



EQUILIBRIUM POINT OF A SUPPLY CHAIN NETWORK COMPRISING DISASTER RELIEF MODEL VIA VARIATIONAL INEQUALITY PROBLEM

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ABSTRACT. In this paper, the supply chain network equilibrium model which composites of the disaster relief part is constructed. In such model, we consider five tiers of decision makers: manufacturers who produce the products for sale and donation, retailers, demand market who can purchase products, freight service providers who transport the relief items (or the products) to demand points and, finally, the demand points. The behavior of all decision makers is considered by using the variational inequality formulation. Furthermore, the qualitative properties of such model are studied. Finally, we give some numerical supply chain examples of such model.

KEYWORDS: supply chain network, equilibrium model, variational inequality, qualitative property.

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1. INTRODUCTION

Mathematical modeling of supply chain is a model in economic which contains many important and interesting researches. Supply chain studies are both industrial and academia. The supply chain is the coordination of organizations, people, activities, information and resource related to production and transportation products from suppliers to customers. It can be seen that, in supply chain, there will be a competitive. Since it has multi decision makers, who are able to make independent decisions. As a result, competitive supply chain networks and supply chain equilibrium models and others are examined and studied. Another interesting thing is supply chain management that researchers present in the mathematical models

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of supply chain networks and analyze in the process such as production, transportation, purchase products, etc. In addition, the optimization of their models is considered such as supplier selection, distribution network design, production coordination, and inventory management, etc. Firstly, the content of the supply chain network consist of multiple manufacturers and retailers, with the manufacturer producing the same product while the retailer purchases the product and sells it to the customers in the market. The study and development of research continues to be ongoing. In 2002, A. Nagurney et. al. [12] proposed the concept of supply chain network equilibrium by mentioning manufacturing enterprises, retailers in supply chain competition, the purchasing behavior of consumers and considered by using the variational inequality and establishing the supply chain network equilibrium model. Later, many authors developed this concept in various fields until now, see [3, 6, 19]. But not only that, in 2016, A. Nagurney [10] studied the freight service provision for disaster relief on the concept of network model by establishing the competitive freight service provision network model for disaster relief. The results which obtained from the research was the equilibrium shipment and price patterns in the freight service provision sector. Freight service provision network model describes the network model related to the humanitarian organizations, who must use the freight service providers for transporting supplier to demand point or victims. It is seen that efficient transportation is essential to humanitarian operations and disaster relief. At the same time, it is well known that the cost of the shipping is a secondary concern. However, the principles of study focus on the center of decision-makers and the competition which involves the freight service provision. But the survival of relief organizations is essential and, at the same time, aims to reduce and save victims' relief. But it all depend on smart financial and budget management, thus resulting in the efficient use of the service is necessary. Larger humanitarian organizations may have their own freight forwarding services. But they do not have the financial resources to maintain the freight fleets. So, they have to purchase the service. The freight service providers want to maximize the profit while the humanitarian organizations have no profit. They also have to compete with others in order to gain business. Hence, their behavior is different from humanitarian organizations that not only requires the responsible use of the source of funds, but under pressure to deliver relief items to respond to disasters in a timely manner. According to study [10], the disaster relief supply chain continues to be developed, see [11, 14].

On the other hand, the variational inequality is well known that it is the powerful tool for using in industry, finance, economics, social and pure and applied science. In the supply chain, the variational inequality plays an important role because of having a form that is easy to apply and consider. There have also been many studies on the variational inequality problem for a long time, then we have many researches on such problem and can use the knowledge of the variational inequality for further application and development. Most of the times, the variational inequality in supply chain will be used for finding the optimization and equilibrium of problem by introducing the variational inequality associated with a supply chain network and considering the variational inequality formulations of equilibrium conditions to obtain the quantitative properties of that equilibrium pattern, see [10, 12, 15] and the references therein.

With all of the above in this article, authors are interested the network model which developed from [10] of A. Nagurney and [12] of A. Nagurney et.al. In our

network model, we consider the manufacturers who want to produce products for sale in the demand market and still want to produce to help victims as well. In this concept, we were inspired by the epidemic situation (COVID-19) which is the deadliest pandemic of the world today and the affect people, economics and social activities around the world. But the most interesting thing, there is a shortage of medical devices such as PPEs, respirators and devices that protect themselves from COVID-19 such as masks and alcohol gels, etc. For this reason, in order for the public to protect themselves initially, government agencies therefore encourage and seek cooperation for people to wear masks and use alcohol gels all the time. But there might be some people who are poor or in a remote area making it impossible to access these devices. The producers therefore allocate relief for such people. As a result of this situation, people have a great demand for such products (or relief items), resulting in a shortage. This is due to the higher demand, but the amount of production remains the same. In addition, in a part of the manufacturer, if the manufacturers have donated the products to the demand points, then we are interested that they can take the cost of the donation to tax-deductible, which is the income that comes back from the donation. For this reason, we are interested in exploring a supply chain that has both sales and donations in a network to determine the equilibrium network model of such supply chain network.

The rest of this paper organized as follows. In section 2, the fundamental concept of our work is proposed. In section 3, we construct the supply chain network comprising disaster relief and propose the variational inequality of each behavior in such network model and consider the variational inequality problem. The qualitative property of the variational inequality problem is considered. The existence and uniqueness of the solution of the variational inequality problem are established. Finally, the algorithm of the network model which is constructed and used for numerical example to understand in the content and network model is shown in section 4 and 5 and the conclusion of results of this paper is proposed in section 6.

2. PRELIMINARIES

Firstly, we will recall the variational inequality problem which was introduced by Kinderlehrer and Stampacchia in 1964 in [4] as follows: Determine $X^* \in K$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in K. \quad (2.1)$$

where X and $F(X)$ are an n -dimensional vector with F is a continuous function from K to \mathbb{R}^n , K is closed and convex, and $\langle \cdot, \cdot \rangle$ denotes the inner product in n -dimensional Euclidean space.

Next, we will present some properties which are used for considering the existence and uniqueness of the solution of the variational inequality problem as follows: throughout of this paper, we let K be a closed and convex set.

Definition 2.1. A mapping $F : K \rightarrow \mathbb{R}^n$ is said to be a monotone if

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0 \quad (2.2)$$

for all $X', X'' \in K$.

Definition 2.2. A mapping $F : K \rightarrow \mathbb{R}^n$ is said to be a strictly monotone if for any two $X', X'' \in K$ and $X' \neq X''$ such that

$$\langle F(X') - F(X''), X' - X'' \rangle > 0. \quad (2.3)$$

By the condition of the strictly monotone, we can obtain the existence and uniqueness of the solution of the variational inequality as the following theorem.

Theorem 2.3. *Suppose that F is a strictly monotone mapping on K . Then, the solution of (2.1) is unique.*

Definition 2.4. A mapping $F : K \rightarrow \mathbb{R}^n$ is said to be a Lipschitz continuous if there exists a real number $L > 0$ such that

$$\|F(X') - F(X'')\| \leq L\|X' - X''\| \quad (2.4)$$

for all $X', X'' \in K$.

For the concept of the Lipschitz continuous will be used to guarantee that an algorithm which is constructed by projected method converges which will be presented again in section 4.

3. MAIN RESULTS

In the above concepts of the variational inequality (2.1), now we will present some example which used the variational inequality problem to consider the equilibrium model and guarantee the existence unique solution of the variational inequality problem.

By the inequality (2.1), since $F : K \rightarrow \mathbb{R}^n$ is a mapping and $X \in K$, which K is a closed and convex set in \mathbb{R}^n . So, if we have $X \in K$, and

$$F(X) = (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X)),$$

where F^i is functions depend on X with $i = 1, 2, 3, 4, 5$, which can formulate in standard form (2.1). Then, we can be solve (find X^*) the unique solution of the problem by using the previous concepts of the variational inequality problem.

The following article, we will propose a supply chain network model which can be formulate in a variational inequality problem and used the concepts of variational inequality for considering the equilibrium model.

In this model, we consider M manufacturers who are involved in the production of product for sale and donation which is denoted by a typical m , the N retailers who can be purchased product from manufacturers and transported to K demand markets. The retailers will be denoted by a typical n , and the demand markets will be denoted by a typical k . In another way, M manufacturers will be delivered some products for donation by L competing freight service providers which denote a typical freight service providers by l , and the manufacturers are interested to deliver the relief items to the O demand points, a typical demand points by o . The structure of the supply chain network model comprising disaster relief is depicted in Figure 1.

The following article we will consider the behavior of all desition makers by starting with the profit maximization for manufacture i .

3.1. Behavior of the manufacturers. Since each manufacturer will produce the products for sale and donation. A manufacturer ships the product to the retailers with the amount of product shipped between manufacturer m and retailer n which denoted by q_{mn} . In another way, a manufacturer still wants to ship the product to the demand points in areas that are scarce and we let q_{lo}^m denotes the amount of the relief items, which has in stock and has prepositioned, that m contracts

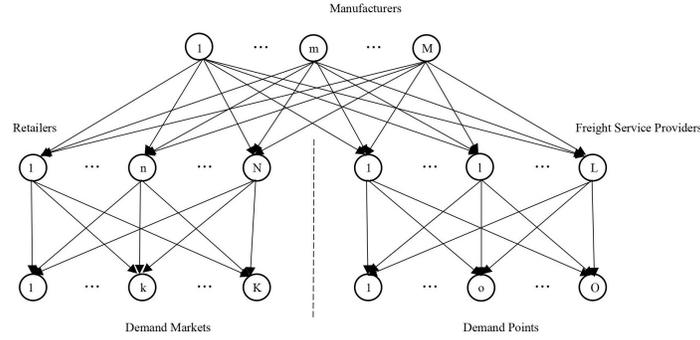


FIGURE 1. Network structure

with freight service provider l to deliver to demand point o . So, we let q_m^1 denotes the nonnegative production output of the product for sale and q_m^2 denotes the nonnegative production output of the product for donation by manufacturers m . We group the product shipments between the manufacturers and the retailers into the column vector $Q^1 \in \mathbb{R}_+^{MN}$. We group the relief item shipments of each manufacturers m into the vector $Q^2 \in \mathbb{R}_+^{MLO}$. We assume that each manufacturers m is faced with a production cost function f_m , which can depend on the entire vector of production outputs, that is,

$$f_m = f_m(Q^1, Q^2) \quad (3.1)$$

for all m . A transaction cost denotes by c_{mn} , which is given by

$$c_{mn} = c_{mn}(q_{mn}) \quad (3.2)$$

for all m, n , that the transaction cost includes the cost of shipping the product. Moreover, each manufacturers are faced with a total cost \hat{c}_{ml} associated with transacting with freight service provider l . This cost includes the cost associated with handling the product until pickup by provider l and interacting with provider l . Observe that the cost associated with a manufacturer in transacting with a freight service provider can depend not only on its own shipments associated with the freight service provider but on those of other manufacturers and the same or other freight service providers. The freight service providers guarantee delivery of the disaster relief items in a timely fashion, given what is known about the disaster landscape, and charge accordingly. So, the quantity produced by manufacturer m must satisfy the following conservation of flow equation:

$$q_m = \sum_{n=1}^N q_{mn} + \sum_{o=1}^O \sum_{l=1}^L q_{lo}^m \quad (3.3)$$

This states that the quantity produced by manufacturer m ; q_m is equal to the sum of the quantities shipped from the manufacturer to all retailers and all demand points. Next, the price charged of the product by manufacturer m to retailer n (supply price) is denoted by ρ_{1mn}^* and the per unit price that freight service provider l charges m for transport to o is denoted by ρ_{lo}^{m*} . Moreover, in the donation of items, the donor can bring the cost of the donation to a tax deduction. Then, it can be seen that when the product manufacturer donates their stuff, they receive a refund from that donation. The amount of refunded will depend on the manufacturer's tax rate levied on the manufacturer's net income. We denote the tax rate levied on the

net yield of manufacturer m as t_m with $t_m \in [0, 1)$ and denote the price of product which produce by manufacturer m to donate into all of demand points as ρ_m^* , where this price does not exceed the lowest price the manufacturer m sells to the retailers, that is, $\rho_m^* \leq \rho_{1mn}, \forall n = 1, 2, \dots, N$. So, these prices which are revealed once the supply chain network equilibrium model for disaster relief is solved.

The total costs, incurred by a manufacturer m , are equal to the sum of their production cost plus the total transaction costs for sale and for donation plus the price charges by freight service provider. The revenue of a manufacturer m is equal to the price that the manufacturer charges for the product (which the retailers are willing to pay) times the total quantity purchased of product from the manufacturer by all the retail outlets and the income received from tax breaks. By the conservation of flow equations (3.3), we can express the criterion of profit maximization for manufacturer m as

$$\begin{aligned} \text{Maximize} \quad & \sum_{n=1}^N \rho_{1mn}^* q_{mn} + t_m \sum_{l=1}^L \sum_{o=1}^O \rho_m^* q_{lo}^m - f_m(Q^1, Q^2) - \sum_{n=1}^N c_{mn}(q_{mn}) \\ & - \sum_{l=1}^L \sum_{o=1}^O \rho_{lo}^{m*} q_{lo}^m - \sum_{l=1}^L \hat{c}_{ml}(q_{lo}^m) \end{aligned} \quad (3.4)$$

subject to:

$$q_{mn} \geq 0 \quad \text{for } n = 1, 2, \dots, N, \quad (3.5)$$

$$q_{lo}^m \geq 0 \quad \text{for } l = 1, 2, \dots, L; o = 1, 2, \dots, O, \quad (3.6)$$

$$q_o = \frac{\sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O q_{lo}^m}{O} \quad (3.7)$$

where q_o is the number of products that received at the demand point o .

Assume that the manufacturers compete in a noncooperative fashion. The production cost functions, the transaction cost functions and the total cost associated with freight service provider for each manufacturer are continuously differentiable and convex. Given that the governing equilibrium concept underlying noncooperative behavior is that of Cournot [1] and Nash [7, 8] which states that each manufacturer will determine their optimal production quantity and shipments. The optimality conditions for all manufacturers can be expressed as the following variational inequality (see Nagurney [9]) : Determine $(Q^1, Q^2) \in \mathbb{R}_+^{MN+MLO}$ satisfying

$$\begin{aligned} & \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q_{mn}} + \frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}} - \rho_{1mn}^* \right] \times [q_{mn} - q_{mn}^*] \\ & + \sum_{m=1}^M \sum_{l=1}^L \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q_{lo}^m} + \frac{\partial \hat{c}_{ml}(q_{lo}^{m*})}{\partial q_{lo}^m} + \rho_{lo}^{m*} - t_m \rho_m^* \right] \times [q_{lo}^m - q_{lo}^{m*}] \geq 0 \end{aligned} \quad (3.8)$$

for all $Q^1 \in \mathbb{R}_+^{MN}, Q^2 \in \mathbb{R}_+^{MLO}$.

The optimality conditions as expressed (3.8) have an economic interpretation that, in the part for sale, a manufacturer will ship a positive amount of the product to a retailer, if the price that the retailer is willing to pay for the product is precisely equal to the manufacturer's marginal production and transaction costs associated with that retailer. But if the manufacturer's marginal production and transaction costs exceed the price what the retailer is willing to pay for the product, then the flow on the link will be zero. In the part for donation, if a manufacturer will ship a positive amount of the product to a demand point. Then, the amount of refund from the tax deduction exceed the total cost of

a manufacturer for donation the product to demand point: but if not so, a manufacturer will be also ship a positive amount of the product to a retailer, where the price charged by manufacturer to retailer, ρ_{1mn}^* , is greater than the manufacturer's marginal production and transaction costs and, moreover, the profit for sale product of a manufacturer is greater than the total cost of a manufacturer for donation the product to demand point, that is, the sum of the first term of inequality (3.8) is greater than the sum of the second term of such inequality.

3.2. Behavior of the retailers. The retailers are involved in transactions both the manufacturers and the consumers which the retailers wish to obtain the product for their retail outlets as well as with the consumers, who are ultimate purchasers of the product.

A retailer n is faced with a handling cost, which may include the display and storage cost associated with the product, and this cost is denoted by c_n . In the simplest case, we would have that c_n is a function of $\sum_{m=1}^M q_{mn}$, that is, the handling cost of a retailer is a function of how much of the product which they have obtained from the various manufacturers. Now, we let the function to depend on the amounts of the product which held by other retailers and we can write

$$c_n = c_n(Q^1) \quad (3.9)$$

for all n . Next, we will denote ρ_{2n}^* for a price with the product at their retail outlet for retailer n , which this price in the model will also be endogenously determined and denotes \tilde{q}_{nk} for the amount of the product, which is purchased by the consumer k from the retailer n . Normally, the retailers want the maximization of profit. Then, the optimization problem of a retailer n is given by

$$\text{Maximize } \rho_{2n}^* \sum_{k=1}^K \tilde{q}_{nk} - c_n(Q^1) - \sum_{m=1}^M \rho_{1mn}^* q_{mn} \quad (3.10)$$

subject to:

$$\sum_{k=1}^K \tilde{q}_{nk} \leq \sum_{m=1}^M q_{mn} \quad (3.11)$$

and the nonnegative constraints: $q_{mn} \geq 0$ and $\tilde{q}_{nk} \geq 0$ for all m and k . Objective function (3.10) express that the difference between the revenues minus the handling cost and the payout to the manufacturers should be maximized and constraint (3.11) expresses that consumers cannot purchase more from a retailer than what held in stock.

Now, we consider the optimality conditions of the retailers assuming that each retailer is faced with the optimization problem (3.10) subject to (3.11), and assume that the variables is nonnegativity. Here, we also assume that the retailers compete in a noncooperative manner given the actions of the other retailers so that each retailer maximizes his profits. Note that, at this point, we consider that retailers seek to determine not only the optimal amounts purchased by the consumers from their specific retail outlet but, also, the amount that they wish to obtain from the manufacturers. Hence, there is a monitor that receives data from manufacturers, retailers and consumers to consider and determine the optimal amount and price that manufacturers sell to retailers, amount and price that retailers sell to consumers, and the price that consumers are willing to purchase so that all parties are satisfied in trading. The monitor must be the one who holds the information as well and is the most confidential so that other party does not know each other's information. Therefore, in equilibrium, all the shipments between the tiers of network agents will have to coincide. Assume that the handling cost for each retailer is continuous and convex. The optimality conditions for all the retailers coincide with the solution of the variational

inequality: Determine $(Q^{1*}, Q^{3*}, \gamma^*) \in \mathbb{R}_+^{MN+NK+N}$ satisfying

$$\begin{aligned} & \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial c_n(Q^{1*})}{\partial q_{mn}} + \rho_{1mn}^* - \gamma_n^* \right] \times [q_{mn} - q_{mn}^*] + \sum_{n=1}^N \sum_{k=1}^K [-\rho_{2n}^* + \gamma_n^*] \times [\tilde{q}_{nk} - \tilde{q}_{nk}^*] \\ & + \sum_{n=1}^N \left[\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K \tilde{q}_{nk}^* \right] \times [\gamma_n - \gamma_n^*] \geq 0 \end{aligned} \quad (3.12)$$

for all $(Q^1, Q^3, \gamma) \in \mathbb{R}_+^{MN+NK+N}$, where γ_n is the term of the Lagrange multiplier associated with constraint (3.11) for retailer n , γ is the N -dimensional column vector of all the multipliers, and Q^3 is the group of the product flows between the retailer's and the demand markets in the NK -dimensional vector.

We now have the economic interpretation of the retailers, optimality conditions. From the second term in inequality (3.12), we see that, if consumers at demand market k purchase the product from a particular retailer n , (that is, \tilde{q}_{nk}^* is positive) then the price charged by retailer n , ρ_{2n}^* , is precisely equal to γ_n^* , which in the third term in the inequality, serves as the price to clear the market from retailer n . Note that, from the second term (3.12), if no product is sold by a particular retailer, then the price associated with holding the product can exceed the price charged to the consumers. Furthermore, from the first term in inequality (3.12), we can infer that, if a manufacturer transacts with a retailer resulting in a positive flow of the product between the two, then the price γ_n^* is precisely equal to the retailer n 's payment to the manufacturer, ρ_{1mn}^* , plus its marginal cost of handling the product from the retailer.

3.3. Behavior of the consumers at the demand markets. In this section, we will describe the consumers located at the demand markets. We are interested in deciding on the consumer's product consumption, where consumers have not only the price charged for the product by the retailers, but also the transaction cost to obtain the product.

We recall \tilde{q}_{nk} denotes the amount of the product which is purchased from retailer n by consumers at demand market k and let \tilde{c}_{nk} denotes the transaction cost associated with obtaining the product by consumers at demand market k from retailer n and assume that the transaction cost is continuous and positive. We can write the general form as follows

$$\tilde{c}_{nk} = \tilde{c}_{nk}(Q^3) \quad (3.13)$$

for all n, k . Further, we let ρ_{3k} denotes the price of the product at demand market k and d_k denotes the demand for the product at demand market k and assume the continuous demand functions as follows

$$d_k = d_k(\rho_3) \quad (3.14)$$

for all k , where ρ_3 is the K -dimensional column vector of demand market prices. By (3.14), the demand for consumers for the product at a demand market depends not only on the price of the product at that demand market but also on the prices of the product at the other demand markets. Thus, consumers at a demand market also compete with consumers at other demand markets.

Next, we will discuss the equilibrium conditions between consumers at demand market k and retailer n that is the price of product at demand market k is relative to the consumers who take the price charged by the retailers for the product for retailer n plus the transaction cost associated with obtaining the product, in making their consumption decisions, and there is still a relationship between the demand for the product at demand market k and the amount of the product which is purchased by the consumers from the retailers. Then, we can write the equilibrium conditions for consumers at demand market k as follows: for

all retailers $n, n = 1, 2, \dots, N$,

$$\rho_{2n}^* + \tilde{c}_{nk}(Q^{3*}) \begin{cases} = \rho_{3k}^* & \text{if } \tilde{q}_{nk}^* > 0, \\ \geq \rho_{3k}^* & \text{if } \tilde{q}_{nk}^* = 0 \end{cases} \quad (3.15)$$

and

$$d_k(\rho_3^*) \begin{cases} = \sum_{n=1}^N \tilde{q}_{nk}^* & \text{if } \rho_{3k}^* > 0, \\ \leq \sum_{n=1}^N \tilde{q}_{nk}^* & \text{if } \rho_{3k}^* = 0. \end{cases} \quad (3.16)$$

In equilibrium price, Condition (3.15) state that if the consumers at demand market k purchase the product from retailer n , then the price charged by retailer for the product plus the transaction cost does not exceed the price that the consumers are willing to pay for the product and Conditions (3.16) state that if the consumers who are willing to pay for the product at the demand market is positive, then the quantities purchased of the product from the retailers will be precisely equal to the demand for that product at the demand market. This condition correspond to the well-known spatial price equilibrium conditions ([9] and the references therein).

In equilibrium on the conditions (3.15) and (3.16) will have to hold for all demand markets k , and these can also be expressed as a variational inequality problem given by: Determine $(Q^{3*}, \rho_3^*) \in \mathbb{R}_+^{NK+K}$ such that

$$\sum_{n=1}^N \sum_{k=1}^K \left[\rho_{2n}^* + \tilde{c}_{nk}(Q^{3*}) - \rho_{3k}^* \right] \times [\tilde{q}_{nk} - \tilde{q}_{nk}^*] + \sum_{k=1}^K \left[\sum_{n=1}^N \tilde{q}_{nk}^* - d_k(\rho_3^*) \right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0 \quad (3.17)$$

for all $(Q^3, \rho_3) \in \mathbb{R}_+^{NK+K}$.

3.4. Behavior of the freight service providers. Since the freight service providers want to be profit-maximizers, thus they have to cover their costs. The cost associated with freight service providers delivering the relief items from manufacturers to demand points. We let c_{mo}^l for the cost associated with freight service provider l delivering the relief items from manufacturer m to demand point o , given by

$$c_{mo}^l = c_{mo}^l(Q^2) \quad (3.18)$$

for all $l = 1, 2, \dots, L$. Assume that the cost functions of the freight service providers are continuously differentiable and convex. Note that the cost functions in (3.18) depend on the freight service provider's shipment quantities and those of the other freight service providers because there may be congestion, competition for labor, etc. In the other hand, the revenue of a freight service provider is equal to the per unit price that a freight service provider charges a manufacturer for transport to demand points times the amount of the relief item that a manufacturer contracts with a freight service provider to deliver to demand points. Therefore, the optimization problem of freight service provider $l; l = 1, 2, \dots, L$, where each freight service providers requires the maximized profits, is illustrated in the following:

$$\text{Maximize } \sum_{m=1}^M \sum_{o=1}^O \rho_{lo}^{m*} q_{lo}^m - \sum_{m=1}^M \sum_{o=1}^O c_{mo}^l(Q^2) \quad (3.19)$$

subject to :

$$q_{lo}^m \geq 0, \quad (3.20)$$

for $o = 1, 2, \dots, O$ and $m = 1, 2, \dots, M$.

We assume that the freight service providers $l; l = 1, 2, \dots, L$, are a noncooperative competition for the disaster relief items and seek to maximize its profits. The optimality

conditions of all freight service providers holding simultaneously must satisfy the variational inequality problem: Determine $Q^{2*} \in \mathbb{R}_+^{MLO}$ such that:

$$\sum_{l=1}^L \left(\sum_{m=1}^M \sum_{o=1}^O \left(\sum_{x=1}^M \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^{2*})}{\partial q_{lo}^m} - \rho_{lo}^{m*} \right) \right) \times [q_{lo}^m - q_{lo}^{m*}] \geq 0 \quad (3.21)$$

for all $q_{lo}^m \in \mathbb{R}_+^{MLO}$.

The economic interpretation of the freight service providers in optimality conditions is that if a freight service provider transacts with a manufacturers to a demand point resulting in a positive flow of the product between three tiers, then the price ρ_{lo}^{m*} is precisely equal to its marginal cost of handling the product by a freight service provider from a manufacturer to a demand point.

All of previous behavior, we have the network equilibrium conditions for the supply chain network comprising disaster relief model as follows.

Definition 3.1. [Supply Chain Network Comprising Disaster Relief Equilibrium] A supply chain network comprising disaster relief equilibrium is one which the product flows between the distinct tiers of the decision-makers coincide and the product flows and prices satisfy the sum of the optimality conditions (3.8), (3.12), (3.17) and (3.21).

We now present the variational inequality formulation of the supply chain network comprising disaster relief equilibrium conditions and then discuss how to find the equilibrium prices.

Theorem 3.2. *The equilibrium conditions governing the supply chain network comprising disaster relief model with competition are equivalent to the solution of the variational inequality problem given by determine $(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \rho_3^*) \in K$ satisfying*

$$\begin{aligned} & \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \frac{\partial f_s(Q^{1*}, Q^{2*})}{\partial q_{mn}} + \frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}} + \frac{\partial c_n(Q^{1*})}{\partial q_{mn}} - \gamma_n^* \right] \times [q_{mn} - q_{mn}^*] \\ & + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{s=1}^M \frac{\partial f_s(Q^{1*}, Q^{2*})}{\partial q_{lo}^m} + \frac{\partial \hat{c}_{mi}(q_{lo}^{m*})}{\partial q_{lo}^m} + \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^{2*})}{\partial q_{lo}^m} - t_m \rho_m^* \right] \times [q_{lo}^m - q_{lo}^{m*}] \\ & + \sum_{n=1}^N \sum_{k=1}^K [\tilde{c}_{nk}(Q^{3*}) + \gamma_n^* - \rho_{3k}^*] \times [\tilde{q}_{nk} - \tilde{q}_{nk}^*] + \sum_{n=1}^N \left[\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K \tilde{q}_{nk}^* \right] \times [\gamma_n - \gamma_n^*] \\ & + \sum_{k=1}^K \left[\sum_{n=1}^N \tilde{q}_{nk}^* - d_k(\rho_3^*) \right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0 \end{aligned} \quad (3.22)$$

for all $(Q^1, Q^2, Q^3, \gamma, \rho_3) \in K$, where

$$K = \{(Q^1, Q^2, Q^3, \gamma, \rho_3) | (Q^1, Q^2, Q^3, \gamma, \rho_3) \in \mathbb{R}_+^{MN+MLO+NK+N+K}\}.$$

Proof. The summation of (3.8), (3.12), (3.17) and (3.21). This imply that (3.22).

In conversely, we will consider that the solution to variational inequality (3.22) satisfies the sum of inequalities (3.8), (3.12), (3.17) and (3.21), that is the equilibrium according to Definition 3.1. In the first term of the inequality (3.22) add $-\rho_{1mn}^* + \rho_{1mn}^*$ and in the second term add $-\rho_{lo}^{m*} + \rho_{lo}^{m*}$ and, in the third term add $-\rho_{2n}^* + \rho_{2n}^*$. This implies that

$$\begin{aligned} & \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \frac{\partial f_s(Q^{1*}, Q^{2*})}{\partial q_{mn}} + \frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}} + \frac{\partial c_n(Q^{1*})}{\partial q_{mn}} - \gamma_n^* - \rho_{1mn}^* + \rho_{1mn}^* \right] \times [q_{mn} - q_{mn}^*] \\ & + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{s=1}^M \frac{\partial f_s(Q^{1*}, Q^{2*})}{\partial q_{lo}^m} + \frac{\partial \hat{c}_{mi}(q_{lo}^{m*})}{\partial q_{lo}^m} + \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^{2*})}{\partial q_{lo}^m} - t_m \rho_m^* - \rho_{lo}^{m*} + \rho_{lo}^{m*} \right] \times [q_{lo}^m - q_{lo}^{m*}] \\ & + \sum_{n=1}^N \sum_{k=1}^K [\tilde{c}_{nk}(Q^{3*}) + \gamma_n^* - \rho_{3k}^* - \rho_{2n}^* + \rho_{2n}^*] \times [\tilde{q}_{nk} - \tilde{q}_{nk}^*] + \sum_{n=1}^N \left[\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K \tilde{q}_{nk}^* \right] \times [\gamma_n - \gamma_n^*] \end{aligned}$$

$$+ \sum_{k=1}^K \left[\sum_{n=1}^N \tilde{q}_{nk}^* - d_k(\rho_3^*) \right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0.$$

Thus,

$$\begin{aligned} & \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q_{mn}} + \frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}} - \rho_{1mn}^* \right] \times [q_{mn} - q_{mn}^*] \\ & + \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial c_n(Q^1)}{\partial q_{mn}} - \gamma_n^* + \rho_{1mn}^* \right] \times [q_{mn} - q_{mn}^*] \\ & + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q_{lo}^m} + \frac{\partial \hat{c}_{ml}(q_{lo}^m)}{\partial q_{lo}^m} + \rho_{lo}^{m*} - t_m \rho_m^* \right] \times [q_{lo}^m - q_{lo}^{m*}] \\ & + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{x=1}^L \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^2)}{\partial q_{lo}^m} - \rho_{lo}^{m*} \right] \times [q_{lo}^m - q_{lo}^{m*}] \\ & + \sum_{n=1}^N \sum_{k=1}^K [\tilde{c}_{nk}(Q^3) + \rho_{2n}^* - \rho_{3k}^*] \times [\tilde{q}_{nk} - \tilde{q}_{nk}^*] \\ & + \sum_{n=1}^N \sum_{k=1}^K [\gamma_n^* - \rho_{2n}^*] \times [\tilde{q}_{nk} - \tilde{q}_{nk}^*] + \sum_{n=1}^N \left[\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K \tilde{q}_{nk}^* \right] \times [\gamma_n - \gamma_n^*] \\ & + \sum_{k=1}^K \left[\sum_{n=1}^N \tilde{q}_{nk}^* - d_k(\rho_3^*) \right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0. \end{aligned} \quad (3.23)$$

We see that (3.23) is equivalent to the price and shipment pattern satisfying the sum of inequalities (3.8), (3.12), (3.17) and (3.21). This completes the proof. \square

From the above theorem, if we can define $X \equiv (Q^1, Q^2, Q^3, \gamma, \rho_3) \in K$ and $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$ where $F^1(X)$ consists of components F_{mn} , with

$$F_{mn}(X) \equiv \sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q_{mn}} + \frac{\partial c_{mn}(q_{mn})}{\partial q_{mn}} + \frac{\partial c_n(Q^1)}{\partial q_{mn}} - \gamma_n,$$

for $m = 1, 2, \dots, M; n = 1, 2, \dots, N$, $F^2(X)$ consists of components F_{lo}^m with

$$F_{lo}^m(X) \equiv \sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q_{lo}^m} + \frac{\partial \hat{c}_{ml}(q_{lo}^m)}{\partial q_{lo}^m} + \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^2)}{\partial q_{lo}^m} - t_m \rho_m,$$

for $m = 1, 2, \dots, M; l = 1, 2, \dots, L, o = 1, 2, \dots, O$, $F^3(X)$ consists of components F_{nk} with

$$F_{nk}(X) \equiv \tilde{c}_{nk}(Q^3) + \gamma_n - \rho_{3k}$$

for $n = 1, 2, \dots, N, k = 1, 2, \dots, K$, $F^4(X)$ consists of components F_n with

$$F_n(X) \equiv \sum_{m=1}^M q_{mn} - \sum_{k=1}^K \tilde{q}_{nk}$$

for $n = 1, 2, \dots, N$, $F^5(X)$ consists of components F_k with

$$F_k(X) \equiv \sum_{n=1}^N \tilde{q}_{nk} - d_k(\rho_3)$$

for $k = 1, 2, \dots, K$. Then, the above functions can be formulated in the standard form (2.1), that is, the variational inequality (3.22) takes on (2.1).

The following article, we will consider the examination of qualitative properties of the equilibrium pattern, that is, the solution to the variational inequality (3.22), equivalently, (2.1).

Since the feasible set underlying the variational inequality problem (3.22) is not compact. So, we can define a rather weak condition to guarantee the existence of a solution pattern. Let

$$K_b = \{(Q^1, Q^2, Q^3, \gamma, \rho_3) | 0 \leq Q^1 \leq b_1, 0 \leq Q^2 \leq b_2, 0 \leq Q^3 \leq b_3, 0 \leq \gamma \leq b_4, 0 \leq \rho_3 \leq b_5\}, \quad (3.24)$$

which b is a positive scalar and $Q^1 \leq b_1, Q^2 \leq b_2, Q^3 \leq b_3, \gamma \leq b_4, \rho_3 \leq b_5$, this mean that $q_{mn} \leq b_1, q_{lo}^m \leq b_2, q_{nk} \leq b_3, \gamma_n \leq b_4$ and $\rho_{3k} \leq b_5$ for all m, n, k, l, o . Then, K_b is a bounded, closed convex subset of $R^{MN+MLO+NK+N+K}$. Since K_b is compact and F is continuous. Therefore, we have the following variational inequality:

$$\langle F(X_b), X - X_b \rangle \geq 0, \quad \text{for all } X \in K_b \quad (3.25)$$

admits at least one solution $X_b^* \equiv (Q^{1b^*}, Q^{2b^*}, Q^{3b^*}, \gamma^{b^*}, \rho_3^{b^*}) \in K_b$. So, by D. Kinderlehrer and G. Stampacchia [4], we obtain the following theorem.

Theorem 3.3. *Variational inequality (2.1) admits a solution if and only if there exists $b > 0$ such that variational inequality (3.25) admits a solution in K_b with*

$$Q^{1b^*} < b_1, Q^{2b^*} < b_2, Q^{3b^*} < b_3, \gamma^{b^*} < b_4, \rho_3^{b^*} < b_5. \quad (3.26)$$

The following proposition is presented for guarantee the existence of solution of the variational inequality problem in Theorem 3.3 which is proved in the same way of Nagurney and Zhao [15].

Proposition 3.4. *Suppose that there exist positive constants S, T and $W > 0$ such that:*

$$\sum_{s=1}^M \frac{\partial f_s(Q^{1*}, Q^{2*})}{\partial q_{mn}^*} + \frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}^*} + \frac{\partial c_n(Q^{1*})}{\partial q_{mn}^*} \geq S; \quad \text{for all } Q^{1*}, Q^{2*}, \text{ with } q_{mn}^* \geq T, \\ \text{or } \tilde{q}_{nk}^* \geq T \quad \text{for all } m, n, k \quad (3.27)$$

$$d_k(\rho_3^*) \leq T; \quad \text{for all } \rho_3 \text{ with } \rho_{3k}^* \geq W, \quad \text{for all } k, \quad (3.28)$$

$$\sum_{m=1}^M q_{mn}^* - \sum_{k=1}^K \tilde{q}_{nk}^* > 0 \quad \text{with } q_{mn}^* \geq T \quad \text{or } \tilde{q}_{nk}^* \geq T \quad \text{for all } m, n, k. \quad (3.29)$$

Then variational inequality (2.1) admits at least one solution.

Proof Choose $b_1 = b_2 = b_3 = b_4 = b > T$ and $b_5 > T_1$ where $T_1 = \max_{nk, Q^3 < b, \gamma < b} \{\rho_{2n} + c_{nk}(Q^3)\}$. If we can prove that:

$$Q^{1b^*} < b, Q^{2b^*} < b, Q^{3b^*} < b, \gamma^{b^*} < b, \rho_{3k}^* < b_5,$$

then, by Theorem 3.3, we obtain the existence of the solution (2.1).

(i.) We will show that $q_{mn}^* < b$ and $\tilde{q}_{nk}^* < b$ for all m, n, k .

Assume that there exist x, y, z such that $q_{xy}^* = b > T$ or $\tilde{q}_{yz}^* = b > T$. From assumption (3.27), we have

$$S \leq \sum_{s=1}^M \frac{\partial f_s(Q^{1*}, Q^{2*})}{\partial q_{mn}^*} + \frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}^*} + \frac{\partial c_n(Q^{1*})}{\partial q_{mn}^*} \leq \gamma_n^*.$$

Since if $\tilde{q}_{nk}^* > 0$ then (3.15), we have $\rho_{2n}^* + \tilde{c}_{nk}(Q^{3*}) \leq \rho_{3k}^*$ and $\gamma_n^* = \rho_{2n}^*$. Then, $\rho_{3k}^* > 0$. By (3.16), imply that

$$\sum_{n=1}^N \tilde{q}_{nk}^* - d_k(\rho_3^*) \leq 0.$$

Hence $d_k(\rho_3^*) > T$. This is a contradiction with the assumption (3.28). Therefore, $q_{mn}^* < b$ and $\tilde{q}_{nk}^* < b$ for all m, n, k .

- (ii.) We will show that $q_{lo}^{m*} < b$. Since (3.7), we know that $\sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O q_{lo}^m = Oq_o$. Then, q_{lo}^m is bounded. So, there exists $b_2 > 0$ such that $q_{lo}^{m*} < b_2 = b$.
- (iii.) We will show that $\gamma^* < b$.

Assume that there exists d such that

$$\gamma_d^* = b > T$$

This imply that

$$\sum_{m=1}^M q_{md}^* \leq \sum_{k=1}^K \tilde{q}_{dk}^*$$

By (3.11), we have

$$\sum_{m=1}^M q_{md}^* = \sum_{k=1}^K \tilde{q}_{dk}^*,$$

which is a contradiction with the assumption (3.29). Therefore, $\gamma^* < b$.

- (iv.) We will show that $\rho_{3k}^* < b_5$.

Since $\tilde{q}_{nk}^* < b$ which b is a positive scalar. We have $\rho_{2n}^* + \tilde{c}_{nk}(Q^{3*}) \geq \rho_{3k}^*$. This imply

$$\rho_{3k}^* \leq T_1 < b_5.$$

The proof is complete.

Under the conditions in Proposition 3.4, it is possible to construct b_1, b_2, b_3, b_4 and b_5 large enough so that the restricted variational inequality (3.25) will satisfy the boundedness condition (3.26) and, thus, existence of a solution to the original variational inequality problem according to Theorem 3.3 will hold.

From the assumptions (3.27), (3.28) and (3.29), it is reasonable from an economic point of view. If a large number of products are shipped between a manufacturer and a retailer, then we expect that the marginal production cost plus the marginal transaction cost plus the marginal handling cost exceed a positive lower bound. If the demand market price at a demand market is high, we can expect that the demand for the product is low at that demand market and below upper bound. Finally, if a large number of products are shipped between a manufacturer and a retailer and between a retailer and a demand market, then we expect that the shipment between a manufacturer and a retailer exceed the shipment between a retailer and a demand market.

Next, we will consider the monotonicity properties of the function F in variational inequality (3.22). Then, we recall the definition of an additive production cost functions introduced in Zhang and Nagurney [18] for considering in the qualitative properties.

Definition 3.5. Suppose that, for each manufacture m , the production cost f_m is additive, that is,

$$f_m(q) = f_m^1(q_m) + f_m^2(\tilde{q}_m), \quad (3.30)$$

which $f_m^1(q_m)$ is the internal production cost that depend on the manufacturer's own output level q_m , which may include the production operation and the facility maintenance, etc., and $f_m^2(\tilde{q}_m)$, is the interdependent part of the production cost that is a function of all the other manufacturers' output levels $\tilde{q}_m = (q_1, \dots, q_{m-1}, q_{m+1}, \dots, q_m)$ and reflects the impact of the other manufacturers' production patterns on manufacturer m 's cost. This interdependent part of the production cost may describe the competition for the resources, consumption of the homogeneous raw materials, etc.

Here, we will consider the qualitative properties of the function F that enters the variational inequality problem and the uniqueness of the equilibrium pattern. So, firstly,

the monotonicity and Lipschitz continuity of F is presented. Moreover, the subsequent section, this concept is used for proving the convergence of the algorithmic scheme.

Lemma 3.6. *Suppose that the production cost functions $f_m, m = 1, 2, \dots, M$, are additive, as defined in Definition 3.5, and $f_m^1, m = 1, 2, \dots, M$, are convex functions. If the $c_{mn}, c_n, \hat{c}_{ml}$ and \hat{c}_{mo} functions are convex, the \tilde{c}_{nk} functions are monotone increasing, and the d_k functions are monotone decreasing functions of the generalized prices, for all m, n, k, l, o . Then the vector function F that enters the variational inequality (3.22) is monotone mapping.*

Proof. Let $X' \equiv (Q^1, Q^2, Q^3, \gamma', \rho'_3)$ and $X'' \equiv (Q^{1''}, Q^{2''}, Q^{3''}, \gamma'', \rho''_3)$ with $X', X'' \in K$. Then,

$$\begin{aligned}
& \langle F(X') - F(X''), X' - X'' \rangle \\
= & \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q'_{mn}} + \frac{\partial c_{mn}(q'_{mn})}{\partial q'_{mn}} + \frac{\partial c_n(Q^1)}{\partial q'_{mn}} - \gamma'_n - \sum_{s=1}^M \frac{\partial f_s(Q^{1''}, Q^{2''})}{\partial q''_{mn}} \right. \\
& \quad \left. - \frac{\partial c_{mn}(q''_{mn})}{\partial q''_{mn}} - \frac{\partial c_n(Q^{1''})}{\partial q''_{mn}} + \gamma''_n \right] \times [q'_{mn} - q''_{mn}] \\
& + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q^{m'}_{lo}} + \frac{\partial \hat{c}_{ml}(q^{m'}_{lo})}{\partial q^{m'}_{lo}} + \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c'_{xy}(Q^2)}{\partial q^{m'}_{lo}} - t_m \rho_m \right. \\
& \quad \left. - \sum_{s=1}^M \frac{\partial f_s(Q^{1''}, Q^{2''})}{\partial q^{m''}_{lo}} - \frac{\partial \hat{c}_{ml}(q^{m''}_{lo})}{\partial q^{m''}_{lo}} - \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c'_{xy}(Q^{2''})}{\partial q^{m''}_{lo}} + t_m \rho_m \right] \times [q^{m'}_{lo} - q^{m''}_{lo}] \\
& + \sum_{n=1}^N \sum_{k=1}^K [\tilde{c}_{nk}(Q^3) + \gamma'_n - \rho'_{3k} - \tilde{c}_{nk}(Q^{3''}) - \gamma''_n + \rho''_{3k}] \times [\tilde{q}'_{nk} - \tilde{q}''_{nk}] \\
& + \sum_{n=1}^N \left[\sum_{m=1}^M q'_{mn} - \sum_{k=1}^K \tilde{q}'_{nk} - \sum_{m=1}^M q''_{mn} + \sum_{k=1}^K \tilde{q}''_{nk} \right] \times [\gamma'_n - \gamma''_n] \\
& + \sum_{k=1}^K \left[\sum_{n=1}^N \tilde{q}'_{nk} - d_k(\rho'_3) - \sum_{n=1}^N \tilde{q}''_{nk} + d_k(\rho''_3) \right] \times [\rho'_{3k} - \rho''_{3k}] \\
= & \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q'_{mn}} - \sum_{s=1}^M \frac{\partial f_s(Q^{1''}, Q^{2''})}{\partial q''_{mn}} \right] \times [q'_{mn} - q''_{mn}] \\
& + \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial c_{mn}(q'_{mn})}{\partial q'_{mn}} - \frac{\partial c_{mn}(q''_{mn})}{\partial q''_{mn}} \right] \times [q'_{mn} - q''_{mn}] \\
& + \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial c_n(Q^1)}{\partial q'_{mn}} - \frac{\partial c_n(Q^{1''})}{\partial q''_{mn}} \right] \times [q'_{mn} - q''_{mn}] \\
& + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{s=1}^M \frac{\partial f_s(Q^1, Q^2)}{\partial q^{m'}_{lo}} - \sum_{s=1}^M \frac{\partial f_s(Q^{1''}, Q^{2''})}{\partial q^{m''}_{lo}} \right] \times [q^{m'}_{lo} - q^{m''}_{lo}] \\
& + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\frac{\partial \hat{c}_{ml}(q^{m'}_{lo})}{\partial q^{m'}_{lo}} - \frac{\partial \hat{c}_{ml}(q^{m''}_{lo})}{\partial q^{m''}_{lo}} \right] \times [q^{m'}_{lo} - q^{m''}_{lo}] \\
& + \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{x=1}^L \sum_{y=1}^O \frac{\partial c'_{xy}(Q^2)}{\partial q^{m'}_{lo}} - \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c'_{xy}(Q^{2''})}{\partial q^{m''}_{lo}} \right] \times [q^{m'}_{lo} - q^{m''}_{lo}] \\
& + \sum_{n=1}^N \sum_{k=1}^K [\tilde{c}_{nk}(Q^3) - \tilde{c}_{nk}(Q^{3''})] \times [\tilde{q}'_{nk} - \tilde{q}''_{nk}] \\
& + \sum_{k=1}^K [-d_k(\rho'_3) + d_k(\rho''_3)] \times [\rho'_{3k} - \rho''_{3k}]
\end{aligned}$$

$$= (I) + (II) + (III) + (IV) + (V) + (VI) + (VII) + (VIII). \quad (3.31)$$

Since f_m for all m are additive and f_m^1 are convex functions, one has

$$(I) = \sum_{m=1}^M \sum_{n=1}^N \left[\sum_{s=1}^M \left(\frac{\partial f_s(Q^1, Q^2)}{\partial q'_{mn}} - \frac{\partial f_s(Q^{1''}, Q^{2''})}{\partial q''_{mn}} \right) \right] \times [q'_{mn} - q''_{mn}] \geq 0$$

and

$$(IV) = \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{s=1}^M \left(\frac{\partial f_s(Q^1, Q^2)}{\partial q'_{lo}^{m'}} - \frac{\partial f_s(Q^{1''}, Q^{2''})}{\partial q''_{lo}^{m''}} \right) \right] \times [q_{lo}^{m'} - q_{lo}^{m''}] \geq 0.$$

The convexity of c_{mn} , c_n , \hat{c}_{ml} and c_{mo}^l for all m, n, l, o , we have

$$(II) = \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial c_{mn}(q'_{mn})}{\partial q'_{mn}} - \frac{\partial c_{mn}(q''_{mn})}{\partial q''_{mn}} \right] \times [q'_{mn} - q''_{mn}] \geq 0,$$

$$(III) = \sum_{m=1}^M \sum_{n=1}^N \left[\frac{\partial c_n(Q^1)}{\partial q'_{mn}} - \frac{\partial c_n(Q^{1''})}{\partial q''_{mn}} \right] \times [q'_{mn} - q''_{mn}] \geq 0,$$

$$(V) = \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\frac{\partial \hat{c}_{ml}(q_{lo}^{m'})}{\partial q_{lo}^{m'}} - \frac{\partial \hat{c}_{ml}(q_{lo}^{m''})}{\partial q_{lo}^{m''}} \right] \times [q_{lo}^{m'} - q_{lo}^{m''}] \geq 0$$

and

$$(VI) = \sum_{m=1}^M \sum_{l=1}^L \sum_{o=1}^O \left[\sum_{x=1}^L \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^2)}{\partial q_{lo}^{m'}} - \sum_{x=1}^L \sum_{y=1}^O \frac{\partial c_{xy}^l(Q^{2''})}{\partial q_{lo}^{m''}} \right] \times [q_{lo}^{m'} - q_{lo}^{m''}] \geq 0.$$

Since \tilde{c}_{nk} for all n, k are assumed to be monotone increasing, and d_k for all k are assumed to be monotone decreasing, we have

$$(VII) = \sum_{n=1}^N \sum_{k=1}^K [\tilde{c}_{nk}(Q^{3'}) - \tilde{c}_{nk}(Q^{3''})] \times [\tilde{q}'_{nk} - \tilde{q}''_{nk}] \geq 0$$

and

$$(VIII) = \sum_{k=1}^K [-d_k(\rho'_3) + d_k(\rho''_3)] \times [\rho'_{3k} - \rho''_{3k}] \geq 0.$$

We see that the right hand side of (3.31) is nonnegative. The proof is completed. \square

Lemma 3.7. *Assume that the conditions of Lemma 3.6 hold. In addition, suppose that one of the five families of convex functions f_m^1 , c_{mn} , c_n , \hat{c}_{ml} and c_{mo}^l for all $m = 1, \dots, M, n = 1, \dots, N, l = 1, \dots, L, o = 1, \dots, O$ is a family of strictly convex functions. Suppose that $\tilde{c}_{nk}, n = 1, \dots, N, k = 1, \dots, K$ and $d_k, k = 1, \dots, K$ are strictly monotone. Then, the vector function F that enters the variational inequality (3.22) is strictly monotone, with respect to (Q^1, Q^2, Q^3, ρ_3) .*

Proof. For any two distinct $(Q^1, Q^2, Q^3, \rho_3), (Q^{1''}, Q^{2''}, Q^{3''}, \rho_3'')$, we must have at least one of the following four cases:

- (i) $Q^1 \neq Q^{1''}$,
- (ii) $Q^2 \neq Q^{2''}$,
- (iii) $Q^3 \neq Q^{3''}$,
- (iv) $\rho_3 \neq \rho_3''$.

Under the condition of the theorem, if (i) holds true, then, at the right-hand side of (3.31), at least one of (I), (II), (III) and (IV) is positive. If (ii) is true, then at least one of (I), (IV), (V) and (VI) is positive. If (iii) is true, then (VII) is positive. In case of (iv), (VIII) is positive. Hence, we can conclude that the right hand side of (3.31) is greater than zero. The proof is completed. \square

Lemma 3.7 has an important implication for the uniqueness of product shipments, Q^1 , the relief item shipments, Q^2 , the retailer shipments, Q^3 and the prices at the demand markets, ρ_3 , at the equilibrium. We note also that no guarantee of a unique $\gamma_n, n = 1, \dots, N$, can be generally expected at the equilibrium.

Theorem 3.8. *Under the conditions of Lemma 3.7, there is a unique product shipment pattern Q^{1*} , a unique relief item shipment pattern Q^{2*} , a unique retail shipment (consumption) pattern Q^{3*} , and a unique demand price vector ρ_3^* satisfying the equilibrium conditions of the supply chain. In other words, if the variational inequality (3.22) admits a solution, that should be the only solution in Q^1, Q^2, Q^3 and ρ_3 .*

Proof. Since the result of Lemma 3.7, we have the strict monotonicity of the vector function F that enters the variational inequality (3.22) and uniqueness follows. By the standard variational inequality theory [4], this theorem holds. \square

The following lemma, if the function F that enters the variational inequality problem (3.22) has some conditions then we can show that F is Lipschitz continuous.

Lemma 3.9. *The function F that enters the variational inequality problem (3.22) is Lipschitz continuous under the following conditions:*

- (1.) Each $f_m, m = 1, \dots, M$, is additive and has a bounded second-order derivative;
- (2.) $c_{mn}, c_n, \hat{c}_{ml}$ and c_{mo}^l have bounded second-order derivatives for all m, n, l, o ;
- (3.) \tilde{c}_{nk} and d_k have bounded first-order derivatives for all n, k .

Proof. The result is direct by applying a mean-value theorem from calculus to the vector function F that enters the variational inequality problem (3.22). Since

$$F'(x) = (F^{1'}(X), F^{2'}(X), F^{3'}(X), F^{4'}(X), F^{5'}(X)),$$

where

$$\begin{aligned} F^{1'}(X) &\equiv \sum_{s=1}^M \frac{\partial^2 f_s(Q^{1*}, Q^{2*})}{\partial^2 q_{mn}} + \frac{\partial^2 c_{mn}(q_{mn}^*)}{\partial^2 q_{mn}} + \frac{\partial^2 c_n(Q^{1*})}{\partial^2 q_{mn}} \\ F^{2'}(X) &\equiv \sum_{s=1}^M \frac{\partial^2 f_s(Q^{1*}, Q^{2*})}{\partial^2 q_{io}^m} + \frac{\partial^2 \hat{c}_{ml}(q_{io}^{m*})}{\partial^2 q_{io}^m} + \sum_{x=1}^L \sum_{y=1}^O \frac{\partial^2 c_{xy}^l(Q^{2*})}{\partial^2 q_{io}^m} \\ F^{3'}(X) &\equiv \frac{\partial \tilde{c}_{nk}(Q^{3*})}{\partial q_{nk}}; \quad F^{4'}(X) \equiv 0; \quad F^{5'}(X) \equiv -\frac{\partial d_k(\rho_3^*)}{\partial \rho_{3k}}. \end{aligned}$$

By the assumption (1.) – (3.), we have there exists L such that $\|F'(x)\| \leq L$. The proof is completed. \square

4. THE ALGORITHM

In this section, an algorithm is presented which can be applied to solve a variational inequality problem that was proposed in the above article. The algorithm is the modified projection method of Korpelevich [5] and is guaranteed to converge which provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists). Then, the algorithm for our supply chain network model comprising disaster relief as follows, where \Im denotes an iteration counter:

Modified projection method for the solution of variational inequality (3.22).

Step 0. Initialization Set $X_0 \equiv (Q_0^1, Q_0^2, Q_0^3, \gamma_0, \rho_{3k}^0) \in \mathcal{K}$. Let $\Im = 1$ and set α such that $0 < \alpha \leq \frac{1}{L}$, where L is the Lipschitz constant for the problem.

Step 1. Computation Compute $X^\Im \equiv (Q_\Im^1, Q_\Im^2, Q_\Im^3, \gamma_\Im, \rho_{3k}^\Im) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\left\langle X^\Im + (\alpha F(X^{\Im-1}) - X^{\Im-1}), X - X^{\Im-1} \right\rangle \geq 0, \tag{4.1}$$

for all $X \in \mathcal{K}$.

Step 2. Convergence verification If $|Q_{\mathfrak{S}}^1 - Q_{\mathfrak{S}-1}^1| \leq \epsilon$, $|Q_{\mathfrak{S}}^2 - Q_{\mathfrak{S}-1}^2| \leq \epsilon$, $|Q_{\mathfrak{S}}^3 - Q_{\mathfrak{S}-1}^3| \leq \epsilon$, $|\gamma_{\mathfrak{S}} - \gamma_{\mathfrak{S}-1}| \leq \epsilon$, $|\rho_{3k}^{\mathfrak{S}} - \rho_{3k}^{\mathfrak{S}-1}| \leq \epsilon$ for all $m = 1, 2, \dots, M$, $n = 1, 2, \dots, N$, $k = 1, 2, \dots, K$, $l = 1, 2, \dots, L$, $o = 1, 2, \dots, O$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathfrak{S} := \mathfrak{S} + 1$, and go to Step 1.

Note that the variational inequality subproblem (4.1) can be solved explicitly and in closed form since the feasible set is that of the nonnegative orthant. Indeed, they yield subproblems in the Q^1, Q^2, Q^3, γ_n and ρ_{3k} variables for all n, k .

Next, we state the convergence result for the modified projection method in this model.

Theorem 4.1. *Assume that the function that enters the variational inequality (3.22) (or (2.1)) satisfies the conditions in Lemma 3.6 and Lipschitz continuous of F . Then the modified projection method described above converges to the solution of the variational inequality (3.22).*

Proof. According to Korprlevich [5], the modified projected method converges to the solution of the variational inequality problem of the form (2.1). We provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. \square

5. NUMERICAL EXAMPLES

In this section, we apply the modified projection method to several numerical examples. The modified projection method was implemented in SCILAB and the computer system used was a ASUS located at the Pibulsongkhram Rajabhat University at Phitsanulok, Thailand. The convergence criterion used was that the absolute value of the product flows and prices between two successive iterations differed by no more than 10^{-4} . For the examples, α was set to 0.005 in the algorithm. The numerical examples had the network structure depicted in Figure 2 and consisted of two manufacturers, two retailers, two freight service providers, two demand markets and two demand points. The concept of this research was inspired by the paper of [12] and [10].

Example 5.1. In this example, we consider the supply chain network in Figure 2. There

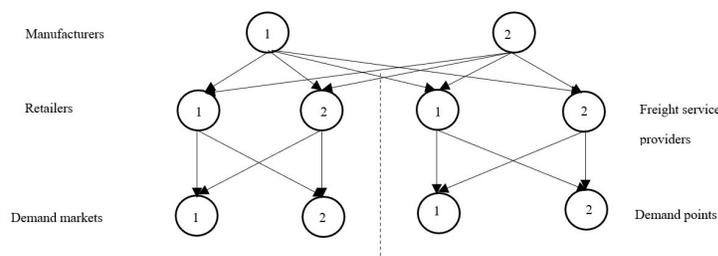


FIGURE 2. Example 6.1

is two manufacturers which each manufacturers transacted some products for sale to two retailers and donation to two freight service providers and then each retailer sent the product to two demand markets and each freight service provider sent the product to two demand point. Here, each manufacturer wish to ship the relief items to demand points 100 items, this implies $q_1 = q_2 = 100$. The data for the first example were constructed for

easy interpretation purposes. The production cost functions for the manufacturers were given by:

$$f_1(Q^1, Q^2) = 0.05(q_1^1 + q_1^2)^2 + 0.2q_1^1, f_2(Q^1, Q^2) = 0.05(q_2^1 + q_2^2)^2 + 0.2q_2^2.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers were given by:

$$\begin{aligned} c_{11}(q_{11}) &= 0.5(q_{11})^2 + 3.5q_{11}, & c_{12}(q_{12}) &= 0.5(q_{12})^2 + 3.5q_{12}, \\ c_{21}(q_{21}) &= 0.5(q_{21})^2 + 3.5q_{21}, & c_{22}(q_{22}) &= 0.5(q_{22})^2 + 3.5q_{22}. \end{aligned}$$

The handling costs of the retailers, in turn, were given by:

$$c_1(Q^1) = 0.5 \left(\sum_{m=1}^2 q_{m1} \right)^2, \quad c_2(Q^1) = 0.5 \left(\sum_{m=1}^2 q_{m2} \right)^2.$$

The transaction costs of the freight service providers associated with transacting with the manufacturers were given by:

$$\begin{aligned} \hat{c}_{11} &= 2.5(q_{11}^1 + q_{12}^1), & \hat{c}_{12} &= 2.25(q_{21}^1 + q_{22}^1), \\ \hat{c}_{21} &= 2.5(q_{11}^2 + q_{12}^2), & \hat{c}_{22} &= 2.25(q_{21}^2 + q_{22}^2). \end{aligned}$$

The total cost associated with freight service provider 1, \hat{c}_{11} , is higher than that for freight service provider 2, \hat{c}_{12} , since it does not have as much experience with the former provider and the transfer cost is higher per unit.

The freight service provider total costs are as follows: For freight service provider 1:

$$\begin{aligned} c_{11}^1 &= 0.01 (q_{11}^1)^2 + 9.67q_{11}^1, & c_{12}^1 &= 0.1 (q_{12}^1)^2 + 14.88q_{12}^1, \\ c_{21}^1 &= 0.1 (q_{11}^2)^2 + 9.67q_{11}^2, & c_{22}^1 &= 0.1 (q_{12}^2)^2 + 14.88q_{12}^2, \end{aligned}$$

and for freight service provider 2:

$$\begin{aligned} c_{11}^2 &= 0.1 (q_{21}^1)^2 + 9.67q_{21}^1, & c_{12}^2 &= 0.1 (q_{22}^1)^2 + 14.88q_{22}^1, \\ c_{21}^2 &= 0.1 (q_{21}^2)^2 + 9.67q_{21}^2, & c_{22}^2 &= 0.01 (q_{22}^2)^2 + 14.88q_{22}^2. \end{aligned}$$

The transaction costs between the retailers and the consumers at the demand markets were given by:

$$\tilde{c}_{11}(Q^3) = \tilde{q}_{11} + 5, \quad \tilde{c}_{12}(Q^3) = \tilde{q}_{12} + 5, \quad \tilde{c}_{21}(Q^3) = \tilde{q}_{21} + 5, \quad \tilde{c}_{22}(Q^3) = \tilde{q}_{22} + 5.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000.$$

Next, assume that the income tax rate as follows.

Net income	Income tax rate
$0 \leq x \leq 5000$	Tax exemption
$5000 < x \leq 10000$	5%
$10000 < x \leq 15000$	10%
$x \geq 15000$	20%

where x denotes for the amount of net income. Assume that the product cost price for donation of the manufacturer m , (ρ_m^*) , is equal to the minimum of the price which the manufacture m sells the product to the retailers.

Therefore, we computed equilibrium solution via the projection method as follows: The projection method converged in 708 iterations, the product shipments between the two manufacturers and two retailers were:

$$Q^{1*} : q_{11}^* = 50; q_{12}^* = 53; q_{21}^* = 55; q_{22}^* = 58.$$

The product shipments between the two manufacturers and two demand points via two freight service providers were:

$$Q^{2*} : \begin{aligned} q_{11}^1 &= 74, & q_{12}^1 &= 6, & q_{21}^1 &= 9, & q_{22}^1 &= 7, \\ q_{11}^2 &= 8, & q_{12}^2 &= 7, & q_{21}^2 &= 10, & q_{22}^2 &= 80. \end{aligned}$$

By the above results, we see that the most of products are shipped to the demand point 1 by the manufacturer 1 via the freight service provider 1 and, also the most of products are shipped to the demand point 2 by the manufacturer 2 via the freight service provider 2 because the shipping cost is cheaper. The product shipments between the two retailers and two demand markets were:

$$Q^{3*} : \tilde{q}_{11}^* = 62; \tilde{q}_{12}^* = 60; \tilde{q}_{21}^* = 59; \tilde{q}_{22}^* = 57.$$

The vector γ^* , which was equal to the prices charged by the retailers ρ_2^* , were:

$$\gamma^* : \gamma_1^* = 185.32; \gamma_2^* = 191.69.$$

The demand prices at demand markets were:

$$\rho_3^* : \rho_{31}^* = 254.96; \rho_{32}^* = 249.96.$$

In this example, we can interpret, in equilibrium, that if the manufacturer 1 and 2 sell the products to the retailers and demand points in above results then the profit of the manufacturer 1 is 1,645.16 and the profit of the manufacturer 2 is 1638.455. If the retailers sell the products to the demand markets in above results then the profit of the retailer 1 is 9000.04 and the profit of the retailer 2 is 7184.34. For the freight service provider, if they sent the product from the manufacturers to the demand points in above results then the profit of the freight service provider 1 is 69.66 and the profit of the freight service provider 2 is 77.

Next, the following example, we want to consider that if the manufacturers need to deliver the product to the demand points more than 100 items then we obtain the results as follows.

Example 5.2. In this example, we assume as all Example 5.1, except that each manufacturer wish to ship the product (relief items) 200 items. So, we obtain the results as:

The projection method converged in 1092 iterations, the product shipments between the two manufacturers and two retailers were:

$$Q^{1*} : q_{11}^* = 54; q_{12}^* = 54; q_{21}^* = 55; q_{22}^* = 54.$$

The product shipments between the two manufacturers and two demand points via two freight service providers were:

$$Q^{2*} : q_{11}^1 = 154, q_{12}^1 = 15, q_{21}^1 = 17, q_{22}^1 = 17, q_{11}^2 = 14, q_{12}^2 = 14, q_{21}^2 = 16, q_{22}^2 = 154.$$

Observe that, in the same way Example 5.1, the most of products are shipped to the demand point 1 by the manufacturer 1 via the freight service provider 1 and, also the most of products are shipped to the demand point 2 by the manufacturer 2 via the freight service provider 2. The product shipments between the two retailers and two demand markets were:

$$Q^{3*} : \tilde{q}_{11}^* = \tilde{q}_{12}^* = 53; \tilde{q}_{21}^* = \tilde{q}_{22}^* = 54.$$

The vector γ^* , which was equal to the prices charged by the retailers ρ_2^* , were:

$$\gamma^* : \gamma_1^* = 197.25; \gamma_2^* = 195.75.$$

The demand prices at demand markets were:

$$\rho_3^* : \rho_{31}^* = 254.56; \rho_{32}^* = 255.75.$$

In this example, we have the following results: if the manufacturer 1 and 2 sell the products to the retailers and demand points in above results then the profit of the manufacturer 1 is 1,780.38 and the profit of the manufacturer 2 is 1,211.11. If the retailers sell the products to the demand markets in above results then the profit of the retailer 1 is 10,612.8 and the profit of the retailer 2 is 5,751. For the freight service provider, if they

sent the product from the manufacturers to the demand points in above results then the profit of the freight service provider 1 is 298.86 and the profit of the freight service provider 2 is 320.56. Moreover, comparing the results with Example 5.1, we can see that the trading of products between manufacturers, retailers and consumers is almost no different from Example 5.1, but the difference is clearly in the delivery of donations. In this case, it can be seen that all freight service providers earn more profits than in Example 5.1 due to sending more products to donate and getting more tax relief.

For the following example, we are interested that if all conditions are the same as Example 5.2 except for the part of the retailers which there is a retailer.

Example 5.3. In this example, we are interested that all assumption is the same Example 5.2, except it has not the retailer 2 see Figure 3. So, we obtain the results as:

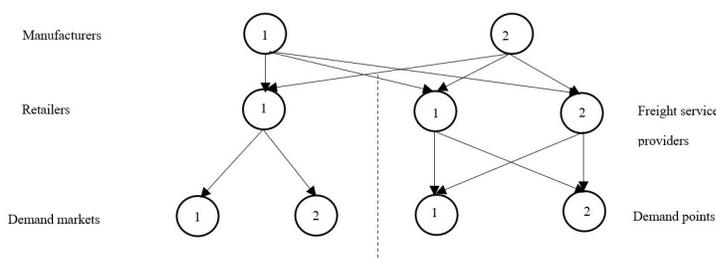


FIGURE 3. Example 5.3

The projection method converged in 832 iterations, the product shipments between the two manufacturers and a retailer were:

$$Q^{1*} : q_{11}^* = q_{21}^* = 57.$$

The product shipments between the two manufacturers and two demand points via two freight service providers were the same as Example 1,

$$Q^{2*} : q_{11}^* = 154, q_{12}^* = 15, q_{21}^* = 17, q_{22}^* = 17, q_{11}^{2*} = 14, q_{12}^{2*} = 14, q_{21}^{2*} = 16, q_{22}^{2*} = 154.$$

The product shipments between a retailer and two demand markets were:

$$Q^{3*} : \tilde{q}_{11}^* = 63; \tilde{q}_{12}^* = 64.$$

The vector γ^* , which was equal to the prices charged by the retailers ρ_2^* , were:

$$\gamma^* : \gamma_1^* = 204.23.$$

The demand prices at demand markets were:

$$\rho_3^* : \rho_{31}^* = 268.51; \rho_{32}^* = 266.71.$$

We have the following results: if the manufacturer 1 and 2 sell the products to the retailers and demand points in above results then the profit of the manufacturer 1 is -17.13 and the profit of the manufacturer 2 is -653.61 . If the retailers sell the products to the demand markets in above results then the profit of the retailer 1 is $9,595.31$. For the freight service provider, if they sent the product from the manufacturers to the demand points in above results then the profit of the freight service provider 1 is 298.86 and the profit of the freight service provider 2 is 320.56. Therefore, comparing the results with Example 5.2, we can see that the manufacturers sent the products to the demand point in the same of Example 5.2 and must sell more products to retailers than Example 5.2 and also the retailers sell more products to consumers than Example 5.2. If we consider the profit of all parties, it can be seen that the manufactures loses while other part have the same profit.

The following example, we will show that if Example 5.2 remains a demand market then the results as follows.

Example 5.4. If all of condition as the same Example 5.2, except this example has a demand market (Figure 4) and let

$$d_1(\rho_3) = -2\rho_{31} + 1000.$$

Then we have the following results. The projection method converged in 241 iterations,

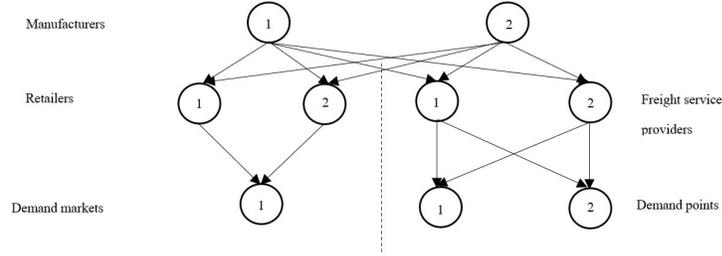


FIGURE 4. Example 5.4

the product shipments between the two manufacturers and two retailers were:

$$Q^1 : q_{11} = q_{12} = 65; q_{21}^* = q_{22}^* = 66.$$

The product shipments between the two manufacturers and two demand points via two freight service providers were:

$$Q^2 : q_{11}^* = 153, q_{12}^* = 15, q_{21}^* = 17, q_{22}^* = 16, q_{11}^2 = 15, q_{12}^2 = 14, q_{21}^2 = 16, q_{22}^2 = 154.$$

The product shipments between the retailers and a demand market were:

$$Q^3 : \tilde{q}_{11}^* = \tilde{q}_{21}^* = 131.$$

The vector γ^* , which was equal to the prices charged by the retailers ρ_2^* , were:

$$\gamma^* : \gamma_1^* = 233.13; \gamma_2^* = 233.03.$$

The demand prices at demand markets were:

$$\rho_3^* : \rho_{31}^* = 369.06.$$

We have the following results: if the manufacturer 1 and 2 sell the products to the retailers and demand points in above results then the profit of the manufacturer 1 is 3,920.25 and the profit of the manufacturer 2 is 3,452.10. If the retailers sell the products to the demand markets in above results then the profit of the retailer 1 is 8,572.24 and the profit of the retailer 2 is 8,557.83. For the freight service provider, if they sent the product from the manufacturers to the demand points in above results then the profit of the freight service provider 1 is 298.69 and the profit of the freight service provider 2 is 314.66. Therefore, comparing the results with Example 5.2, we can see that the manufacturers and retailers have to sell products in the larger quantities and also the price charged by the retailers and demand prices increase. But at the same time, the profits of all parties have also increased. For the donation part, it is not much different from Example 5.2.

The following example, we will show that if Example 5.1 remains a freight service provider then the results as follows.

Example 5.5. If all of condition as the same Example 5.1, except this example has a freight service provider (Figure 5). Then we have the following results. The projection method converged in 755 iterations, the product shipments between the two manufacturers and two retailers were:

$$Q^1 : q_{11}^* = q_{12}^* = 48; q_{21}^* = 60; q_{22}^* = 62.$$

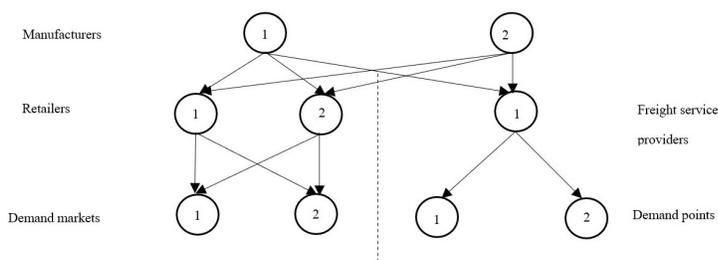


FIGURE 5. Example 5.5

The product shipments between the two manufacturers and two demand points via two freight service providers were:

$$Q^{2*} : q_{11}^{1*} = 99, q_{12}^{1*} = 16, q_{11}^{2*} = 2, q_{12}^{2*} = 85.$$

The product shipments between the retailers and a demand market were:

$$Q^{3*} : \tilde{q}_{11}^* = \tilde{q}_{12}^* = 61; \tilde{q}_{21}^* = \tilde{q}_{22}^* = 59.$$

The vector γ^* , which was equal to the prices charged by the retailers ρ_2^* , were:

$$\gamma^* : \gamma_1^* = 189.91; \gamma_2^* = 192.23.$$

The demand prices at demand markets were:

$$\rho_3^* : \rho_{31}^* = \rho_{32}^* = 251.39.$$

We have the following results: if the manufacturer 1 and 2 sell the products to the retailers and demand points in above results then the profit of the manufacturer 1 is 4,596.87 and the profit of the manufacturer 2 is 1,490.05. If the retailers sell the products to the demand markets in above results then the profit of the retailer 1 is 7,266.62 and the profit of the retailer 2 is 6,269.94. For the freight service provider, if they sent the product from the manufacturers to the demand points in above results then the profit of the freight service provider 1 is 846.51. Therefore, comparing the results with Example 5.1, we can see that the manufacturers 1 produce more products for donations and less for sale and the manufacturers 2 produce more products for sale and less for donations. In the margins, only the manufacturer 1 and the freight service provider 1 gain more profit.

Remark 5.6. By all of the previous examples, we see that if the network has a demand market (Example 5.4) then all of the parties have more profitable, but also have to produce more and consumers are willing to pay more.

6. CONCLUSION

In this paper, we present the generalized supply chain network, that is, the supply chain network comprising disaster relief which is a combination between a supply chain network and a competitive freight service provider network. Firstly, we proposed a model which was satisfy our supply chain network and considered the behavior of the manufacturers, where the manufacturers want to sell the product and donate as well, the retailers, the demand markets and the freight service provider. So, we obtain the supply chain network comprising disaster relief equilibrium model and the variational inequality which was equivalent to such supply chain network. The existence and uniqueness of the solution of the variational inequality was proposed. Finally, the algorithm which was a tool for using the computing our example was presented and some examples were presented for illustrative in the above articles.

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