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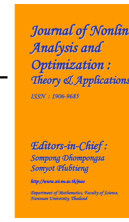

COMMON FIXED POINTS FOR WEAKLY COMPATIBLE MAPS IN INTUITIONISTIC FUZZY METRIC SPACES USING PROPERTY (S-B)

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ABSTRACT. In this note we prove a common fixed point theorem for weakly compatible maps in intuitionistic fuzzy metric spaces using property (S-B), which generalize the results of Kumar and Vats [26], Alaca, Turkoglu and Yildiz [5]

KEYWORDS :Intuitionistic fuzzy metric space, weakly compatible mappings, and (S-B) property.
AMS Subject Classification: 47H10, 54H25

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [45] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. A fuzzy set A in X is a function with domain X and values in [0, 1]. The concept of fuzzy set corresponds to the degree of nearness between two objects. Deng [9], Erceg [11], Fang [12], George and Veeramani [14], Kaleva and Seikkala [20], Kramosil and Michalek [23] have introduced the concept of fuzzy metric spaces in different ways. Atanassove [6] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Intuitionistic fuzzy sets deals with both degree of nearness and non-nearness. Park [32] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric spaces due to George and Veeramani [14]. Further, Alaca et al. [4] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space, as park with the help of continuous t-norms and continuous t-conorms, as a generalization of fuzzy metric space due to Kramosil and Michalek [24]. Further Coker [8] introduced the concept of intuitionisitc fuzzy topological spaces. Turkoglu et. al. [42] gave a generalization of Jungck's common fixed point

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theorem [17] to intuitionistic fuzzy metric spaces. They first formulated the definition of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [31]. Recently, many authors have also studied the fixed point and common fixed point theorems in intuitionistic fuzzy metric space (See [2], [39], [40], [41]). Sharma and Bamboria [38] defined a property (S-B) for self maps and obtained some common fixed point theorems for such mappings under strict contractive conditions. The class of (S-B) maps contains the class of non compatible maps. Kamran [21] obtained some coincidence and fixed point theorems for hybrid strict contractions. Sharma and Sharma [37] proved common fixed point theorem in intuitionistic fuzzy metric spaces under (S-B) property.

2. PRELIMINARIES

The concept of triangular norms (t-norm) and triangular conorms (t- conorm) are known as the axiomatic skeletons that we use for characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [29] in study of statistical metric spaces.

Definition 2.1:[36] A binary operation * : $[0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-norm if * is satisfying the following conditions:

(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a * 1 = a \,\forall\, a \in [0,1]$;
(iv) $a * b \leqslant c * d$ whenever $a \leqslant c$ and $b \leqslant d$; $\forall\, a,b,c,d \in [0,1]$

Definition 2.2:[36] A binary operation $\Diamond$: $[0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a \,\forall\, a \in [0,1]$;
(iv) $a \Diamond b \leqslant c \Diamond d$ whenever $a \leqslant c$ and $b \leqslant d$; $\forall\, a,b,c,d \in [0,1]$

Alaca et al. [4] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [24] as follows;

Definition 2.3:[4] A 5 tuple $(X, M, N, *, \Diamond)$ is said to an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions;

(i) $M(x,y,t) + N(x,y,t) \leqslant 1 \,\forall\, x,y \in X$ and $t > 0$;
(ii) $M(x,y,t) = 0 \,\forall\, x,y \in X$;
(iii) $M(x,y,t) = 1$ for all $x,y \in X$ and t > 0
(iv) $M(x,y,t) = M(y,x,t) \,\forall\, x,y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \leqslant M(x,z,t+s)$ for all $x,y,z \in X$ and s, t > 0;
(vi) For all $x,y \in X, M(x,y,.) : [0,\infty) \longrightarrow [0,1]$ is continuous;
(vii) $\lim_{t \longrightarrow \infty} M(x,y,t) = 1$ for all $x,y \in X$ and t > 0;
(viii) $N(x,y,0) = 1$ for all $x,y \in X$;
(ix) $N(x,y,t) = 0$ for all x, y $\in X$ and t > 0 iff $x = y$;
(x) $N(x,y,t) = N(y,x,t)$ for all $x,y \in X$ and t > 0;
(xi) $N(x,y,t) \Diamond N(y,z,s) \geqslant N(x,z,t+s)$ for all $x,y,z \in X$ and s, t > 0;

(xii) For all $x, y \in X, N(x, y, .) : [0, \infty) \longrightarrow [0, 1]$ is continuous;
(xiii) $\lim_{t \longrightarrow \infty} N(x, y, t) = 0$ for all $x, y \in X$;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Example 2.1:[26] Let $X = \{1/n : n \varepsilon N\} u\{0\}$ with * continuous t-norm and $\lozenge$ continuous t-conorm defined by $a * b = ab$ and $a \lozenge b$ = min $\{1, a + b\}$ respectively for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define(M, N) by

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|} & \text{if } t > 0. \\ 0 & \text{if } t = 0. \end{cases} \; and\, N(x,y,t) = \begin{cases} \frac{|x-y|}{t+|x-y|} & \text{if } t > 0. \\ 1 & \text{if } t = 0. \end{cases}$$

Then $(X, M, N$, *, $\lozenge)$ is an intuitionistic fuzzy metric space.

Remark 2.1:[27] Every fuzzy metric space $(X, M$, *)is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M$, *, $\lozenge)$ such that t-norm * and t-conorm $\lozenge$ are associated as $x \lozenge y = 1 - ((1 - x) * (1 - y)) \forall x, y \in$ X.

Remark 2.2:[26] An intuitionistic fuzzy metric spaces with continuous t-norm * and continuous t-conorm $\lozenge$ defined by a * $a \geqslant a$ and $(1-a)\lozenge(1-a) \leqslant (1-a) \forall a \in [0, 1]$. Then for all $x, y \in X, M(x, y,$ *) is non- decreasing and $N(x, y, \lozenge)$ is non-increasing.

Alaca, Turkoglu, and Yildiz [4] introduced the following notions:

Definition 2.4: Let $(X, M, N$, *,$\lozenge)$ be an intuitionistic fuzzy metric space. Then

(i) a sequence $\{x_n\}$ is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n \longrightarrow \infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n \longrightarrow \infty} N(x_{n+p}, x_n, t) = 0$

(ii) a Sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0, \lim_{n \longrightarrow \infty} M(x_n, x, t) = 1$, $\lim_{n \longrightarrow \infty} N(x_n, x, t) = 0$

Since * and $\lozenge$ are continuous, the limit is uniquely determined from (v) and (xi) of definitions 3 respectively.

Definition 2.5: An intuitionistic fuzzy metric space $(X, M, N$, *, $\lozenge)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Turkoglu, Alaca and Yildiz [43] introduced the notions of compatible mappings in intuitionistic fuzzy metric space, akin to the concept of compatible mappings introduced by Jungck [18] in metric spaces.

Definition 2.6: A pair of self mappings (f, g) of a intuitionistic fuzzy metric space $(X, M, N$, *, $\lozenge)$ is said to be compatible if $\lim_{n \longrightarrow \infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n \longrightarrow \infty} N(fgx_n, gfx_n, t = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \longrightarrow \infty} fx_n = \lim_{n \longrightarrow \infty} gx_n = z$ for some $z \in X$.

Definition 2.7: A pair of self mappings (f, g) of a intuitionistic fuzzy metric space

$(X, M, N, *, \diamondsuit)$ is said to be non-compatible if $\lim_{n\longrightarrow\infty} M(fgx_n, gfx_n, t) \neq 1$ or non- existent and $\lim_{n\longrightarrow\infty} N(fgx_n, gfx_n, t) \neq 0$ or non- existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\longrightarrow\infty} fx_n = \lim_{n\longrightarrow\infty} gx_n = z$ for some $z \in X$.

In 1998, Jungck and Rhoades [19] introduced the concept of weakly compatible maps as follows.

Definition 2.8: Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Example 2.2: Let $X = R$ and define $f, g : R \longrightarrow R$ by $fx = x/3$ and $gx = x^2 \,\forall\, x \in R$. Here 0 and 1/3 are two coincidence points for the maps$f$ and g. Note that $f$and g commute at 0, i.e.$fg(0) = gf(0) = 0$.But $fg(1/3) = f(1/9) = 1/27$ and $gf(1/3) = g(1/9) = 1/81$ and so f and g are not weakly compatible maps on R.

Remark 2.3: Weakly compatible maps need not be compatible.

Aamri and Moutawakil [1] generalized the concept of non compatibility in metric spaces by defining the notion of E.A. property and proved common fixed point theorems. Using this property Sharma and Bamboria [38] defined $(S-B)$ property in fuzzy metric space.

Definition 2.9:[38] A pair of self mappings (S, T) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$ is said to satisfy the $(S-B)$ property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\longrightarrow\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\longrightarrow\infty} N(Sx_n, Tx_n, t) = 0$.

Example 2.3: Let $X = [0, \infty)$ consider $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space, where M and N are two fuzzy sets defined by $M(x, y, t) = t/[t + d(x, y)]$ and $N(x, y, t) = d(x, y)/[t + d(x, y)]$ where d is usual metric. Define $T, S : X \longrightarrow [0, \infty)$ by $Tx = x/5$ and $Sx = 2x/5$ for all x in X. Consider $x_n = 1/n$. Now, $\lim_{n\longrightarrow\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\longrightarrow\infty} N(Sx_n, Tx_n, t) = 0$. Therefore S and T satisfy property $(S - B)$.

Now we state two lemmas which are useful in proving our main results.

Lemma 2.1:[3] Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1), M(x, y, kt) \geqslant M(x, y, t)$ and $N(x, y, kt) \leqslant N(x, y, t)$ then $x = y$.

Lemma 2.2:[3] Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that;
$M(y_{n+2}, y_{n+1}, kt) \geqslant M(y_{n+1}, y_n, t), N(y_{n+2}, y_{n+}, kt) \leqslant N(y_{n+1}, y_n, t)$ for all $t > 0$ and $n = 1, 2, \ldots$ then $\{y_n\}$ is a Cauchy sequence in X.

Alaca et al [5] proved the following theorem.

Theorem A: Let A, B, S and T be self maps of a complete intuitionistic fuzzy metric spaces $(X, M, N, *, \diamondsuit)$ with continuous t-norm * and continuous t-conorm $\diamondsuit$ defined by $a * a \geqslant a$ and $(1 - a) \leqslant (1 - a) \diamondsuit (1 - a)$ for all $a \in [0, 1]$ satisfying the following conditions;

$(A.1)$ $A(X) \subset T(X), B(X) \subset S(X)$;
$(A.2)$ S and T are continuous;
$(A.3)$ The pairs $\{A, S\}$ and $\{B, T\}$ are compatible maps;
$(A.4)$ For all$x, y \in X, t > 0$ and $k \in (0, 1)$ such that

$M(Ax, By, kt) \geqslant M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t)*$
$M(Ax, Ty, t)$
$N(Ax, By, kt) \leqslant N(Sx, Ty, t) \lozenge N(Ax, Sx, t) \lozenge N(By, Ty, t) \lozenge N(By, Sx, 2t) \lozenge N(Ax, Ty, t).$

Then A, B, S and T have a unique common fixed point in X.

Kumar and Vats [26] proved the following.

Theorem B: Let A, B, S and T be self maps of a complete intuitionistic fuzzy metric spaces$(X, M, N, *, \lozenge)$ with continuous t-norm * and continuous t-conorm $\lozenge$ defined by $a* a \geqslant a$ and$(1-a)\lozenge(1-a) \leqslant (1-a)$ for all $a \in [0, 1]$ satisfying the following conditions;

$(B.1)$ $A(X) \subset T(X), B(X) \subset S(X)$
$(B.2)$ $\forall x, y \in X, t > 0$ and $k \in (0, 1)$ such that
$M(Ax, By, kt) \geqslant M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) *$
$M(Ax, Ty, t$
$N(Ax, By, kt) \leqslant N(Sx, Ty, t) \lozenge N(Ax, Sx, t) \lozenge N(By, Ty, t) \lozenge N(By, Sx, 2t) \lozenge N(Ax, Ty, t)$

Then
(i) A and S have a point of coincidence;
(ii) B and T have a point of coincidence.
Moreover, if the pairs$\{A, S\}$ and $\{B, T\}$ are weakly compatible maps, then A, B, S and T have a unique common fixed point in X.

3. MAIN RESULTS

Now, we prove our main result which generalizes the theorems A and B.

Theorem 3.1. *Let A, B, S and T be self maps of a intuitionistic fuzzy metric spaces $(X, M, N, *, \lozenge)$ with continuous t-norm * and continuous t-conorm $\lozenge$ defined by $a * a \geqslant a$ and $(1-a)\lozenge(1-a) \leqslant (1-a)$ for all $a \in [0, 1]$satisfying the following conditions;*
(3.1) A (X) ⊂ T (X), B (X) ⊂ S (X)
(3.2) pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible.
(3.3) pairs $\{A, S\}$ or $\{B, T\}$ satisfies the property (S-B)
(3.4) for all $x, y \in X, t > 0$ and $k \in (0, 1)$such that
*M (Ax, By, kt) ⩾ M (Sx, Ty, t) * M (Ax, Sx, t) * M (By, Ty, t) * M (By, Sx, 2t) * M (Ax, Ty, t)*
N (Ax, By, kt) ⩽ N (Sx, Ty, t) ◊ N (Ax, Sx, t) ◊ N (By, Ty, t) ◊ N (By, Sx, 2t) ◊ N (Ax, Ty, t)
(3.5) one of A(X), B(X), S(X) or T(X) is a closed subset of X.
Then A, B, S and T have a unique common fixed point in X.

Proof. Suppose that (B, T) satisfies the property $(S - B)$, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Tx_n = z$ for some $z \in X$.

Since$B(X) \subset S(X)$ there exists a sequence $\{x_n\} \in$ X such that

$\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z$

Now we shall show that $\lim_{n\longrightarrow\infty} Ay_n = z$

From (3.4), we have;

$M(Ay_n, Bx_n, kt) \geqslant M(Sy_n, Tx_n, t) * M(Ay_n, Sy_n, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Sy_n, 2t)$
$* M(Ay_n, Tx_n, t)$
and
$N(Ay_n, Bx_n, kt) \leqslant N(Sy_n, Tx_n, t) \lozenge N(Ay_n, Sy_n, t) \lozenge N(Bx_n, Tx_n, t) \lozenge N(Bx_n, Sy_n, 2t) \lozenge N(Ay_n, Tx_n, t)$

Proceeding limit as $n \longrightarrow \infty$, we have;

$\lim_{n\longrightarrow\infty} M(Ay_n, z, kt) \geqslant \lim_{n\longrightarrow\infty}[M(z, z, t) * M(Ay_n, z, t) * M(z, z, t) * M(z, z, 2t)$
$* M(Ay_n, z, t)]$
and
$\lim_{n\longrightarrow\infty} N(Ay_n, z, kt) \leqslant \lim_{n\longrightarrow\infty}[N(z, z, t) \lozenge N(Ay_n, z, t) \lozenge N(z, z, t) \lozenge (z, z, 2t) \lozenge N(Ay_n, z, t)]$,

$\lim_{n\longrightarrow\infty} M(Ay_n, z, kt) \geqslant \lim_{n\longrightarrow\infty}[1 * M(Ay_n, z, t) * 1 * 1 * M(Ay_n, z, t)]$
and
$\lim_{n\longrightarrow\infty} N(Ay_n, z, kt) \leqslant \lim_{n\longrightarrow\infty}[0 \lozenge N(Ay_n, z, t) \lozenge 0 \lozenge 0 \lozenge N(Ay_n, z, t)]$

It follows that

$\lim_{n\longrightarrow\infty} M(Ay_n, z, kt) \geqslant \lim_{n\longrightarrow\infty} M(Ay_n, z, t)$
and
$\lim_{n\longrightarrow\infty} N(Ay_n, z, kt) \leqslant \lim_{n\longrightarrow\infty} N(Ay_n, z, t)$

and we deduce that $\lim_{n\longrightarrow\infty} Ay_n = z$

Suppose that $S(X)$ is a closed subset of X .Then $z = Su$ for some $u \in X$.
Subsequently, we have,

$\lim_{n\longrightarrow\infty} Ay_n = \lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Tx_n = \lim_{n\longrightarrow\infty} Sy_n = z = Su.$

Now, we shall show that $Au = Su$

From (3.4) we have,

$M(Au, Bx_n, kt) \geqslant M(Su, Tx_n, t) * M(Au, Su, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Su, 2t)$
$* M(Au, Tx_n, t)$
and
$N(Au, Bx_n, kt) \leqslant N(Su, Tx_n, t) \lozenge N(Au, Su, t) \lozenge N(Bx_n, Tx_n, t) \lozenge N(Bx_n, Su, 2t) \lozenge N(Au, Tx_n, t)$

Taking the limit as $n \longrightarrow \infty$, we have,

$M(Au,Su,kt) \geqslant M(Su,Su,t) * M(Au,Su,t) * M(Su,Su,t) * M(Su,Su,2t) * M(Au,Su,t)$
and
$N(Au,Su,kt) \leqslant N(Su,Su,t) \Diamond N(Au,Su,t) \Diamond N(Su,Su,t) \Diamond N(Su,Su,2t) \Diamond N(Au,Su,t)$,

$M(Au,Su,kt) \geqslant 1 * M(Au,Su,t) * 1 * 1 * M(Au,Su,t)$
and
$N(Au,Su,t) \leqslant 0 * N(Au,Su,t) * 0 * 0 * N(Au,Su,t)$,

$M(Au,Su,kt) \geqslant M(Au,Su,t)$
and
$N(Au,Su,kt) \leqslant N(Au,Su,t)$

Therefore by lemma 2.1, we have; $Au = Su$.

i.e. u is a coincidence point of A and S

The weak compatibility of A and S implies that $ASu = SAu$ and then $AAu = ASu = SAu = SSu$ on the other hand since $A(X) \subset T(X)$, there exists a point $v \in X$ such that $Au = Tv$. We claim that $Tv = Bv$.

Suppose that $Tv \neq Bv$. Then (3.4) implies that.

$M(Au,Bv,kt) \geqslant M(Su,Tv,t) * M(Au,Su,t) * M(Bv,Tv,t) * M(Bv,Su,2t) * M(Au,Tv,t)$
and
$N(Au,Bv,kt) \leqslant N(Su,Tv,t) \Diamond N(Au,Su,t) \Diamond N(Bv,Tv,t) \Diamond N(Bv,Su,2t) \Diamond N(Au,Tv,t)$,

$M(Au,Bv,kt) \geqslant M(Au,Au,t) * M(Au,Au,t) * M(Bv,Au,t) * M(Bv,Au,2t) * M(Au,Au,t)$
and
$N(Au,Bv,kt) \leqslant N(Au,Au,t) \Diamond N(Au,Au,t) \Diamond N(Bv,Tu,t) \Diamond N(Bv,Au,2t) \Diamond N(Au,Au,t)$,

$M(Au,Bv,kt) \geqslant 1 * 1 * M(Bv,Au,t) * M(Bv,Au,2t) * 1$
and
$N(Au,Bv,kt) \leqslant 0 \Diamond 0 \Diamond N(Bv,Au,t) \Diamond N(Bv,Au,2t) \Diamond 0$

Thus, it follows that $M(Au,Bv,kt) \geqslant M(Au,Bv,t)$
and
$N(Au,Bv,kt) \leqslant N(Au,Bv,t)$

Therefore by lemma 2.1 we have; $Au = Bv$. Hence $Tv = Bv$.

i.e. v is a coincidence point of B and T.

Thus we have: $Au = Su = Tv = Bv$.

The weak compatibility of B and T implies that

$BTv = TBv$ and $TTv = TBv = BTv = BBv$.

Let us show that Au is a common fixed point of A, B, S and T.

Suppose that $AAu \neq Au$. Then using (3.4) we have;

$M(AAu, Bv, kt) \geqslant M(SAu, Tv, t) * M(AAu, SAu, t) * M(Bv, Tv, t) * M(Bv, SAu, 2t)$
$*M(AAu, Tv, t)$
and
$N(AAu, Bv, kt) \leqslant N(SAu, Tv, t) \Diamond N(AAu, SAu, t) \Diamond N(Bv, Tv, t) \Diamond N(Bv, SAu, 2t) \Diamond N(AAu, Tv, t),$

$M(AAu, Au, kt) \geqslant M(AAu, Au, t) * M(AAu, AAu, t) * M(Bv, Bv, t) * M(Au, AAu, 2t)$
$* M(AAu, Au, t)$
and
$N(AAu, Au, kt) \leqslant N(AAu, Au, t) \Diamond N(AAu, AAu, t) \Diamond N(Bv, Bv, t) \Diamond N(Au, AAu, 2t) \Diamond N(AAu, Au, t),$

$M(AAu, Au, kt) \geqslant M(AAu, Au, t) * 1 * 1 * M(Au, AAu, 2t) * M(AAu, Au, t)$
and
$N(AAu, Au, kt) \leqslant N(AAu, Au, t) \Diamond 0 \Diamond 0 \Diamond N(Au, AAu, 2t) \Diamond N(AAu, Au, t)$

$M(AAu, Au, kt) \geqslant M(AAu, Au, t)$
and
$N(AAu, Au, kt) \leqslant N(AAu, Au, t)$

Therefore by lemma 2.1, we have; $AAu = Au$.

Since $AAu = SAu$, therefore,$Au = AAu = SAu$

i.e. Au is a common fixed point of A and S.

Similarly, we prove that Bv is a common fixed point of B and T. Since $Au = Bv$, we conclude that Au is a common fixed point of A, B, S and T. The proof is similar when $T(X)$ is assumed to be a closed subset of X. The cases in which $A(X)$ or $B(X)$ is a closed subset of X are similar to the cases in which $T(X)$ or $S(X)$ respectively is closed subset of X, since $A(X) \subset T(X)$ and $B(X) \subset S(X)$. Uniqueness follows easily.

□

Theorem 3.2. *Let A, B and S be self maps of a intuitionistic fuzzy metric spaces $(X, M, N, *, \Diamond)$ with continuous t-norm * and continuous t-conorm $\Diamond$ defined by $a * a \geqslant a$ and $(1-a) \Diamond (1-a) \leqslant (1-a)$ for all $a \in [0,1]$ satisfying the following conditions;*
(3.21) A (X) ⊂ S (X), B (X) ⊂ S (X)
(3.22) pairs $\{A, S\}$ and $\{B, S\}$ are weakly compatible.
(3.23) pairs $\{A, S\}$ or $\{B, S\}$ satisfies the property (S-B)
(3.24) for all $x, y \in X, t > 0$ and $k \in (0,1)$ such that
$M^2(Ax, By, kt) \geqslant [M(Ax, Sx, t) * M(By, Sx, t)]^2$
$N^2(Ax, By, kt) \leqslant [N(Ax, Sx, t) \Diamond N(By, Sx, t)]^2$
(3.25) one of A(X), B(X), S(X) or T(X) is a closed subset of X.
Then A, B and S have a unique common fixed point in X.

Proof. Suppose that (B, S) satisfies the property $(S-B)$, then there exists a sequence $\{x_n\}$ in XSuch that $\lim_{n \longrightarrow \infty} Bx_n = \lim_{n \longrightarrow \infty} Sx_n = z$ for some $z \in X$.

Since $B(X) \subset S(X)$ there exists a sequence$\{y_n\} \in X$ such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z$

Now, we shall show that $\lim_{n\longrightarrow\infty} Ay_n = z$

From (3.24), we have; (at $x = y_n, y = x_n$)

$[M(Ay_n, Bx_n, kt)]^2 \geqslant [M(Ay_n, Sy_n, t) * M(Bx_n, Sy_n, t)]^2$

$[N(Ay_n, Bx_n, kt)]^2 \leqslant [N(Ay_n, Sy_n, t) \lozenge N(Bx_n, Sy_n, t)]^2$

Proceeding limit as $n \longrightarrow \infty$, we have;

$[\lim_{n\longrightarrow\infty} M(Ay_n, z, kt)]^2 \geqslant [\lim_{n\longrightarrow\infty} M(Ay_n, z, t) * M(z, z, t)]^2$
and
$[\lim_{n\longrightarrow\infty} N(Ay_n, z, kt)]^2 \leqslant [\lim_{n\longrightarrow\infty} N(Ay_n, z, t) \lozenge N(z, z, t)]^2$

$[\lim_{n\longrightarrow\infty} M(Ay_n, z, kt)]^2 \geqslant [\lim_{n\longrightarrow\infty} M(Ay_n, z, t) * 1]^2$
and
$[\lim_{n\longrightarrow\infty} N(Ay_n, z, kt)]^2 \leqslant [\lim_{n\longrightarrow\infty} N(Ay_n, z, t) \lozenge 0]^2$

Thus, it follows that,

$\lim_{n\longrightarrow\infty} M(Ay_n, z, kt) \geqslant \lim_{n\longrightarrow\infty} M(Ay_n, z, t)$
and
$\lim_{n\longrightarrow\infty} N(Ay_n, z, kt) \leqslant \lim_{n\longrightarrow\infty} N(Ay_n, z, t)$

and we deduce that $\lim_{n\longrightarrow\infty} Ay_n = z$.

Suppose that $S(X)$ is a closed subset of X. then $z = Su$. For some $u \in X$, subsequently we have;

$\lim_{n\longrightarrow\infty} Ay_n = \lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z = Su$

Now, we shall show that $Au = Su$.

From (3.24) we have (at $x = u, y = x_n$)

$[M(Au, Bx_n, kt)]^2 \geqslant [M(Au, Su, t) * M(Bx_n, Su, t)]^2$
and
$[N(Au, Bx_n, kt)]^2 \leqslant [N(Au, Su, t) \lozenge N(Bx_n, Su, t)]^2$

Taking the limit as $n \longrightarrow \infty$, we have;

$[M(Au, Su, kt)]^2 \geqslant [M(Au, Su, t) * M(Su, Su, t)]^2$
and
$[N(Au, Su, kt)]^2 \leqslant [N(Au, Su, t) \lozenge N(Su, Su, t)^2$

$[M(Au, Su, kt)]^2 \geqslant [M(Au, Su, t) * 1]^2$

and
$[N(Au, Su, kt)]^2 \leqslant [N(Au, Su, t) \lozenge 0]^2$

Thus, it follows that,

$M(Au, Su, kt) \geqslant M(Au, Su, t)$
and
$N(Au, Su, kt) \leqslant N(Au, Su, t)$

Therefore, by lemma 2.1, we have; $Au = Su$.

i.e. u is a coincidence point of A and S.

The weak compatibility of A and S implies that $ASu = SAu$ and then $AAu = ASu = SAu = SSu$. On the other hand, since $A(X) \subset S(X)$, there exist a point $v \in X$ such that $Au = Sv$. We claim that $Sv = Bv$.

i.e. v is a coincidence point of S and B.

Using (3.24), we have;

$[M(Au, Bv, kt)]^2 \geqslant [M(Au, Su, t) * M(Bv, Su, t)]^2$
and
$[N(Au, Bv, kt)]^2 \leqslant [N(Au, Su, t) \lozenge N(Bv, Su, t)]^2$

$[M(Au, Bv, kt)]^2 \geqslant [M(Au, Au, t) * M(Bv, Au, t)]^2$
and
$[N(Au, Bv, kt)]^2 \leqslant [N(Au, Au, t) \lozenge N(Bv, Au, t)]^2$

$[M(Au, Bv, kt)]^2 \geqslant [1 * M(Au, Bv, t)]^2$
and
$[N(Au, Bv, kt)]^2 \leqslant [0 \lozenge N(Au, Bv, t)]^2$

Thus, it follows that,

$M(Au, Bv, kt) \geqslant M(Au, Bv, t)$
and
$N(Au, Bv, kt) \leqslant N(Au, Bv, t)$

Therefore, by lemma 2.1, we have; $Au = Bv$

Thus $Au = Su = Sv = Bv$.

The weak compatibility of B and S implies $BSv = SBv$ and then$BBv = BSv = SBv = SSv$.

Let us show that Au is a common fixed point of A, B and S.

In view of (3.24), it follows that

$[M(AAu, Bv, kt)]^2 \geqslant [M(AAu, SAu, t) * M(Bv, SAu, t)]^2$
and
$[N(AAu, Bv, kt)]^2 \leqslant [N(AAu, SAu, t) \diamondsuit N(Bv, SAu, t)]^2$

$[M(AAu, Bv, kt)]^2 \geqslant [M(AAu, AAu, t) * M(Au, AAu, t)]^2$
and
$[N(AAu, Bv, kt)]^2 \leqslant [N(AAu, AAu, t) \diamondsuit N(Au, AAu, t)]^2$

$[M(AAu, Bv, kt)]^2 \geqslant [1 * M(Au, AAu, t)]^2$
and
$[N(AAu, Bv, kt)]^2 \leqslant [0 \diamondsuit N(Au, AAu, t)]^2$

That, it follows that,

$M(AAu, Au, kt) \geqslant M(Au, AAu, t)$
and
$N(AAu, Au, kt) \leqslant N(Au, AAu, t)$

Therefore, by lemma 2.1, we have; $AAu = Au$.

Thus $AAu = Au = SAu$ and Au is a common fixed point of A and S.

Similarly, we prove that Bv is a common fixed point of B and S.

Since $Au = Bv$, we conclude that Au is a common fixed point of A, B and S.

If $Az = Bz = Sz = z$ and $Aw = Bw = Sw = w$, then by (3.24), we have;

$[M(Az, Bw, kt)]^2 \geqslant [M(Az, Sz, t) * M(Bw, Sz, t)]^2$
and
$[N(Az, Bw, kt)]^2 \leqslant [N(Az, Sz, t) \diamondsuit N(Bw, Sz, t)]^2$

$[M(z, w, kt)]^2 \geqslant [M(z, z, t) * M(w, z, t)]^2$
and
$[N(z, w, kt)]^2 \leqslant [N(z, z, t) \diamondsuit N(w, z, t)]^2$

$[M(z, w, kt)]^2 \geqslant [1 * M(z, w, t)]^2$
and
$[N(z, w, kt)]^2 \leqslant [0 \diamondsuit N(z, w, t)]^2$

Thus, it follows that,

$M(z, w, kt) \geqslant M(z, w, t)$
and
$N(z, w, kt) \leqslant N(z, w, t)$

Therefore, by lemma 2.1, we have;$z = w$.

Hence the common fixed point is unique.

This completes the proof of the theorem.

□

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