$) and for every $y_\alpha \in T(x_\alpha)$, there exist $y \in T(x)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$, such that $y_\beta \longrightarrow y$.

(iii) T is closed if and only if for any net x_α, $x_\alpha \longrightarrow x$, and any net y_α, $y_\alpha \in T(x_\alpha)$, with $y_\alpha \longrightarrow y$ one has $y \in T(x)$.

(iv) If T is closed and T is compact, then T is u.s.c..

(v) If T is u.s.c. and for each $x \in X, T(x)$ is closed set, then T is closed.

(vi) If X is compact and T is u.s.c. and for any $x \in X, T(x)$ is compact, then $T(X)$ is compact.

Theorem 1.3. ([9]) (Kakutani-Fan-Glicksberg Fixed point Theorem) Let S be a non-empty, compact and convex subset of a locally convex topological vector space. Let $F : S \longrightarrow 2^S$ be a mapping with nonempty compact and convex values. Then F has a fixed point.

2. MAIN RESULTS

In this section we first introduce a nonlinear scalarization function for a moving cone in the setting of topological vector spaces (t.v.s. for short) and then some important properties of the nonlinear scalarization function will be presented. Finally, we establish an existence theorem for a solution of (GQEP) and we show that under our assumptions the solution set (GQEP) is compact.

Let E be a topological vector space with its zero vector θ and the topological dual space E^*. By a cone $P \neq \{\theta\}$ we understand a closed convex subset of E such that $\lambda P \subseteq P$ for all $\lambda \geq 0$ and $P \cap -P = \{\theta\}$. Given a cone $P \subseteq E$, we define a *partial ordering* $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. We shall write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int}P$ if P has nonempty interior.

Definition 2.1. Let E be a t.v.s. and $C : E \longrightarrow 2^E$ a multi-valued map and for all $x \in E$, $C(x)$ be a solid cone (that is, $intC(x)$ is nonempty). Let $e : E \longrightarrow E$ be a map with $e(x) \in C(x)$ for all $x \in E$. The nonlinear scalarization function $\xi : E \times E \longrightarrow \mathbb{R}$ is defined as follows:

$$\xi(x, y) = \inf\{r \in \mathbb{R} : y \in re(x) - C(x)\}.$$

In the following we establish some important properties of the nonlinear scalarization function which generalize Propositions 2.3 and 2.4 in [5] from locally convex spaces to topological vector spaces by presenting a new proof. Moreover if we take $C(x) = P$ (P is a cone with $e \in intP$) and define $e(x) = e$ for all $x \in E$, then it collapses to the corresponding result given in [7]. It is worthwhile to note that, we will not assume $int(\bigcap_{x\in X} C(x)) \neq \emptyset$ as considered in [6]. The condition $int(\bigcap_{x\in X} C(x)) \neq \emptyset$ is too restrictive even for finite dimensional t.v.s., for example, take $X = \mathbb{R}$ and consider $C(x) = [0, \infty)$ for $x \in Q$(the rational numbers) and $C(x) = (-\infty, 0]$ for $x \in Q^c$(the irrational numbers) then $int(\bigcap_{x\in X} C(x)) = \emptyset$.

Proposition 2.2. *Let E be a t.v.s., $C : E \longrightarrow 2^E$ a multi-valued map and for all $x \in E$, $C(x)$ be a solid cone. Let $e : E \longrightarrow E$ be a function with $e(x) \in C(x)$, for all $x \in E$. Then the following assertions, for each $r \in \mathbb{R}$ and $y \in E$, are satisfied.*

(i) $\xi(x, y) = \inf\{r \in \mathbb{R} : y \in re(x) - C(x)\} = \min\{r \in \mathbb{R} : y \in re(x) - C(x)\}$;
(ii) $\xi(x, y) \leq r \iff y \in re(x) - C(x)$;
(iii) $\xi(x, y) < r \iff y \in re(x) - \text{int}C(x)$;
(iv) *If* $y_1 \preceq y_2$, *then* $\xi(x, y_1) \leq \xi(x, y_2)$;
(v) *For each fixed $x \in E$ the function $z \longrightarrow \xi(x, z)$ is continuous, positively homogeneous and subadditive on E;*
(vi) *For each fixed $x \in E$ the function $z \longrightarrow \xi(x, z)$ is bounded on some neighborhood of zero.*

Proof. (i) It is obvious from $e(x) \in intC(x)$ that , for each $y \in E$, the set

$$A(y) = \{r \in \mathbb{R} : y \in re(x) - C(x)\}$$

is nonempty. The nonlinear scalarization function ξ is well-defined, otherwise there exist a $y \in E$ and a sequence $\{r_n\}$ of real numbers such that

$$y \in r_n e(x) - C(x) \text{ and } r_n \longrightarrow -\infty.$$

Hence

$$\frac{1}{-r_n}y \in e(x) + C(x),$$

and so

$$-e(x) \in intC(x).$$

Thus

$$e(x) \in C(x) \cap -C(x) = \{\theta\},$$

and hence $\theta = e(x) \in intC(x)$ and so $C(x) = E$ which is contradicted by $C(x) \cap -C(x) = \{\theta\}$. Finally the infimum is attainable for each $y \in E$. Indeed, by the property of the infimum there exists a sequence $\{r_n\}$ of real numbers such that $r_n \longrightarrow \xi(x,y)$ with $y \in r_n e(x) - C(x)$. Hence it follows from the continuities of the scalar multiplication and the vector addition defined on E and the closedness of $C(x)$ that $\xi(x,y)e(x) - y \in C(x)$.

To see (ii), if $y \notin re(x) - C(x)$ then assume $r < \xi(x,y)$. The converse is obvious by the definition of $\xi(x,y)$.

To verify (iii) let $y \in re(x) - \mathrm{int}C(x)$. Since $re(x) - \mathrm{int}C(x)$ is an open set and the sequence $\{y + \frac{1}{n}e(x)\}$ is convergent to y, there exists a natural number n_0 such that

$$y + \frac{1}{n}e(x) \in re(x) - \mathrm{int}C(x), \quad \forall n > n_0.$$

Then

$$y \in (r - \frac{1}{n})e(x) - \mathrm{int}C(x), \quad \forall n > n_0,$$

and so for all $n > n_0$ we have

$$\xi(x,y) \leq r - \frac{1}{n} < r.$$

Conversely, if $\xi(x,y) < r$ and $re(x) - y \in C(x) \backslash intC(x)$, then

$$ne(x) - y \in C(x) \backslash intC(x),\ \forall n \in N,\ n > r,$$

and so

$$e(x) - \frac{1}{n}y = \frac{1}{n}(ne(x) - y) \in C(x) \backslash intC(x),\ \forall n \in N,\ n > r.$$

Hence $e(x) \in C(x) \backslash intC(x)$, which is a contradiction. This completes the proof of (iii).

To prove (iv), let $y_1 \preceq y_2$. If $re(x) - y_2 \in C(x)$, then it follows from $y_1 \preceq y_2$ that $re(x) - y_1 \in C(x)$ and so the proof of (iv) is finished. It follows from (i) and (ii) that, for all $r \in \mathbb{R}$,

$$\xi^{-1}(x,.)(r,\infty) = \{z \in E : \xi(x,z) \in (r,\infty)\} = (re(x) - C(x))^c,$$

and similarly

$$\xi^{-1}(x,.)(-\infty,r) = re(x) - \mathrm{int}C(x),$$

are open sets and then the function $z \longrightarrow \xi(x,z)$ is continuous. Also if t is a positive real number and $y \in E$, then

$$\xi(x,ty) = \inf\{r \in \mathbb{R} : ty \in re(x) - C(x)\} = \inf\{r \in \mathbb{R} : y \in \frac{r}{t}e(x) - C(x)\}$$

$$= t\inf\{\frac{r}{t} \in \mathbb{R} : y \in \frac{r}{t}e(x) - C(x)\} = t\inf\{r^{'} \in \mathbb{R} : x \in r^{'}e(x) - C(x)\} = t\xi(x,y),$$

and so the function $z \longrightarrow \xi(x,z)$ is positively homogenous. If $y, z \in E$ with $y \in re(x) - C(x)$ and $z \in se(x) - C(x)$ then $y + z \in (r+s)e(x) - C(x)$ and so

$$\xi(x, y+z) \leq \xi(x,y) + \xi(x,z).$$

Hence the function $z \longrightarrow \xi(x,z)$ is subadditive and so the proof of (v) is completed. Finally, if we take $r = 1$ in (ii) then $\xi(x,y) \leq 1$ for all $y \in e(x) - C(x)$ and especially for some symmetric neighborhood U of zero (note that $e(x) \in intC(x)$). Hence, for each $y \in U$, by (v), we have $-\xi(x,-y) \leq \xi(x,y) \leq 1$ and so $\xi(x,-y) \geq -1$ and hence since $U = -U$ we obtain $|\xi(x,y)| \leq 1$ for each $y \in U$, and so the proof completes. □

Theorem 2.3. *Let E be a topological vector space and K be a closed subset of E. Let $C : K \longrightarrow 2^E$ be a multi-valued map such that, for each $x \in K$, $C(x)$ is a solid convex cone, and let $e : K \longrightarrow E$ be a continuous map with $e(x) \in intC(x)$ for all $x \in E$. Define $W : K \longrightarrow 2^E$ by $W(x) = E \setminus intC(x)$, for all $x \in K$.*

(i) If the multi-valued map W is closed, then the function $(x,y) \longrightarrow \xi(x,y)$ is upper semicontinuous on $K \times K$.
(ii) If the multi-valued map C is closed, then the function $(x,y) \longrightarrow \xi(x,y)$ is lower semicontinuous on $K \times K$.

Proof. (i) Let λ be an arbitrary real number. We shall prove that

$$A(\lambda) = \{(x,y) \in K \times K : \xi(x,y) \geq \lambda\}$$

is closed.

For the purpose, let $\{(x_\alpha, z_\alpha)\}_{\alpha \in I} \in A(\lambda)$ be a net and $(x_\alpha, z_\alpha) \longrightarrow (x,z)$.
Since $(x_\alpha, z_\alpha) \in A(\lambda)$, it follows from Proposition 2.2 (iii) that

$$z_\alpha \notin \lambda e(x_\alpha) - intC(x_\alpha), \quad \forall \alpha \in I,$$

and so

$$z_\alpha \in \lambda e(x_\alpha) - W(x_\alpha), \quad \forall \alpha \in I.$$

This means

$$\lambda e(x_\alpha) - z_\alpha \in W(x_\alpha), \quad \forall \alpha \in I.$$

Since W has a closed graph, the map $x \longrightarrow e(x)$ is continuous and $(x_\alpha, z_\alpha) \longrightarrow (x,z)$, we get $\lambda e(x) - z \in W(x)$ and so $\lambda e(x) - z \notin intC(x)$. Hence it follows from Proposition 2.2 (iii) that $\xi(x,z) \geq \lambda$, which shows that $A(\lambda)$ is closed.

(ii) Let λ be an arbitrary real number. We show that

$$B(\lambda) = \{(x,y) \in K \times K : \xi(x,y) \leq \lambda\}$$

is closed. For that purpose, we use a similar way as given for (i). Let $\{(x_\alpha, z_\alpha)\}_{\alpha \in I} \in B(\lambda)$ be a net and $(x_\alpha, z_\alpha) \longrightarrow (x,z)$. Now the result follows from Proposition 2.2(ii), continuity of the map e and being closed of the graph $G_r(C)$ of C. □

We need the following definition and lemma in the sequel.

Definition 2.4. ([10]) Let X and Y be linear spaces and $C \subseteq Y$ a convex cone of Y. The function $f : X \longrightarrow Y$ is called $C-$ quasi-convex if for any $y \in Y$ the set

$$\{x \in X : f(x) \in y - C\},$$

is a convex subset of X.

Lemma 2.5. *([5]) Let E, Z and X be topological vector spaces. Let $C : X \longrightarrow 2^X$ be a multi-valued map so that for every $x \in X, C(x)$ is a proper, closed and convex cone with a nonempty interior $intC(x)$. Assume that $intC(.)$ has a continuous selection $e(.)$. $Y \subset E$ and $D \subset Z$ be nonempty convex sets. Let $g : E \longrightarrow X$ and $f : Y \times D \times Y \longrightarrow X$ be two functions. If, for each $y \in Y$, $z \in Z$, the function*

$v \longrightarrow f(y,z,v)$ is $C(g(y))-$ quasi-convex, then the function $v \longrightarrow \xi(g(y), f(y,z,v))$ is $\mathbb{R}+-$ quasi-convex.

Now, we are ready, by using the nonlinear scalarization function, present the first existence theorem of (GVQEP) without any assumption of monotonicity and semi-continuity on the multi-valued functions.

Theorem 2.6. *Let E, Z and X be locally convex topological vector spaces. Let $C : X \longrightarrow 2^X$ be a multi-valued map so that for every $x \in X, C(x)$ is a proper, closed and convex cone with a nonempty interior $intC(x)$. Assume that $intC(.)$ has a continuous selection $e(.)$. Define a multi-valued map $W : X \longrightarrow 2^X$ by $W(x) = X \backslash intC(x)$, for $x \in X$. Let $Y \subset E$ and $D \subset Z$ be nonempty compact convex sets. Let $Q : Y \longrightarrow 2^Y$ be a lower semi-continuous function and $V : Y \longrightarrow 2^D$ be multi-valued function. Let $g : E \longrightarrow X$ and $f : Y \times D \times Y \longrightarrow X$ be two functions. Suppose all the following conditions are satisfied*

(i)The multi-valued functions C, W, V and Q are closed ;
(ii)f and g are continuous on $Y \times D \times Y$ and E, respectively;
(iii) For each $y \in Y$ and $z \in D$ the function $v \in f(y,z,v)$ is $C(g(y))-$ quasi-convex;
(iv) For each $y \in Y$, $V(y)$ $(Q(y)$, respectively) is a nonempty and convex subset of D (Y, respectively).
Then, there exists $\overline{y} \in Q(\overline{y})$ and $\overline{z} \in V(\overline{y})$ such that

$$f(\overline{y}, \overline{z}, \overline{y}) - f(\overline{y}, \overline{z}, y) \notin int\ C(g(\overline{y})), \quad \forall y \in Q(\overline{y}).$$

Furthermore the solution set of (GVQEP) is a closed subset of Y and so compact.

Proof. Define $F : Y \times D \longrightarrow 2^Y$ by

$$F(y,z) = \{u \in Q(y) : \xi(g(y), f(y,z,u)) = \min_{v \in Q(y)} \xi(g(y), f(y,z,v))\},$$

where ξ is the mapping given in Lemma 2.5. It follows from lemma 2.5 and (iii) that, for each $(y,z) \in Y \times D$, the set $F(y,z)$ is convex. Moreover F is closed. Indeed, let $u_\alpha \in F(y_\alpha, z_\alpha)$ and $(u_\alpha, y_\alpha, z_\alpha) \longrightarrow (u, y, z)$ we shall show that $u \in F(y,z)$. For that purpose, let $w \in Q(y)$. Since $y_\alpha \longrightarrow y$ and Q is lower semi-continuous it follows from Lemma 1.2(i) that there is a net $w_\alpha \in Q(y_\alpha)$ with $w_\alpha \longrightarrow w$ and then using the definition of the multi-valued function F and $u_\alpha \in F(y_\alpha, z_\alpha)$ we deduce

$$\xi(g(y_\alpha), f(y_\alpha, z_\alpha, u_\alpha)) \leq \xi(g(y_\alpha), f(y_\alpha, z_\alpha, w_\alpha)),$$

and by taking limit the inequality (note ξ, g, f are continuous and $(y_\alpha, z_\alpha, w_\alpha)$ is a convergent net) and applying our assumptions

$$\xi(g(y), f(y,z,u)) \leq \xi(g(y), f(y,z,w))$$

and since $w \in Q(y)$ was an arbitrary element we get

$$\xi(g(y), f(y,z,u)) = \min_{w \in Q(y)} \xi(g(y), f(y,z,v))$$

and hence $u \in F(y,z)$. So F has a closed graph and so F is upper semi-continuous, note that F has a closed graph and the values of F are closed subsets of the compact set Y (see Lemma 1.2 (iv)).

Now we define $W : Y \times D \longrightarrow 2^{Y \times D}$, for each $(y,z) \in Y \times D$ by

$$W(y,z) = F(y,z) \times V(y).$$

Since F and V are upper semi-continuous, note that V has closed graph with closed values of the compact set Y, then W is an upper semi-continuous function. Further it has closed convex values, since F and V have closed convex values.

Hence W fulfils all the assumptions of Theorem 1.3 and so there is $(\overline{y}, \overline{z}) \in W(\overline{y}, \overline{z})$ and then $\overline{z} \in V(\overline{y})$ and $\overline{y} \in F(\overline{y}, \overline{z})$. So

$$\xi(g(\overline{y}), f(\overline{y}, \overline{z}, \overline{y})) = \min_{w \in Q(\overline{y})} \xi(g(\overline{y}), f(\overline{y}, \overline{z}, v)).$$

Then

$$\xi(g(\overline{y}), f(\overline{y}, \overline{z}, \overline{y})) \leq \xi(g(\overline{y}), f(\overline{y}, \overline{z}, v)), \quad \forall v \in Q(\overline{y}),$$

and so it follows from Proposition 2.2 (v)(in fact ξ is subadditive) that

$$0 \leq \xi(g(\overline{y}), f(\overline{y}, \overline{z}, \overline{y})) - \xi(g(\overline{y}), f(\overline{y}, \overline{z}, v)) \leq \xi(g(\overline{y}), f(\overline{y}, \overline{z}, v) - (g(\overline{y}), f(\overline{y}, \overline{z}, v)), \quad \forall v \in Q(\overline{y})$$

and so it follows from Proposition 2.2 (iii), by taking $r = 0$, that

$$f(\overline{y}, \overline{z}, \overline{y}) - f(\overline{y}, \overline{z}, v) \notin -intC(g(\overline{y})), \quad \forall v \in Q(\overline{y}).$$

Since f, g are continuous, W closed, Q lower semi-continuous through Lemma 1.2 one can see that the solution set (GQEP) is closed and so compact (note the solution set of (GQEP) is a subset of the compact set Y). This completes the proof. $\square$

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