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CONTROLLABILITY OF STOCHASTIC IMPULSIVE NEUTRAL INTEGRODIFFERENTIAL SYSTEMS WITH INFINITE DELAY

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ABSTRACT. The paper is concerned with the controllability of stochastic impulsive neutral integrodifferential systems with infinite delay in an abstract space. Sufficient conditions for controllability are obtained by means of semigroup theory and Banach contraction principle. An example is provided to illustrate the theory.

KEYWORDS: Stochastic impulsive neutral integrodifferential system; Infinite delay; Mild solution; Banach fixed point theorem.
AMS Subject Classification: 93B05, 34A37.

1. INTRODUCTION

Controllability is one of the fundamental concept in mathematical control theory and plays an important role in both deterministic and stochastic control systems. It is illustrious that controllability of deterministic systems are widely used in many fields of science and technology. The deterministic models often fluctuate due to noise, which is random or atleast appears to be so. Therefore, we must move from deterministic problems to stochastic problems. Many systems in physics and biology exhibit impulsive dynamical behavior due to sudden jumps at certain instants during the dynamical process. Differential equations involving impulsive effects occur in many applications: pharmacokinetics, the radiation of electromagnetic waves, population dynamics [8], the abrupt increase of glycerol in fed-batch culture, bio-technology, nanoelectronics, etc., and for basic theory refer [16, 26]. The theory of impulsive integrodifferential equations with their applications in fields such as mechanics, electrical engineering, medicine, ecology and so on have become an active areas of investigation since the theory provides a natural framework for mathematical modeling of many physical phenomena. Moreover, impulsive control which based on the theory of impulsive integrodifferential equations has gained renewed interest recently for its promising applications towards controlling systems exhibiting chaotic behavior (see [29]). As the generalization of classic impulsive integrodifferential equations, impulsive neutral stochastic functional integrodifferential equations have attracted the researchers great interest.

Byszewski [9] introduced the nonlocal initial conditions into the initial value problems and argued that the corresponding models more accurately describe the phenomena since

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more information was taken into account at the onset of the experiment, thereby reducing the ill effects incurred by a single (possibly erroneous) initial measurement.

The controllability of nonlinear deterministic systems represented by evolution equations in abstract spaces have been studied by several authors [2, 3, 5]. Balachandran and Anandhi [1] and Balachandran et al. [7] discussed the controllability of neutral functional integrodifferential systems in abstract spaces. Li et al. [18] studied the controllability of impulsive functional differential systems in Banach spaces. Chang [10] and Park et al. [22] investigated the controllability of impulsive functional differential systems and impulsive neutral integrodifferential systems with infinte delay in Banach spaces. The controllability of nonlinear stochastic systems in finite and infinite-dimensional spaces have been studied by many authors [15, 19]. Balachandran and Karthikeyan [4] and Balachandran et al. [6] derived the sufficient conditions for controllability of stochastic integrodifferential systems in finite dimensional spaces. Park et al. [23] discussed the controllability of neutral stochastic functional integrodifferential infinite delay systems in abstract space. Sakthivel et al. [25] investigated the controllability of non-linear impulsive stochastic systems. Subalakshmi and Balachandran [27, 28] studied the controllability of semilinear stochastic functional integrodiffferential systems and approximate controllability of nonlinear stochastic impulsive integrodifferential systems in Hilbert spaces. Based on [21], Hu and Ren [14] proved the existence results for impulsive neutral stochastic functional integrodifferential equations with infinite delays in the phase space $\mathcal{B}_h$. Motivated by these literatures, in this paper we study the controllability of stochastic impulsive neutral integrodifferential system with infinite delay in the phase space $\mathcal{B}_h$ which is an untreated topic sofar.

Consider the following class of stochastic impulsive neutral integrodifferential equation with infinite delay and nonlocal conditions

$$\begin{aligned} d\Big[x(t)-g\Big(t,x_t,\int_0^t c(t,s,x_s)ds\Big)\Big] &= \Big[Ax(t)+f\Big(t,x_t,\int_0^t e(t,s,x_s)ds\Big)+Bu(t)\Big]dt \\ &\quad + \int_{-\infty}^t \sigma(t,s,x_s)dw(s),\ t\in J:=[0,a],\ t\neq\tau_k, \\ \triangle x(\tau_k) &= x(\tau_k^+)-x(\tau_k^-)=I_k(x(\tau_k^-)), \quad k=1,2,\cdots,m, \\ x(s)+\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(s) &= \phi(s)\in\mathcal{L}_2(\Omega,\mathcal{B}_h),\ for\ a.e.\ s\in J_0:=(-\infty,0] \end{aligned} \tag{1.1}$$

where $0<t_1<t_2<t_3<\cdots<t_p\leq a\ \ (p\in N)$. Here, the state variable $x(\cdot)$ takes values in a real separable Hilbert space H with innerproduct $(\cdot,\cdot)$ and norm $|\cdot|$ and A is the infinitesimal generator of a strongly continuous semigroup of a bounded linear operator $\mathcal{S}(t),\ t\geq 0$ in H. The control function $u(\cdot)$ takes values in $L^2(J,U)$ of admissible control functions for a separable Hilbert space U and B is a bounded linear operator from U into H. Let K be another separable Hilbert space with innerproduct $(\cdot,\cdot)_K$ and the norm $|\cdot|_K$. Suppose $\{w(t):t\geq 0\}$ is a given K-valued Wiener process with a finite trace nuclear covariance operator $Q\geq 0$. We employ the same notation $|\cdot|$ for the norm $\mathcal{L}(K,H)$, where $\mathcal{L}(K,H)$ denotes the space of all bounded linear operators from K into H. Further, $c:D\times\mathcal{B}_h\longrightarrow H, e:D\times\mathcal{B}_h\longrightarrow H, g:J\times\mathcal{B}_h\times H\longrightarrow H, f:J\times\mathcal{B}_h\times H\longrightarrow H,\ \sigma: D\times\mathcal{B}_h\longrightarrow\mathcal{L}_Q(K,H)$ and $h_1:\mathcal{B}_h^p\longrightarrow\mathcal{B}_h$ are measurable mappings in H-norm, $\mathcal{L}_Q(K,H)$-norm and $\mathcal{B}_h$-norm respectively. Here $\mathcal{L}_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operators from K into H which will be defined in Section 2 and $D=\{(t,s)\in J\times J: s\leq t\}$. Here, $I_k\in C(H,H)\ (k=1,2,\cdots,m)$ are bounded functions. Furthermore, the fixed times τ_k satisfies $0=\tau_0<\tau_1<\tau_2<\cdots<\tau_m<\tau_{m+1}=a$, $x(\tau_k^+)$ and $x(\tau_k^-)$ denote the right and left limits of $x(t)$ at $t=\tau_k$. And $\triangle x(\tau_k)=x(\tau_k^+)-x(\tau_k^-)$ represents the jump in the state x at time τ_k, where I_k determines the size of the jump. The histories $x_t:\Omega\longrightarrow\mathcal{B}_h$, $t\geq 0$, which are defined by setting $x_t=\{x(t+s), s\in(-\infty,0]\}$, belong to the phase space $\mathcal{B}_h$, which will be defined in Section 2. The initial data $\phi=\{\phi(t):-\infty<t\leq 0\}$ is an $\mathcal{F}_0$-measurable, $\mathcal{B}_h$-valued random variables independent of $\{w(t):t\geq 0\}$ with finite second moment.

2. PRELIMINARIES

Throughout the paper $(H, |\cdot|)$ and $(K, |\cdot|_K)$ denote real separable Hilbert spaces.

Let $(\Omega, \mathcal{F}, P; \mathbf{F})$ $\{\mathbf{F} = \{\mathcal{F}_t\}_{t\geq 0}\}$ be a complete filtered probability space satisfying that $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. An H-valued random variable is an $\mathcal{F}$-measurable function $x(t)\colon\ \Omega \longrightarrow H$ and the collection of random variables $S = \{x(t,\omega) : \Omega \longrightarrow H \backslash t \in J\}$ is called a stochastic process. Generally, we just write $x(t)$ instead of $x(t,\omega)$ and $x(t) : J \longrightarrow H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t) : t \geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $Tr(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies that $Qe_i = \lambda_i e_i$. So, actually, $w(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t = \sigma\{w(s) : 0 \leq s \leq t\}$ is the σ-algebra generated by w and $\mathcal{F}_a = \mathcal{F}$. Let $\Psi \in \mathcal{L}(K, H)$ and define

$$|\Psi|_Q^2 = Tr(\Psi Q \Psi^*) = \sum_{i=1}^{\infty} |\sqrt{\lambda_i}\Psi e_i|^2.$$

If $|\Psi|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K, H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi : K \longrightarrow H$. The completion $\mathcal{L}_Q(K, H)$ of $\mathcal{L}(K, H)$ with respect to the topology induced by the norm $|\cdot|_Q$ where $|\Psi|_Q^2 = (\Psi, \Psi)$ is a Hilbert space with the above norm topology. For more details refer to Da Prato [11].

Now, we present the abstract phase space $\mathcal{B}_h$. Assume that $h : (-\infty, 0] \longrightarrow (0, \infty)$ is a continuous function with $l = \int_{-\infty}^{0} h(t)dt < \infty$. For any $b > 0$, define

$$\mathcal{B}_h = \Big\{\Psi : (-\infty, 0] \longrightarrow H : (E|\Psi(\theta)|^2)^{1/2} \text{ is a bounded and measurable function on } [-b, 0] \\ \text{and } \int_{-\infty}^{0} h(s) \sup_{s\leq\theta\leq 0} (E|\Psi(\theta)|^2)^{1/2} ds < \infty\Big\}.$$

If $\mathcal{B}_h$ is endowed with the norm

$$\|\Psi\|_{\mathcal{B}_h} = \int_{-\infty}^{0} h(s) \sup_{s\leq\theta\leq 0} (E|\Psi(\theta)|^2)^{1/2} ds, \quad \forall \ \ \Psi \in \mathcal{B}_h,$$

then $(\mathcal{B}_h, \|\cdot\|_{\mathcal{B}_h})$ is a Banach space [13].

Let A be the infinitesimal generator of an analytic semigroup $\mathcal{S}(t)$ in H. Then, $(A - \alpha I)$ is invertible and generates a bounded analytic semigroup for $\alpha > 0$ large enough. Therefore, we can assume that the semigroup $\mathcal{S}(t)$ is bounded and the generator A is invertible. We suppose that $0 \in \rho(A)$, which is the resolvent set of A. It follows that $(-A)^{\alpha},\ 0 < \alpha \leq 1$ can be defined as a closed linear invertible operator with its domain $D(-A)^{\alpha}$ being dense in H. We denote by H_{α} the Banach space $D(-A)^{\alpha}$ endowed with the norm $\|x\|_{\alpha} = \|(-A)^{\alpha}x\|$, which is equivalent to the graph norm of $(-A)^{\alpha}$. Furthermore, we have $H_{\beta} \subset H_{\alpha},\ 0 < \alpha < \beta$ and the embedding is continuous. For semigroup theory literature we refer [24].

Lemma 2.1. *The following two properties hold:*

(i) *If $0 < \beta < \alpha \leq 1$, then $H_{\alpha} \subset H_{\beta}$ and the embedding is compact whenever the resolvent operator of A is compact.*

(ii) *For every $0 < \alpha \leq 1$, there exists C_{α} such that*

$$\|(-A)^{\alpha}\mathcal{S}(t)\| \leq \frac{C_{\alpha}}{t^{\alpha}}, \quad t > 0. \tag{2.1}$$

Let $J_1 = (-\infty, a]$. Now, we define the mild solution of (1.1) as in [14].

Definition 2.2. A stochastic process $x : J_1 \times \Omega \longrightarrow H$ is called a mild solution of (1.1) if

(a) $x(t)$ is measurable and $\mathcal{F}_t$-adapted, for each $t \geq 0$;

(b) $\triangle x(\tau_k) = x(\tau_k^+) - x(\tau_k^-) = I_k(x(\tau_k^-)),\ k = 1, 2, \cdots, m$;

(c) $x(t) \in H$ and for every $0 \leq s < t$, the function $A\mathcal{S}(t-s)g(s, x_s, \int_0^s c(s,\eta,x_\eta)d\eta)$ is integrable such that the following integral equation is satisfied

$$\begin{aligned} x(t) \;=\; & \mathcal{S}(t)\big[\phi(0) - \big(h_1(x_{t_1}, x_{t_2}, x_{t_3}, \cdots, x_{t_p})\big)(0) - g(0, x_0, 0)\big] + g\Big(t, x_t, \int_0^t c(t,s,x_s)ds\Big) \\ & + \int_0^t A\mathcal{S}(t-s)g\Big(s, x_s, \int_0^s c(s,\eta,x_\eta)d\eta\Big)ds + \int_0^t \mathcal{S}(t-s)Bu(s)ds \\ & + \int_0^t \mathcal{S}(t-s)f\Big(s, x_s, \int_0^s e(s,\eta,x_\eta)d\eta\Big)ds + \int_0^t \mathcal{S}(t-s)\Big(\int_{-\infty}^s \sigma\big(s,\eta,x_\eta\big)dw(\eta)\Big)ds \\ & + \sum_{0<\tau_k<t} \mathcal{S}(t-\tau_k)I_k(x(\tau_k^-)), \quad \text{for a.e. } t \in J; \end{aligned} \tag{2.2}$$

(d) $x_0(\cdot) + \big(h_1(x_{t_1}, x_{t_2}, x_{t_3}, \cdots, x_{t_p})\big)(0) = \phi \in \mathcal{L}_2(\Omega, \mathcal{B}_h)$ on J_0 satisfies $\|\phi\|^2_{\mathcal{B}_h} < \infty$.

Definition 2.3. The system (1.1) is said to be controllable on the interval $[0,a]$, if for every initial function $\phi \in \mathcal{L}_2(\Omega, \mathcal{B}_h)$ and $x_1 \in H$, there exists a control $u \in L^2(J,U)$ such that the solution $x(\cdot)$ of (1.1) satisfies $x(a) = x_1$.

In order to establish our result we assume the following hypotheses:

(H1) A is the infinitesimal generator of an analytic semigroup $\mathcal{S}(t)$ in H, $0 \in \rho(A)$ and there exists a positive constant M such that

$$\|\mathcal{S}(t)\|^2 \leq M \text{ for all } t \geq 0.$$

(H2) The linear operator $W : L^2(J,U) \longrightarrow H$ defined by

$$Wu = \int_0^a \mathcal{S}(a-s)Bu(s)ds$$

is invertible with inverse operator W^{-1} taking values in $L^2(J,U) \setminus kerW$ and there exist positive constants M_b and M_W such that

$$\|B\|^2 \leq M_b \; and \; \|W^{-1}\|^2 \leq M_W.$$

(H3) There exists a constant $M_c > 0$ such that for all $x, y \in \mathcal{B}_h, (t,s) \in D$,

$$E\Big|\int_0^t \Big[c(t,s,x) - c(t,s,y)\Big]ds\Big|^2 \leq M_c\|x-y\|^2_{\mathcal{B}_h}.$$

(H4) There exist constants $0 < \beta < 1$ and M_g such that g is H_β -valued, $(-A)^\beta g$ is continuous and for $t \in J,\ x_1, x_2 \in \mathcal{B}_h,\ y_1, y_2 \in H$ such that

$$E\big|(-A)^\beta g(t,x_1,y_1) - (-A)^\beta g(t,x_2,y_2)\big|^2 \leq M_g\Big[\|x_1-x_2\|^2_{\mathcal{B}_h} + E|y_1-y_2|^2\Big].$$

(H5) The function $\sigma : D \times \mathcal{B}_h \longrightarrow \mathcal{L}_Q(K,H)$ is continuous and there exist positive constants $M_\sigma, \tilde{M}_\sigma$ for all $(t,s) \in D$ and $x, y \in \mathcal{B}_h$ such that

$$E|\sigma(t,s,x) - \sigma(t,s,y)|^2_Q \leq M_\sigma\|x-y\|^2_{\mathcal{B}_h},$$
$$E|\sigma(t,s,x)|^2_Q \leq \tilde{M}_\sigma.$$

(H6) For each $\phi \in \mathcal{B}_h$,

$$\mathcal{H}(t) = \lim_{b \longrightarrow \infty} \int_{-b}^0 \sigma(t,s,\phi)dw(s)$$

exists and is continuous. Further, there exists a positive constant $M_\mathcal{H}$ such that

$$|\mathcal{H}(s)|^2_Q \leq M_\mathcal{H}.$$

(H7) For each $(t,s) \in D$, $x, y \in \mathcal{B}_h$, the function $e : D \times \mathcal{B}_h \longrightarrow H$ is continuous and there exist constants $M_e,\ \tilde{M}_e$ such that

$$E\Big|\int_0^t \Big[e(t,s,x) - e(t,s,y)\Big]ds\Big|^2 \leq M_e\|x-y\|^2_{\mathcal{B}_h}$$

and $\tilde{M}_e = \sup_{(t,s)\in D}\big(|\int_0^t e(t,s,0)ds|^2\big)$.

(H8) The function $f : J\times\mathcal{B}_h\times H \longrightarrow H$ is continuous and there exist positive constants $M_f, \tilde{M}_f$ for $t\in J, x_1,x_2\in\mathcal{B}_h,\ y_1,y_2\in H$ such that

$$E\Big|f(t,x_1,y_1)-f(t,x_2,y_2)\Big|^2 \le M_f\Big[\|x_1-x_2\|^2_{\mathcal{B}_h}+E|y_1-y_2|^2\Big]$$

and $\tilde{M}_f=\sup_{t\in J}|f(t,0,0)|^2$.

(H9) The function $h_1 : \mathcal{B}_h^p \longrightarrow \mathcal{B}_h$ is continuous and there exist positive constants $M_h,\ \tilde{M}_h$ for $x,y\in\mathcal{B}_h,\ s\in(-\infty,0]$ such that

$$E\|\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(s)-\big(h_1(y_{t_1},y_{t_2},y_{t_3},\cdots,y_{t_p})\big)(s)\|^2\le M_h\|x-y\|^2_{\mathcal{B}_h},$$

$$E\|\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(s)\|^2\le\tilde{M}_h.$$

(H10) $I_k\in C(H,H)$ and there exist positive constants $\beta_k,\ \tilde{\beta}_k$ such that for all $x,y\in H$,

$$E|I_k(x)-I_k(y)|^2\le\beta_k\|x-y\|^2_{\mathcal{B}_h},\quad k=1,2,\cdots m,$$

$$E|I_k(x)|^2\le\tilde{\beta}_k\quad k=1,2,\cdots m.$$

(H11) There exists a constant $\nu>0$ such that

$$\begin{aligned}
\nu &= 12l^2\Big\{\big(1+6MM_bM_Wa^2\big)\Big[\Big(M_0^2+\frac{(C_{1-\beta}a^\beta)^2}{2\beta-1}\Big)M_g(1+2M_c)+2a^2MM_\sigma\\
&\quad +Ma^2M_f(1+M_e)+mM\sum_{k=1}^m\beta_k\Big]+6M^2M_bM_Wa^2M_h\Big\}<1,\\
\tilde{M} &= 7\Big\{(1+9MM_bM_Wa^2)\Big[2MM_0^2\big[M_g\|\widehat{\phi}\|^2_{\mathcal{B}_h}+c_2\big]+2Ma^2[2M_f\tilde{M}_e+\tilde{M}_f]\\
&\quad +2\Big(M_0^2+\frac{\big(C_{1-\beta}a^\beta\big)^2}{2\beta-1}\Big)[2M_gc_1+c_2]+2Ma^2[M_{\mathcal{H}}+Tr(Q)\tilde{M}_\sigma]\\
&\quad +mM\sum_{k=1}^m\tilde{\beta}_k\Big]+9MM_bM_Wa^2(|x_1|^2+ME|\phi(0)|^2+M\tilde{M}_h)\Big\},\text{ where}\\
M_0 &= \|(-A)^{-\beta}\|, c_1=\sup_{(t,s)\in D}\Big|\int_0^t c(t,s,0)ds\Big|^2\text{ and }c_2=\sup_{t\in J}|(-A)^\beta g(t,0,0)|^2.
\end{aligned}$$

We now consider the space

$$\begin{aligned}
\mathcal{B}_a=\Big\{&x:J_1\longrightarrow H, x_k\in C(J_k,H)\text{ and there exist }x(\tau_k^-)\text{ and }x(\tau_k^+)\text{ with}\\
&x(\tau_k)=x(\tau_k^-),\ x_0+\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(0)=\phi\in\mathcal{B}_h, k=0,1,2,\cdots,m\Big\},
\end{aligned}$$

where x_k is the restriction of x to $J_k=(\tau_k,\tau_{k+1}]$ and $C(J_k,H)$ denotes the space of all continuous H-valued stochastic processes $\{\xi(t):t\in J_k\}, k=0,1,2,\cdots,m$. Set $\|\cdot\|_a$ be a seminorm in $\mathcal{B}_a$ defined by

$$\|x\|_a=\|x_0\|_{\mathcal{B}_h}+\sup_{0\le s\le a}\Big(E|x(s)|^2\Big)^{1/2},\ x\in\mathcal{B}_a.$$

We give a useful lemma appeared in [17].

Lemma 2.4. *Assume that $x\in\mathcal{B}_a$, then for $t\in J, x_t\in\mathcal{B}_h$. Moreover,*

$$l(E|x(t)|^2)^{1/2}\le\|x_t\|_{\mathcal{B}_h}\le l\sup_{0\le s\le t}(E|x(s)|^2)^{1/2}+\|x_0\|_{\mathcal{B}_h},$$

where $l=\int_{-\infty}^0 h(s)ds<\infty$.

3. CONTROLLABILITY RESULT

Theorem 3.1. *If the conditions* $(H1)-(H11)$ *are satisfied then the system* (1.1) *is controllable on* J *provided that*

$$7l^2\Big\{\big(1+9MM_bM_Wa^2\big)\Big[8\Big(M_0^2+\frac{(C_{1-\beta}a^\beta)^2}{2\beta-1}\Big)M_g(1+2M_c)+8Ma^2M_f(1+2M_e)\Big]\Big\}<1 \quad (3.1)$$

Proof: Using the hypothesis $(H2)$ for an arbitrary function $x(\cdot)$, define the control

$$\begin{aligned} u_x^a(t) \;=\;& W^{-1}\Big\{x_1-\mathcal{S}(a)\big[\phi(0)-\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(0)-g(0,x_0,0)\big] \\ & -g\Big(a,x_a,\int_0^a c(a,s,x_s)ds\Big)-\int_0^a A\mathcal{S}(a-s)g\Big(s,x_s,\int_0^s c(s,\eta,x_\eta)d\eta\Big)ds \\ & -\int_0^a\mathcal{S}(a-s)f\Big(s,x_s,\int_0^s e(s,\eta,x_\eta)d\eta\Big)ds-\int_0^a\mathcal{S}(a-s)\Big[\mathcal{H}(s)+\int_0^s\sigma(s,\eta,x_\eta)dw(\eta)\Big]ds \\ & -\sum_{0<\tau_k<a}\mathcal{S}(a-\tau_k)I_k(x(\tau_k^-))\Big\}(t). \end{aligned} \quad (3.2)$$

Consider the mapping $\Phi:\mathcal{B}_a\longrightarrow\mathcal{B}_a$ defined by

$$(\Phi x)(t)=\begin{cases}\phi(t)-\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(t), & t\in J_0,\\[2mm] \mathcal{S}(t)\big[\phi(0)-\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(0)-g(0,x_0,0)\big] & \\ +g\Big(t,x_t,\int_0^t c(t,s,x_s)ds\Big)+\int_0^t A\mathcal{S}(t-s)g\Big(s,x_s,\int_0^s c(s,\eta,x_\eta)d\eta\Big)ds & \\ +\int_0^t\mathcal{S}(t-s)Bu_x^a(s)ds+\int_0^t\mathcal{S}(t-s)f\Big(s,x_s,\int_0^s e(s,\eta,x_\eta)d\eta\Big)ds & \\ +\int_0^t\mathcal{S}(t-s)\Big[\mathcal{H}(s)+\int_0^s\sigma(s,\eta,x_\eta)dw(\eta)\Big]ds+\sum_{0<\tau_k<t}\mathcal{S}(t-\tau_k)I_k(x(\tau_k^-)), & \\ & \text{for a.e. } t\in J.\end{cases} \quad (3.3)$$

We shall show that the operator Φ has a fixed point, which is then a solution of system (1.1). Clearly, $(\Phi x)(a)=x_1$. For

$$\begin{aligned} E\Big|\int_0^t A\mathcal{S}(t-s)g\Big(s,x_s,\int_0^s c(s,\eta,x_\eta)d\eta\Big)ds\Big|^2 &\le E\Big|\int_0^t(-A)^{1-\beta}\mathcal{S}(t-s)(-A)^\beta g\Big(s,x_s,\int_0^s c(s,\eta,x_\eta)d\eta\Big)ds\Big|^2 \\ &\le \int_0^t\frac{2C_{1-\beta}^2}{(t-s)^{2(1-\beta)}}\Big[M_g(1+2M_c)\|x_s\|_{\mathcal{B}_h}^2+2M_gc_1+c_2\Big]ds, \end{aligned}$$

then from the Bochner theorem [20], it follows that $A\mathcal{S}(t-s)g\Big(s,x_s,\int_0^s c(s,\eta,x_\eta)d\eta\Big)$ is integrable on J.
For $\phi\in\mathcal{B}_h$, define

$$\widehat{\phi}(t)=\begin{cases}\phi(t)-\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(t), & t\in J_0,\\ \mathcal{S}(t)\Big[\phi(0)-\big(h_1(x_{t_1},x_{t_2},x_{t_3},\cdots,x_{t_p})\big)(0)\Big], & t\in J,\end{cases} \quad (3.4)$$

then $\widehat{\phi}(t)\in\mathcal{B}_a$. Set

$$x(t)=z(t)+\widehat{\phi}(t),\quad t\in J_1.$$

It is clear that x satisfies (2.2) if and only if z satisfies $z_0=0$ and

$$\begin{aligned} z(t) \;=\;& -\mathcal{S}(t)g(0,\widehat{\phi}_0,0)+g\Big(t,z_t+\widehat{\phi}_t,\int_0^t c(t,s,z_s+\widehat{\phi}_s)ds\Big) \\ & +\int_0^t A\mathcal{S}(t-s)g\Big(s,z_s+\widehat{\phi}_s,\int_0^s c(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)ds \end{aligned}$$

$$
\begin{aligned}
&+\int_0^t \mathcal{S}(t-s)f\Big(s,z_s+\widehat{\phi}_s,\int_0^s e(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)ds\\
&+\int_0^t \mathcal{S}(t-s)\Big(\mathcal{H}(s)+\int_0^s \sigma\big(s,\eta,z_\eta+\widehat{\phi}_\eta)dw(\eta)\Big)ds\\
&+\int_0^t \mathcal{S}(t-s)Bu^a_{z+\widehat{\phi}}(s)ds+\sum_{0<\tau_k<t}\mathcal{S}(t-\tau_k)I_k(z(\tau_k^-)+\widehat{\phi}(\tau_k^-)),\ \ t\in J,
\end{aligned}
$$

where,

$$
\begin{aligned}
u^a_{z+\widehat{\phi}}(t)\ &= W^{-1}\Big\{x_1-\mathcal{S}(a)\big[\phi(0)-\big(h_1((z+\widehat{\phi})_{t_1},(z+\widehat{\phi})_{t_2},\cdots,(z+\widehat{\phi})_{t_p})\big)(0)\\
&\quad -g(0,z_0+\widehat{\phi}_0,0)\big]-g\Big(a,z_a+\widehat{\phi}_a,\int_0^a c(a,s,z_s+\widehat{\phi}_s)ds\Big)\\
&\quad -\int_0^a A\mathcal{S}(a-s)g\Big(s,z_s+\widehat{\phi}_s,\int_0^s c(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)ds\\
&\quad -\int_0^a \mathcal{S}(a-s)f\Big(s,z_s+\widehat{\phi}_s,\int_0^s e(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)ds\\
&\quad -\int_0^a \mathcal{S}(a-s)\Big[\mathcal{H}(s)+\int_0^s \sigma\big(s,\eta,z_\eta+\widehat{\phi}_\eta)dw(\eta)\Big]ds\\
&\quad -\sum_{0<\tau_k<a}\mathcal{S}(a-\tau_k)I_k(z(\tau_k^-)+\widehat{\phi}(\tau_k^-))\Big\}(t).
\end{aligned}
$$

Let $\mathcal{B}_a^0=\{y\in\mathcal{B}_a: y_0=0\in\mathcal{B}_h\}$. For any $y\in\mathcal{B}_a^0$, we have

$$
\|y\|_a=\|y_0\|_{\mathcal{B}_h}+\sup_{0\le s\le a}\Big(E|y(s)|^2\Big)^{1/2}=\sup_{0\le s\le a}\Big(E|y(s)|^2\Big)^{1/2}
$$

and thus $(\mathcal{B}_a^0,\|\cdot\|_a)$ is a Banach space. Set

$$
\mathcal{B}_q=\{y\in\mathcal{B}_a^0:\ y(0)=0,\ \|y\|_a^2\le q\}\ \text{for some}\ \ q\ge 0,
$$

then $\mathcal{B}_q\subseteq\mathcal{B}_a^0$ is a bounded closed convex set, and for $z\in\mathcal{B}_q$, we have

$$
\begin{aligned}
\|z_t+\widehat{\phi}_t\|^2_{\mathcal{B}_h}\ &\le\ 2\big(\|z_t\|^2_{\mathcal{B}_h}+\|\widehat{\phi}_t\|^2_{\mathcal{B}_h}\big)\\
&\le\ 4\big(l^2\sup_{0\le s\le t}E|z(s)|^2+\|z_0\|^2_{\mathcal{B}_h}+l^2\sup_{0\le s\le t}E|\widehat{\phi}(t)|^2+\|\widehat{\phi}_0\|^2_{B_h}\big)\\
&\le\ 4l^2\big(q+2M(E|\phi(0)|^2+M_h)\big)+4\|\widehat{\phi}\|^2_{\mathcal{B}_h}\\
&:=\ q\prime.
\end{aligned}
\tag{3.5}
$$

Let the operator $\widehat{\Phi}:\mathcal{B}_a^0\longrightarrow\mathcal{B}_a^0$ defined by

$$
(\widehat{\Phi}z)(t)=\begin{cases}
0, & t\in J_0,\\
-\mathcal{S}(t)g(0,\widehat{\phi}_0,0)+g\Big(t,z_t+\widehat{\phi}_t,\int_0^t c(t,s,z_s+\widehat{\phi}_s)ds\Big) & \\
+\int_0^t A\mathcal{S}(t-s)g\Big(s,z_s+\widehat{\phi}_s,\int_0^s c(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)ds & \\
+\int_0^t \mathcal{S}(t-s)f\Big(s,z_s+\widehat{\phi}_s,\int_0^s e(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)ds & \\
+\int_0^t \mathcal{S}(t-s)\Big(\mathcal{H}(s)+\int_0^s \sigma\big(s,\eta,z_\eta+\widehat{\phi}_\eta)dw(\eta)\Big)ds & \\
+\int_0^t \mathcal{S}(t-s)Bu^a_{z+\widehat{\phi}}(s)ds+\sum\limits_{0<\tau_k<t}\mathcal{S}(t-\tau_k)I_k(z(\tau_k^-)+\widehat{\phi}(\tau_k^-)),\ t\in J. &
\end{cases}
\tag{3.6}
$$

Obviously, the operator Φ has a fixed point which is equivalent to prove that $\widehat{\Phi}$ has a fixed point. Since all the functions involved in the operator are continuous therefore $\widehat{\Phi}$ is continuous.

From our assumptions we have

$$\begin{aligned}
E|u^a_{z+\widehat{\phi}}|^2 \quad &\leq \quad 9M_W\Big\{|x_1|^2 + ME|\phi(0)|^2 + M\tilde{M}_h + 2MM_0^2\Big[M_g\|\widehat{\phi}\|^2_{B_h} + c_2\Big] \\
&+2\Big(M_0^2 + \frac{\big(C_{1-\beta}a^\beta\big)^2}{2\beta-1}\Big)\Big[M_g(1+2M_c)q\prime + 2M_gc_1 + c_2\Big] \\
&+2Ma^2[M_f(1+2M_e)q' + 2M_f\tilde{M}_e + \tilde{M}_f] + 2Ma^2[M_{\mathcal{H}} + Tr(Q)\tilde{M}_\sigma] \\
&+mM\sum_{k=1}^{m}\tilde{\beta}_k\Big\} := \mathcal{G} \text{ and} \\
E|u^a_{z+\widehat{\phi}} - u^a_{w+\widehat{\phi}}|^2 \quad &\leq \quad 6M_W\Big\{MM_h + \Big(M_0^2 + \frac{\big(C_{1-\beta}a^\beta\big)^2}{2\beta-1}\Big)M_g(1+M_c) + Ma^2M_f(1+M_e) \\
&+2a^2MM_\sigma + mM\sum_{k=1}^{m}\beta_k\Big\}\|z_t - w_t\|^2_{\mathcal{B}_h}.
\end{aligned}$$

Step 1: $\widehat{\Phi}(B_q) \subseteq B_q$ for some $q > 0$.

We claim that there exists a positive integer q such that $\widehat{\Phi}(B_q) \subseteq B_q$. If it is not true, then for each positive number q, there exists a function $z^q(\cdot) \in B_q$, but $\widehat{\Phi}(z^q) \notin B_q$, i.e. $|(\widehat{\Phi}z^q)(t)|^2 > q$ for some $t \in J$. However, on the other hand from $(H1) - (H11)$ we have

$$\begin{aligned}
q \quad &< \quad E|(\widehat{\Phi}z^q)(t)|^2 \\
&\leq \quad 7\Big\{2MM_0^2\Big[M_g\|\widehat{\phi}\|^2_{B_h} + c_2\Big] + 2\Big(M_0^2 + \frac{\big(C_{1-\beta}a^\beta\big)^2}{2\beta-1}\Big)\Big[M_g(1+2M_c)q\prime + 2M_gc_1 + c_2\Big] \\
&+MM_ba^2\mathcal{G} + 2Ma^2[M_f(1+2M_e)q' + 2M_f\tilde{M}_e + \tilde{M}_f] + 2Ma^2[M_{\mathcal{H}} + Tr(Q)\tilde{M}_\sigma] \\
&+mM\sum_{k=1}^{m}\tilde{\beta}_k\Big\} \\
&\leq \quad 7\Big\{(1 + 9MM_bM_Wa^2)\Big[2MM_0^2[M_g\|\widehat{\phi}\|^2_{B_h} + c_2] + 2\Big(M_0^2 + \frac{\big(C_{1-\beta}a^\beta\big)^2}{2\beta-1}\Big)[M_g(1+2M_c)q\prime \\
&+2M_gc_1 + c_2] + 2Ma^2[M_f(1+2M_e)q' + 2M_f\tilde{M}_e + \tilde{M}_f] + 2Ma^2[M_{\mathcal{H}} + Tr(Q)\tilde{M}_\sigma] \\
&+mM\sum_{k=1}^{m}\tilde{\beta}_k\Big] + 9MM_bM_Wa^2\Big(|x_1|^2 + ME|\phi(0)|^2 + M\tilde{M}_h\Big)\Big\}
\end{aligned}$$

$$\begin{aligned}
q \quad &\leq \quad \tilde{M} + 7\Big\{(1 + 9MM_bM_Wa^2)\Big[\Big(M_0^2 + \frac{\big(C_{1-\beta}a^\beta\big)^2}{2\beta-1}\Big)\Big[2M_g(1+2M_c)q\prime\Big] \\
&+2Ma^2[M_f(1+2M_e)q']\Big]\Big\},
\end{aligned}$$

where $\tilde{M}$ is independent of q. Dividing both sides by q and noting that

$$q' = 4l^2\Big(q + 2M(E|\phi(0)|^2 + M_h)\Big) + 4\|\widehat{\phi}\|^2_{\mathcal{B}_h} \longrightarrow \infty \text{ as } q \longrightarrow \infty$$

and thus we have

$$7l^2\Big\{\big(1+9MM_bM_Wa^2\big)\Big[8\Big(M_0^2 + \frac{(C_{1-\beta}a^\beta)^2}{2\beta-1}\Big)M_g(1+2M_c) + 8Ma^2M_f(1+2M_e)\Big]\Big\} \geq 1.$$

This contradicts (3.1). Hence $\widehat{\Phi}(B_q) \subseteq B_q$, for some positive number q.

Step 2: $\widehat{\Phi} : \mathcal{B}^0_a \longrightarrow \mathcal{B}^0_a$ is a contraction mapping.

Let $z, w \in \mathcal{B}_a^0$ then we have

$$
\begin{aligned}
E\Big|\widehat{\Phi}z(t)-\widehat{\Phi}w(t)\Big|^2 \leq\ & E\Big|g\Big(t,z_t+\widehat{\phi}_t,\int_0^t c(t,s,z_s+\widehat{\phi}_s)ds\Big)-g\Big(t,w_t+\widehat{\phi}_t,\int_0^t c(t,s,w_s+\widehat{\phi}_s)ds\Big)\Big|^2\\
&+E\Big|\int_0^t \mathcal{S}(t-s)B\big(u^a_{z+\widehat{\phi}}(s)-u^a_{w+\widehat{\phi}}(s)\big)ds\Big|^2\\
&+E\Big|\int_0^t A\mathcal{S}(t-s)\Big(g\Big(s,z_s+\widehat{\phi}_s,\int_0^s c(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)\\
&\qquad -g\Big(s,w_s+\widehat{\phi}_s,\int_0^s c(s,\eta,w_\eta+\widehat{\phi}_\eta)d\eta\Big)\Big)ds\Big|^2\\
&+E\Big|\int_0^t \mathcal{S}(t-s)\Big(f\Big(s,z_s+\widehat{\phi}_s,\int_0^s e(s,\eta,z_\eta+\widehat{\phi}_\eta)d\eta\Big)\\
&\qquad -f\Big(s,w_s+\widehat{\phi}_s,\int_0^s e(s,\eta,w_\eta+\widehat{\phi}_\eta)d\eta\Big)\Big)ds\Big|^2\\
&+E\Big|\int_0^t\mathcal{S}(t-s)\Big(\int_0^s\sigma\big(s,\eta,z_\eta+\widehat{\phi}_\eta\big)dw(\eta)-\int_0^s\sigma\big(s,\eta,w_\eta+\widehat{\phi}_\eta\big)dw(\eta)\Big)ds\Big|^2\\
&+E\Big|\sum_{0<\tau_k<t}\mathcal{S}(t-\tau_k)\Big(I_k(z(\tau_k^-)+\widehat{\phi}(\tau_k^-))-I_k(w(\tau_k^-)+\widehat{\phi}(\tau_k^-))\Big)\Big|^2\\
\leq\ & 6\Big\{(1+6MM_bM_Wa^2)\Big[\Big(M_0^2+\frac{(C_{1-\beta}a^\beta)^2}{2\beta-1}\Big)M_g(1+2M_c)+2a^2MM_\sigma\\
&+Ma^2M_f(1+M_e)+mM\sum_{k=1}^m\beta_k\Big]+6M^2M_bM_Wa^2M_h\Big\}\|z_t-w_t\|^2_{\mathcal{B}_h}\\
\leq\ & 12\Big\{(1+6MM_bM_Wa^2)\Big[\Big(M_0^2+\frac{(C_{1-\beta}a^\beta)^2}{2\beta-1}\Big)M_g(1+2M_c)+2a^2MM_\sigma\\
&+Ma^2M_f(1+M_e)+mM\sum_{k=1}^m\beta_k\Big]+6M^2M_bM_Wa^2M_h\Big\}\\
&\times\Big[l^2\sup_{0\leq s\leq t}E|z(s)-w(s)|^2+\|z_0-w_0\|^2_{\mathcal{B}_h}\Big]\\
\leq\ & \nu\sup_{0\leq s\leq a}E|z(s)-w(s)|^2,
\end{aligned}
$$

since $\nu<1$ by $(H11)$ and we have used the fact that $z_0=0, w_0=0$. Taking the supremum over t, we get

$$\|\widehat{\Phi}z-\widehat{\Phi}w\|_a^2\leq\nu\|z-w\|_a^2,$$

and so $\widehat{\Phi}$ is a contraction. Hence by Banach fixed point theorem there exists a unique fixed point $x\in\mathcal{B}_a$ such that $(\Phi x)(t)=x(t)$. This fixed point is then the solution of the system (1.1) and clearly, $x(a)=(\Phi x)(a)=x_1$ which implies that the system (1.1) is controllable on J. □

4. EXAMPLE

Consider the following partial neutral integrodifferential equation of the form

$$d\Big[v(t,y)\quad+\quad\int_{-\infty}^t r_1(t,y,s-t)G_1(v(s,y))ds+\int_0^t\int_{-\infty}^s\mu_1(s-\xi)G_2(v(\xi,y))d\xi ds\Big]$$

$$\begin{aligned}
&= \Big[\frac{\partial^2}{\partial y^2}v(t,y)+\int_{-\infty}^{t} r_2(t,y,s-t)F_1(v(s,y))ds+\int_0^t\int_{-\infty}^{s}\mu_2(s-\xi)F_2(v(\xi,y))d\xi ds \\
&\quad +c(y)u(t)\Big]dt+\int_{-\infty}^{t}\mu_3(s-t)v(s,y)d\beta(s), y\in[0,\pi], t\in J=[0,a], t\neq\tau_k, \\
v(t,0) &= v(t,\pi)=0,\ t\geq 0, \\
v(t,y) &+ \sum_{i=0}^{p}\int_0^{\pi}k_i(y,\eta)v(t_i,\eta)d\eta=\phi(t,y),\ t\in(-\infty,0], y\in[0,\pi], \\
\triangle v(\tau_i)(y) &= \int_{-\infty}^{\tau_i}q_i(\tau_i-s)v(s,y)ds,\ y\in[0,\pi]. \qquad (4.1)
\end{aligned}$$

where $0<\tau_1<\tau_2<\cdots<\tau_n<a$ are prefixed numbers, $\phi\in\mathcal{B}_h,\ p\in N$ and $0< t_0<t_1<t_2<\cdots<t_p<a$. We take $H=K=U=L^2([0,\pi])$ with the norm $|\cdot|_{L^2}$. Define the operator $A:H\longrightarrow H$ by $A\omega=\omega''$ with domain

$D(A)=\{\omega(\cdot)\in H:\omega,\omega'$ are absolutely continuous and $\omega''\in H,\omega(0)=\omega(\pi)=0\}$.

Then

$$A\omega=\sum_{n=1}^{\infty}n^2(\omega,\omega_n)\omega_n,\ \omega\in D(A),$$

where $\omega_n(s)=\sqrt{\frac{2}{\pi}}\sin ns,\ n=1,2,\cdots$ is the orthogonal set of eigenvectors of A. It is well known that the A is the infinitesimal generator of an analytic semigroup $\mathcal{S}(t),\ t\geq 0$ in H and is given by

$$\mathcal{S}(t)\omega=\sum_{n=1}^{\infty}e^{-n^2t}(\omega,\omega_n)\omega_n,\ \omega\in H.$$

By [12], $(-A)^{-3/4}\omega=\sum_{n=1}^{\infty}\frac{1}{(\sqrt{n})^3}(\omega,\omega_n)\omega_n$, for every $\omega\in H$ and $\|(-A)^{-3/4}\|$ is bounded. Let $\|(-A)^{-3/4}\|=\lambda$. The operator $(-A)^{3/4}\omega$ is given by

$$(-A)^{3/4}\omega=\sum_{n=1}^{\infty}(\sqrt{n})^3(\omega,\omega_n)\omega_n$$

on the space $D((-A)^{3/4})=\{\omega\in H:\sum_{n=1}^{\infty}(\sqrt{n})^3(\omega,\omega_n)\omega_n\in H\}$. Since the semigroup $\mathcal{S}(t)$ is analytic there exists a constant $M>0$ such that $\|\mathcal{S}(t)\|^2\leq M$ and satisfies $(H1)$.

We assume the following conditions hold:

(a) $B:L^2([0,\pi])\longrightarrow H$ is a bounded linear operator defined by

$$Bu(y)=c(y)u,\ 0\leq y\leq\pi,\ u\in L^2(J,U).$$

(b) The linear operator $W:L^2(J,U)\longrightarrow H$ is defined by

$$Wu=\int_0^a\mathcal{S}(a-s)c(y)u(s)ds$$

has an inverse operator W^{-1} defined on $L^2(J,U)\setminus kerW$ and satisfies condition $(H2)$.

(c) $\beta(t)$ denotes an one-dimensional standard Brownian motion.

(d) The function $k_i:[0,\pi]\times[0,\pi]\longrightarrow R$ are C^2-functions, for each $i=1,2,\cdots,p$.

(e) $q_i : R \longrightarrow R$ are continuous functions and $\beta_i = \int_{-\infty}^{0} h(s)q_i^2(s)ds < \infty$ for $i = 1, 2, \cdots, m$

Let $h(s) = e^{4s}, s < 0$, then $l = \int_{-\infty}^{0} h(s)ds = 1/4$ and define

$$\|\varphi\|_{\mathcal{B}_h} = \int_{-\infty}^{0} h(s) \sup_{\theta\in[s,0]} |\varphi(\theta)|_{L^2} ds.$$

Hence for $(t, \varphi) \in [0, a] \times \mathcal{B}_h$, where $\varphi(\theta)(y) = \varphi(\theta, y),\ (\theta, y) \in (-\infty, 0] \times [0, \pi]$. Set

$$v(t)(y) = v(t, y), \quad g(t, \varphi, N_1\varphi)(y) = \int_{-\infty}^{0} r_1(t, y, \theta)G_1(\varphi(\theta)(y))d\theta + N_1\varphi(y),$$

$$f(t, \varphi, N_2\varphi)(y) = \int_{-\infty}^{0} r_2(t, y, \theta)F_1(\varphi(\theta)(y))d\theta + N_2\varphi(y)$$

$$\text{and} \ \ \sigma(t, s, \varphi)(y) = \int_{-\infty}^{0} \mu_3(\theta)\varphi(\theta)(y)d\theta,$$

where

$$N_1\varphi(y) = \int_0^t \int_{-\infty}^{0} \mu_1(s-\theta)G_2(\varphi(\theta)(y))d\theta ds,\ N_2\varphi(y) = \int_0^t \int_{-\infty}^{0} \mu_2(s-\theta)F_2(\varphi(\theta)(y))d\theta ds.$$

Then the above equation can be written in the abstract form as the system (1.1). Moreover

$$g([0, a] \times \mathcal{B}_h \times L^2) \subseteq D((-A)^{3/4})$$

and

$$|(-A)^{3/4}g(t, \varphi_1, u_1)(y) - (-A)^{3/4}g(t, \varphi_2, u_2)(y)|^2 \leq M_g\big[\|\varphi_1 - \varphi_2\|^2_{\mathcal{B}_h} + |u_1 - u_2|^2\big]$$

for some constant $M_g > 0$ depending on r_1, μ_1 and G_1, G_2 and $|u_1-u_2|^2 = |N_1\varphi_1 - N_1\varphi_2|^2 \leq M_c\|\varphi_1 - \varphi_2\|^2_{\mathcal{B}_h}$ for $M_c > 0$. Further, other assumptions $(H5) - (H11)$ are satisfied such that $\frac{7}{2}\big\{\big(1 + 9MM_bM_Wa^2\big)\big[(\lambda^2 + 2C^2_{1/4}(\sqrt{a})^3)M_g(1 + 2M_c) + Ma^2M_f(1 + 2M_e)\big]\big\} < 1$ and it is possible to choose q_i, k_i in such a way that the constant $\nu < 1$. Hence by Theorem (3.1) the system (4.1) is controllable on J. $\square$

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