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A THREE LAYER SUPPLY CHAIN COORDINATION POLICIES FOR PRICE SENSITIVE AND EXPONENTIALLY DECLINING DEMAND WITH RECOMMENDED RETAIL PRICE BY MANUFACTURER

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ABSTRACT. The article develops an integrated supply chain coordination for multi-channel and multi-echelon supply, in which a single manufacturer, multiple non-competitive distributors, and non-competitive retailers are work together as members of the supply chain. The formulation of this model is based on deterministic exponential decreasing and price-dependent demand on the retailer's end. We formulated the model in two different scenarios, first, one is decentralized,d and the second one is centralized. The integrated profit function has been derived for each supply chain member, incorporating sharing holding costs among the distributors and retailers. We optimized selling price, economical order quantity, wholesale price, and profits for every echelon supply chain member in the finite and certain time horizon for decentralized and centralized scenarios respectively. Finally, we have done sensitivity analysis for some key parameters to examine their influence on the model's outputs. On the basis of numerical studies, we have also proposed managerial insights.

KEYWORDS:Inventory, holding cost, net profit, multi-channel multi-echelon supply chain, coordination.
AMS Subject Classification: :90B05, 90B30, 90B50.

1. Introduction

Due to globalization of market, growing of business competition, growth of population, awareness of consumer and legislative pressure have encouraged business industrialist and organizations to work together with their up stream, down stream members and customers. Furthermore better coordination among all upstream and

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downstream members, make the performance of entire supply chain is effective and efficient. Consequently collaboration and coordination among all echelon supply chain members are very important for an efficient supply chain. Otherwise due to the lack of coordination among each members they would optimizes its own local objectives ignoring impact on whole supply chain, may causes of lower profit.

In the previous decades, a several number of articles have been developed on supply chain management. Some relevant articles are mentioned here. Parlar and weng [17]applied the quantity discount policy in a supplier relationship with considering linear demand of buyer. They analyzed that quantity discount scheme can be very useful for obtaining more profit. Weng [30] presented a model in which they determine optimal lot size, optimal quantity discount policies. Also they analyze the effects of quantity discount on increasing demand of consumer. For development of models, generally price demand relationship are used by authors but Lau and Lau[2] developed a inventory model by considering different demand curve functions and investigate the effects on the single echelon supply chain system along with multi echelon system.

Most of the previously published literatures had adopted trade credit policy among suppliers and retailers only but Huang [29] adopted trade credit policy among not only suppliers and retailers but also retailers and customers in their supply chain model. Change *et al.*[8] developed an inventory model for deteriorating items assuming that a suppliers offers to purchaser a permission of delay in payment when the purchased order quantity is large. Chunge and Liao[13] developed a supply chain model based on trade credit period for exponential deteriorating items in which they assumed the condition that the supplier offers permission of delay in payment which is depends on order quantity.

Karim And Suzuki [18] provided a literature review on warranty claim data analysis in following topics:

- **(1):** Age based claims analysis,
- **(2):** Aggregated warranty claim analysis,
- **(3):** Two dimensional warranty cost analysis,
- **(4):** Warranty cost analysis etc.

Li and Liu, [12] developed optimal supply chain coordination using optimal quantity discount scheme considering probabilistic demand of single product in multiple time interval. Ding and Chen [9] developed a three layer supply chain model for short life cycle product. They highlighted, the coordination issues of three layer supply chain and suggested that three layer supply chain can be fully coordinated with certain contract of revenue sharing among manufacturer and supplier as long as supplier and retailers. Cachon and Lariviere [10] provided a two layer supply chain model with revenue sharing contracts. In this study it has been assumed that retailer's have to pay not only a wholesale price per unit of product to supplier but also pay a fixed percentage of revenue.

Crook Russel and Combs [26] suggested in their inventory model that collaborative environment in supply chain management create much better platform for each supply chain members to grow. They analyzed in this study that, how a weak member is benefited from strong a member in collaborative supply chain management.

Jain *et al.*[11] developed literature review on supply chain management and focused some issues on supply chain management. They gave a classification of more than 5889 published articles and try to find the status of literature on supply chain management. Kadadevaramath *et al.*[24] developed three layer supply chain coordination model by using four particle swarm optimization algorithm. They optimize their objectives by using the following various limitations:

(1): Ordering capacity of vendor
(2): Production capacity of plant
(3): Demand depends on various parameters etc.

Barron *et al.* [14] modified the model of Kadadevaramath *et al.*[24]. In this study optimality can be optimized by using integer linear programing solver technique in place of particle swarm optimization algorithm Kadadevaramath*et al.*[24]. Further they removed the following limitations of Kadadevaramath *et al.*[24] model:

(1): single product model is converted into multiple products model,
(2): single time interval is converted into multiple interval.

Barron*et al.*[15] proposed a vendor managed EOQ inventory model for multi products in which they considered multiple restrictions for optimizing total cost. It is more advance in the following three aspects than previously published works:

(1): The total cost is less than recently research work,
(2): The number of evaluations of the total cost function is less than recently research work,
(3): Computational time is less than recently research work.

Daya *et al.*[16] developed a three stage supply chain model, which formed by single supplier single manufacturer and multiple retailers. In this study they proposed a derivative free solution procedure to derive a optimal solution considering all inputs are constant. They optimized setup cost, holding cost, raw material cost and ordering cost along with the profits of each echelon member. Sarkar and Majumdar [7] developed integrated supply chain coordination for vendor and buyer, based on the following two different approaches:

(1): demand is a function of lead time which depends on probability distribution,
(2): demand is free from lead time.

They optimizes lead time and ordering cost for buyer and reorder point and setup cost for vendor. They also suggested that discrete investment gives better results instead of continuous investment and it may be reduce the setup cost. Modak *et al.*[20] presented two layer dual-channel supply chain, incorporating social responsibility in two different scenario first one is centralize and another one is decentralize. The development of this study is based on the following two different approaches:

(1): price dependent retail demand function,
(2): price dependent e-tail demand function.

After investigation they suggested in the centralized scenario model outputs are better than decentralized.

Pal *et al.*[5] proposed three layer production inventory model considering with three stage credit policy in which supplier provides the certain credit period to manufacturer, manufacturer a provide certain credit period to retailer and retailer also offers credit period to customers. They optimized replenishment lot size, and production rate for manufacturer. Sana [25] presented a three stage supply chain production inventory model which contains, a supplier, a manufacturer and a retailer. During the production he assume that perfect and imperfect both items are produced. They optimized production rate and replenishment rate per unit time for maximization of average profit. Pal *et al.*[6] developed perfect and imperfect three layer production inventory model consisting supplier, manufacturer and retailer. They assumed that the imperfect products are reworkable and rework process is started after end of regular production. They optimized order size of raw material, production rate per unit, production cost per unit and lead time.

Zhao and Chen[23] focused on the pricing strategies of a two-echelon supply chain for single manufacturer and two retailers. They developed price decision model considering the sensitivity of the retail quantity to the wholesale price of manufacturer and sales prices of the retailers. Khedlekar *et al.*[27] developed a production inventory model for deteriorating items. For this they designed two cases, first one is production without disruption and another one is production with disruption system allowing with shortage. Khedlekar *et al.*[28] developed continuous two layer supply chain inventory model by considering price and stock dependent demand for deteriorating items. Revenue sharing on preservation technology are also considered by authors.

Modak *et al.*[21] proposed a two layer supply chain formed by single manufacturer and single retailer for single product. They consider demand function as a function of quality, warranty, and sales price of the product. They optimized profit functions of the manufacturer and retailer under two the scenarios, centralized and decentralized. Nigwal *et al.*[4] developed a three layer multi channel reverse supply chain inventory model for used product in which a single remanufacturer multi-collector and multi-retailer work together as supply chain members. Gupta *et al.*[19] developed an imperfect production inventory model in which they consider imperfect production with and without disruption allowing with and without shortage. Modak *et al.*[22] developed a multi-channel, multi-echelon inventory model for single product incorporating single manufacturer more than one retailers and distributors as the members of the chain. The profit functions of each members have been formulated and optimized. The formulation of profit functions are based on demand of retailer's end.

In this model we considered a three layer multi-channel and multi-echelon supply chain model consisting a single manufacturer, more than one distributors and retailers. It is shown in the Figure (1). At starting the manufacturer provides the fixed lot size of the products to k^{th} (k=1 2...n) distributors and k^{th} (k=1 2...n) distributors supplies the products to jk^{th} (j=1 2 3....n_k), (k=1 2....n) retailers, where each retailer is associated to a certain distributor according to the geographical conditions. Since requisition of products is to be made at retailer's end therefor the total demand of all retailers is fulfilled by all distributors and the total demand of all distributors are fulfilled by the single manufacturer. Manufacturer and distributors assimilates EOQ delivery policy. In this paper we considered random order cycle time for manufacturer which is equally applicable for all distributors as well as all retailers.

The objective of this research is to find optimal retail price, initial order size for retailers in decentralized and centralized situation considering retailer's price sensitive and time dependent demand with sharing holding cost. We will also determine which coordination policy can be adopted that maximize model outputs.

2. Notations and Assumptions

Following notations are used in this model.

p^m : Maximum retail price determined by manufacturer,

D^r_{jk} : jk^{th}retailer's demand (per unit time) depends on retailer's price and time t in decentralized policy,

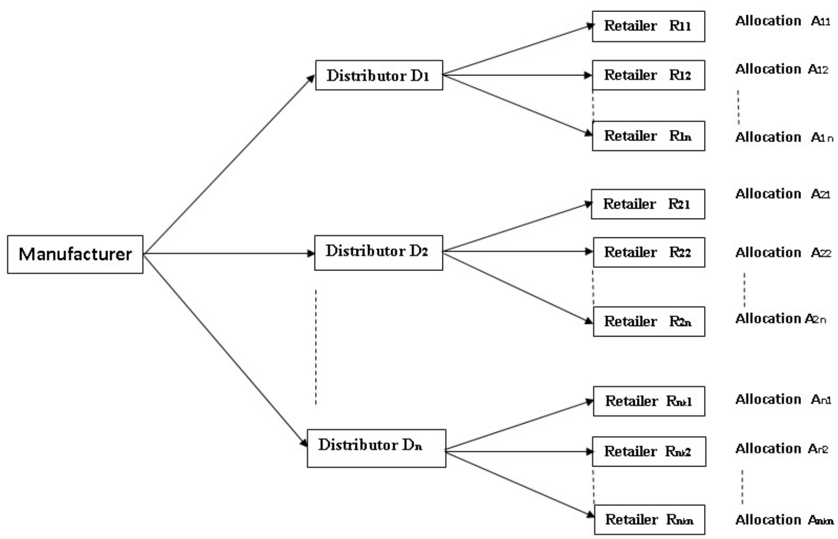

FIGURE 1. Supply Chain Distribution Network

D_k^d : k^{th} distributors's demand (per unit time) in decentralized policy,
D^m : Manufacture's demand (per unit time) in decentralized policy,
p_{jk}^r : Retailing price per unit product of jk^{th} retailer in decentralized policy,
p_{jk}^{rc} : Retailing price per unit product of jk^{th} retailer in centralized policy,
w_k^d : k^{th} Distributor's wholesale price per unit product in decentralized policy,
w^m : Manufacturer's wholesale price per unit product in decentralized policy,
c : Production cost per unit product,
NP_{jk}^r : Net profit of jk^{th} retailer in decentralized policy,
NP_k^d : Net profit of k^{th} distributor in decentralized policy,
NP^m : Net profit of manufacturer in decentralized policy,
n : Total number of distributors,
n^r : Total number of retailers,
NP^c : Net profit of whole channel in centralized policy,
β : Difference coefficient of $(p_{jk}^r - p^m)$ which may be positive or negative,
η : Price sensitive factor of demand function,
T : Total time horizon,
Q_{jk}^r : Initial demand of jk^{th} retailer,
Q_k^d : Initial demand of k^{th} distributor,
Q^m : Initial demand of manufacturer,
λ : Sharing coefficient of holding cost,
h : Holding cost per unit per unit time.

Assumptions: The following assumptions are made in this model

- Demand of product in the market is D_{jk}^r at the rate per unit time t; where $D_{jk}^r = a_{jk}e^{-\alpha t} - \eta p_{jk}^r + \beta(p^m - p_{jk}^r)$, is nonnegative exponential function of t and p_{jk}^r, where a_{jk} is demand scale parameter, β is difference coefficient of p^m and p_{jk}^r, $\alpha > 0$, $a_{jk} > 0$, $\beta > 0$, $\eta > 0$, and $0 \leq t \leq T$,
- Holding cost is constant and it is shared by retailers and distributors,
- The lead time is zero, and replenishment rate is infinite, however the planning horizon is finite,

- $a_k = \sum_{j=1}^{n_k} a_{jk}$ and $a = \sum_{j=1}^{n_k}\sum_{j=1}^{n} a_{jk}$,
- There is no competitive environment between retailers and distributors because each retailer's and distributors are associated according different geographical areas.
- We used the forward and backward substitution method to find the optimal decision variables.

3.

The study has been developed under the following two situations:

3.1. **Decentralized Policy.** In this scenario the all supply chain members are independent to take their decisions to optimize their objectives and manufacturer is a leader of supply chain. Therefore, firstly manufacturer announce the wholesale price of product, and letter distributors and retailers optimize their decision variables. Formulation of model is based on deterministic demand of retailer's end. Therefore firstly proposed model of retailer could be formulated as

3.1.1. *Mathematical Model for Retailers.* Since manufacturer manufactures the product, he absolutely knows all those cost which are related to the production. Therefore manufacturer can lead the supply chain of the product and also determine the maximum retail price at which the product is expected to be sold. This retail price of the product is called manufacturer's determined retail price (MDRP). The MDRP generally printed on the packet or tag of the product. It can be easily searched by the customer. In generally according to the market conditions consumers are satisfied or dissatisfied with MDRP.Initially we assume that the certain lot of product is distributed by manufacturer to n distributors d_1, d_2, d_3,...d_n. Distributors d_1, d_2, d_3,...d_n supply certain lot of the products to n_1 n_2 n_3...n_k retailers respectively. As per assumptions $jk^{th}t$ retailer receives the stock, at time t, $0 \leq t \leq T$. The rate of changes in the jk^{th} retailer's inventory level is balanced by demand. At any time t the following nonlinear equation may represent the inventory status of jk^{th} retailer

$$\begin{aligned}\frac{dI^r_{jk}(t)}{dt} &= -D^r_{jk}, \quad \text{where} \quad 0 \leq t \leq T. \\ &= -\left(a_{jk}e^{-\alpha t} - \eta p^r_{jk} + \beta(p^m - p^r_{jk})\right),\end{aligned} \tag{3.1}$$

where $j = 1, 2, 3, ..., n_k$ and $k = 1, 2, 3, ..., n$, with boundary condition $I^r_{jk}(t)$ =0, at $t = T$. solution of equation (3.1) gives

$$I^r_{jk}(t) = \frac{a_{jk}}{\alpha}(e^{-\alpha t} - e^{-\alpha T}) + (\eta + \beta)p^r_{jk}(t - T) + \beta p^m(T - t) \tag{3.2}$$

The initial inventory level $I^r_{jk}(0)$ for jk^{th} retailer at time $t = 0$, where $t \in [0, T]$ is

$$I^r_{jk}(0) = Q^r_{jk} = \frac{a_{jk}}{\alpha}(1 - e^{-\alpha T}) - (\eta + \beta)p^r_{jk}T + \beta p^m T \tag{3.3}$$

The sales revenue SR^r_{jk} in replenishment time period $[0, T]$ can be formulated as

$$SR^r_{jk} = \int_0^T p^r_{jk} D^r_{jk} dt$$

$$SR^r_{jk} = p^r_{jk}\left(\frac{a_{jk}}{\alpha}(1 - e^{-\alpha T}) - (\eta + \beta)p^r_{jk}T + \beta P^m T\right) \tag{3.4}$$

Purchase cost PC^r_{jk} of jk^{th}retailer can be formulated as

$$PC^r_{jk} = \int_0^T w^d_k D^r_{jk} d$$

$$PC^r_{jk} = w^d_k \left(\frac{a_{jk}}{\alpha}(1 - e^{-\alpha T}) - (\eta + \beta)p^r_{jk}T + \beta p^m T\right) \tag{3.5}$$

The inventory holding cost IHC^r_{jk} per unit of per unit time of jk^{th}retailer is

$$IHC^r_{jk} = h\int_0^T I^r_{jk}(t)dt$$

$$IHC^r_{jk} = h\int_0^T \frac{a_{jk}}{\alpha}(e^{-\alpha t} - e^{-\alpha T}) + (\eta + \beta)p^r_{jk}(t - T) + \beta p^m(T - t)dt \tag{3.6}$$

$$IHC^r_{jk} = h\left(\frac{a_{jk}}{\alpha^2}(1 - e^{-\alpha t} - T\alpha e^{-\alpha T}) - (\eta + \beta)p^r_{jk}\frac{T^2}{2} + \beta p^m\frac{T^2}{2}\right) \tag{3.7}$$

The net profit of jk^{th} retailer must be after subtraction of purchasing cost and sharing holding costs from sales revenue. Hence the net profit function NP^r_{jk} of jk^{th} retailer is

$$\begin{aligned} NP^r_{jk} = (p^r_{jk} - w^d_k)&\left[\frac{a_{jk}}{\alpha}(1 - e^{-\alpha T}) - (\eta + \beta)p^r_{jk}T + \beta p^m T\right] \\ - h\lambda&\left[\frac{a_{jk}}{\alpha^2}(1 - e^{-\alpha t} - T\alpha e^{-\alpha T}) - (\eta + \beta)p^r_{jk}\frac{T^2}{2} + \beta p^m\frac{T^2}{2}\right] \end{aligned} \tag{3.8}$$

According to the Taylor's theorem for small value of α the exponential function e^{-at} can be approximated by $1 - \alpha T + \frac{\alpha^2 T^2}{2}$ i.e $e^{-\alpha T} \approx 1 - \alpha T + \frac{\alpha^2 T^2}{2}$. Substituting the approximated value into the equation (3.8) we have

$$\begin{aligned} NP^r_{jk} = (p^r_{jk} - w^d_k)&\left[a_{jk}\left(T - \frac{\alpha T^2}{2}\right) - (\eta + \beta)p^r_{jk}T + \beta p^m T\right] \\ - h\lambda&\left[\frac{a_{jk}}{2}(T^2 - \alpha T^3) - (\eta + \beta)p^r_{jk}\frac{T^2}{2} + \beta p^m\frac{T^2}{2}\right] \end{aligned} \tag{3.9}$$

Proposition 3.1. *The optimal selling price p^{r*}_{jk} of jk^{th} retailer associated with k^{th} distributor's wholesale price w^d_k is p^{r*}_{jk} and where*

$$p^{r*}_{jk} = \frac{w^d_k}{2} + \frac{\beta p^m}{2(\eta + \beta)} + \frac{\lambda h T}{4} + \frac{a_{jk(1-e^{-\alpha T})}}{2\alpha(\eta + \beta)T} \tag{3.10}$$

Proof. At an optimal point, NP^r_{jk}, $\frac{\partial NP^r_{jk}}{\partial p^r_{jk}}$ must vanish i.e.

$$-(p^r_{jk} - w^d_k)(\eta + \beta)T + \left[\frac{a_{jk}}{\alpha}(1 - e^{-\alpha T}) - (\eta + \beta)p^r_{jk}T + \beta p^m T\right] + h\lambda(\eta + \beta)\frac{T^2}{2} = 0 \tag{3.11}$$

optimal value of p^r_{jk} is given by following equation

$$p^r_{jk} = \frac{w^d_k}{2} + \frac{\beta p^m}{2(\eta + \beta)} + \frac{\lambda h T}{4} + \frac{a_{jk}(1 - e^{-\alpha T})}{2\alpha(\eta + \beta)T} \tag{3.12}$$

□

Proposition 3.2. *NP^r_{jk} shows concavity in p^r_{jk} if $\eta > 0$ and $\beta > 0$.*

Proof. Second order partial derivatives of NP^r_{jk} in p^r_{jk} is

$$\frac{\partial^2 NP^r_{jk}}{\partial p^{r2}_{jk}} = -2(\eta + \beta)T. \tag{3.13}$$

Hence NP^r_{jk} is a concave function in p^r_{jk} if $\eta > 0$ and $\beta > 0$.
By using backward substitution method the optimal demand of the product at jk^{th} (j=1 2 3 ...n_k ,k=1 2 3 ...n) retailer's fag end is

$$D^{r*}_{jk} = a_{jk}e^{-\alpha T} - \frac{c(\eta+\beta)}{8} - \frac{(1-e^{-\alpha T})}{2\alpha T}\left(\frac{a}{4n^r} + \frac{a_k}{2n_j} + a_{jk}\right) + \frac{15\beta p^m}{8} \tag{3.14}$$

□

3.1.2. *Mathematical Model for Distributors.* There are n^{th} distributors d_1, d_2, d_3,...d_n and demand at k^{th} distributor's end is the sum of all jk^{th} retailer's demand. Hence the demand of k^{th} distributors can be written as

$$D^d_k = \sum_{j=1}^{n_k} D^r_{jk} = a_k e^{-\alpha t} - (\eta+\beta)\sum_{j=1}^{n_k} p^r_{jk} + n_j\beta p^m$$

Therefor the rate of changes in the k^{th} distributor's inventory is balanced by demand. At any time t following nonlinear equation represents the inventory status of k^{th} distributor

$$\begin{aligned} \frac{dI^d_k(t)}{dt} &= -D^d_k \quad \text{where} \quad 0 \le t \le T, \\ \frac{dI^d_k(t)}{dt} &= -\left(a_k e^{-\alpha t} - (\eta+\beta)\sum_{j=1}^{n_k} p^r_{jk} + n_j\beta p^m\right) \text{where} \quad 0 \le t \le T. \end{aligned} \tag{3.15}$$

with boundary condition $I_m(t_s) = 0$, at $t = T$. Now we derived the net profit function for k^{th} distributor during a time interval of length $[0, T]$. The net profit function for k^{th} distributor must be after subtraction of purchasing cost and sharing holding costs from sales revenue. The solution of equation (3.15) gives

$$I^d_k(t) = \frac{a_k}{\alpha}(e^{-\alpha t} - e^{-\alpha T}) + (\eta+\beta)\sum_{j=1}^{n_k} p^r_{jk}(t-T) + \beta p^m n_j(T-t) \tag{3.16}$$

At the initial time $t = 0$ the inventory level for k^{th} retailer is, where $t \in [0, T]$

$$I^d_k(0) = Q^d_k = \frac{a_k}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_k} p^r_{jk}T + \beta p^m n_j T \tag{3.17}$$

The sales revenue SR^d_k in the replenishment time period $[0, T]$ can be formulated as

$$SR^d_k = \int_0^T w^d_k D^d_k dt$$

$$SR^d_k = w^d_k\left(\frac{a_k}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_k} p^r_{jk}T + \beta p^m T\eta_j\right) \tag{3.18}$$

Purchase cost of k^{th} distributor in the interval [0, T] is

$$PC^d_k = \int_0^T w^m D^d_k dt$$

$$PC^r_{jk} = w^m\left(\frac{a_k}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_k} p^r_{jk}T + \beta p^m T\eta_j\right) \tag{3.19}$$

The inventory holding cost IHC^r_{jk} per unit per unit time is

$$IHC^d_k = h\int_0^T I^d_k(t)dt$$

$$IHC^d_k = h\left(\frac{a_k}{\alpha^2}(1-e^{-\alpha t}-T\alpha e^{-\alpha T})-(\eta+\beta)\sum_{j=1}^{n_k}p^r_{jk}\frac{T^2}{2}+\beta p^m\eta_j\frac{T^2}{2}\right) \quad (3.20)$$

Hence the net profit function for k^{th} distributor per unit time is

$$\begin{aligned} NP^d_k = (w^d_k - w^m)&\left[\frac{a_k}{\alpha}(1-e^{-\alpha T})-(\eta+\beta)\sum_{j=1}^{n_k}p^r_{jk}T+\beta p^m T\eta_j\right] \\ &- h(1-\lambda)\left[\frac{a_k}{\alpha^2}(1-e^{-\alpha t}-T\alpha e^{-\alpha T})-(\eta+\beta)\sum_{j=1}^{n_k}p^r_{jk}\frac{T^2}{2}+\beta p^m\eta_j\frac{T^2}{2}\right] \end{aligned} \quad (3.21)$$

where p^r_{jk} is given by (3.10)

Proposition 3.3. *The optimal wholesale price of k^{th} distributor associated with manufacturer's wholesale price w^m is w^{d*}_k, where*

$$w^{d*}_k = \frac{w^m}{2}+\frac{a_k(1-e^{-\alpha T})}{2\alpha(\eta+\beta)Tn_j}-\frac{\lambda hT}{2}+\frac{\beta p^m}{2(\eta+\beta)}+\frac{Th}{4} \quad (3.22)$$

Proof. Partial differentiation of equation (3.21) gives

$$\begin{aligned} \frac{\partial NP^d_k}{\partial w^d_k} = &-(w^d_k-w^m)n_k(\eta+\beta)\frac{T}{2}+\frac{a_k}{2\alpha}(1-e^{-\alpha T})-w^d_k n_k(\eta+\beta)\frac{T}{2}+\beta p^m n_k\frac{T}{2} \\ &-(\eta+\beta)\lambda h n_k\frac{T^2}{4}+h(1-\lambda)n_k(\eta+\beta)\frac{T^2}{4} \end{aligned} \quad (3.23)$$

If w^{d*}_k is an optimal value of w^d_k then $\frac{\partial NP^d_k}{\partial w^d_k}=0$ i.e.

$$\begin{aligned} &-(w^d_k-w^m)n_k(\eta+\beta)\frac{T}{2}+\frac{a_k}{2\alpha}(1-e^{-\alpha T})-w^d_k n_k(\eta+\beta)\frac{T}{2}+\beta p^m n_k\frac{T}{2} \\ &-(\eta+\beta)\lambda h n_k\frac{T^2}{4}+h(1-\lambda)n_k(\eta+\beta)\frac{T^2}{4}=0 \end{aligned} \quad (3.24)$$

solution of equation (3.24) gives

$$w^{d*}_k = \frac{w^m}{2}+\frac{a_k(1-e^{-\alpha T})}{2\alpha(\eta+\beta)Tn_k}-\frac{\lambda hT}{2}+\frac{\beta p^m}{2(\eta+\beta)}+\frac{Th}{4} \quad (3.25)$$

for optimality of NP^d_k at point $w^d_k = w^{d*}_k$, we have $\frac{\partial^2 NP^d_k}{\partial {w^d_k}^2} = -n_j(\eta+\beta)\frac{T}{2}$ for $\beta > 0$ and $\eta > 0$. Hence the optimal values of NP^d_k exists at w^{d*}_k □

3.1.3. *Mathematical Model for Manufacturer.* Manufacturer provides the initial lot of product to all distributors according to their demands. Therefore demand of product at manufacturer end is equal to the sum of all k^{th} distributor's demand. Hence the demand of manufacturer can be written as

$$D^m = \sum_{k=1}^{n} D^d_k = ae^{-\alpha t}-(\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n}p^r_{jk}+n^r\beta p^m$$

Hence the rate of changes in the manufacturer's inventory is balanced by demand of all distributors. At any time T the following nonlinear equation may represent the inventory status:

$$\begin{aligned}\frac{dI_k^d(t)}{dt} &= -D^m \quad \text{where} \quad 0 \le t \le T,\\ \frac{dI_k^d(t)}{dt} &= -\left(ae^{-\alpha t} - (\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n} p_{jk}^r + n^r\beta P_m\right) \text{where} \quad 0 \le t \le T.\end{aligned} \tag{3.26}$$

with boundary condition $I_m(t_s) = 0$, at $t = T$. The solution of equation (3.26) gives

$$I^m(t) = \frac{a}{\alpha}(e^{-\alpha t} - e^{-\alpha T}) + (\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n} p_{jk}^r(t-T) + \beta p^m n^r(T-t) \tag{3.27}$$

The initial inventory level for manufacturer at time $t = 0$, where $t \in [0, T]$ is

$$I^m(0) = Q^m = \frac{a}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n} p_{jk}^r T + \beta p^m n^r T \tag{3.28}$$

Now we derived the net profit function of manufacturer during a time interval of length $[0, T]$. The net profit function of manufacturer after can be obtain, after subtraction of production cost per unit from sales revenue. The sales revenue of manufacturer in the replenishment time period $[0, T]$ can be formulated as

$$SR^m = \int_0^T w^m D^m dt$$

$$SR^m = w^m\left(\frac{a}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_j}\sum_{k=1}^{n} p_{jk}^r T + \beta p^m T n^r\right) \tag{3.29}$$

Manufacturing cost of product for manufacturer is

$$PC^d = c\int_0^T D^m dt$$

$$PC^m = c\left(\frac{a_k}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n} p_{jk}^r T + \beta p^m T n^r\right) \tag{3.30}$$

Hence the net profit function NP^m of manufacturer is

$$NP^m = (w^m - c)\left[\frac{a}{\alpha}(1 - e^{-\alpha T}) - (\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n} p_{jk}^r T + \beta p^m T n^r\right] \tag{3.31}$$

where p_{jk}^r is given by (3.10)

Proposition 3.4. *The optimal wholesales price of manufacturer associated with production cost of unit product is w^{m*}, where*

$$w^{m*} = \frac{c}{2} + \frac{a(1 - e^{-\alpha T})}{2\alpha(\eta+\beta)Tn^r} \quad - \frac{hT}{4} + \frac{\beta p^m}{2(\eta+\beta)} \tag{3.32}$$

Proof. Partial differentiation of equation (3.31) gives

$$\frac{\partial NP^m}{\partial w^m} = -(w^m - c)(\eta+\beta)n^r\frac{T}{4} + \frac{a(1-e^{-\alpha T})}{4\alpha} - (\eta+\beta)w^m n^r\frac{T}{4} + n^r\beta p^m\frac{T}{4} - (\eta+\beta)hn^r\frac{T^2}{8} \tag{3.33}$$

If w^{m*} is an optimal value of w^m then $\frac{\partial NP^m}{\partial w^m} = 0$ i.e.

$$-(w^m - c)(\eta+\beta)n^r\frac{T}{4} + \frac{a(1-e^{-\alpha T})}{4\alpha} - (\eta+\beta)w^m n^r\frac{T}{4} + n^r\beta p^m\frac{T}{4} - (\eta+\beta)hn^r\frac{T^2}{8} = 0 \tag{3.34}$$

where p^r_{jk} is given by (3.10), equation (3.34) yields

$$w^{m*} = \frac{c}{2} + \frac{a(1-e^{-\alpha T})}{2\alpha(\eta+\beta)Tn^r} - \frac{hT}{4} + \frac{\beta p^m}{2(\eta+\beta)} \tag{3.35}$$

for optimality of NP^m at point $w^m = w^{m*}$, we have $\frac{\partial^2 NP^d_{kk}}{\partial w^{d^2}_k} = -n^r(\eta+\beta)\frac{T}{2}$, for $\beta > 0$ and $\eta > 0$. Hence optimal profit NP^m exists at w^{m*} □

Proposition 3.5. *If p^m is an optimum suggested price and w^{m*} is a wholesale price given by manufacturer, also w^{d*}_k is an optimum wholesales price given by distributors , then optimal selling price is given by*

$$(i)\ p^{r*}_{jk} = \frac{c}{8} + \frac{a(1-e^{-\alpha T})}{2\alpha(\eta+\beta)}\left(\frac{a}{4n^r} + \frac{a_k}{2n_k} + a_{jk}\right) + \frac{7\beta p^m}{8(\eta+\beta)}, \tag{3.36}$$

where j=1 2 3....n_k, and k=1 2 3....n,

$$(ii)\ \ w^{d*}_k - w^{m*} > 0,$$

where k=1 2 3....n,

$$(iii)\ \ p^{r*}_{jk} - w^{d*}_k > 0$$

Proof. (i) Substituting the values of w^{d*}_k and w^{m*} from equations (3.25) and (3.35) respectively into the equation (3.12) we get p^{r*}_{jk} in terms of T and other parameters, which obvious. (ii) It is obvious from equation (3.12) and equation (3.25) and model stability. (iii) It is also obvious from part (i) and equation (3.25). □

3.2. **Centralized Policy.** In the centralized scenario all supply chain members work together as a single unit and cooperate to each other. In this scenario only manufacturer can take all decisions about supply chain and which are equally applicable on all supply chain members. The mathematical model can be formulated as following

3.2.1. *Mathematical Model.* In this scenario manufacturer is a leader of whole supply chain and he is a single decision maker, therefore he can take all decisions to optimize profit of whole chain. If p^{rc}_{jk} is a retail price of jk^{th} retailer, w^d_j is a whole sale price of k^{th} distributor, w^m is a whole sale price of manufacturer, c is a manufacturing cost, IHC^r_{jk} is a holding cost of jk^{th} retailer and IHC^d_k a is holding cost of k^{th} distributor, then the profit function is

$$\begin{aligned} NP^c &= \sum_{j=1}^{n_k}\sum_{k=1}^{n}\left[\left(p^{rc}_{jk}-w^d_j\right)D^r_{jk}-\lambda(IHC^r_{jk})\right] \\ &+\sum_{k=1}^{n}\left[\left(w^r_k-w^m\right)D^d_k-(1-\lambda)IHC^d_k\right]+(w^m-c) \\ &=\sum_{j=1}^{n_k}\sum_{k=1}^{n}\left[\left(p^{rc}_{jk}-c\right)D^r_{jk}-\lambda(IHC^r_{jk})\right] \end{aligned}$$

$$\begin{aligned} NP^c = &\sum_{j=1}^{n_k}\sum_{k=1}^{n}\left(p^{rc}_{jk}-c\right)\left(\frac{a_{jk}}{\alpha}(1-e^{-\alpha T})-(\eta+\beta)p^{rc}_{jk}T+\beta p^m T\right) \\ &-\sum_{j=1}^{n_k}\sum_{k=1}^{n}h\left(\frac{a_{jk}}{\alpha^2}(1-e^{-\alpha t}-T\alpha e^{-\alpha T})-(\eta+\beta)p^{rc}_{jk}\frac{T^2}{2}+\beta p^m\frac{T^2}{2}\right) \end{aligned} \tag{3.37}$$

By using backward substitution method the optimal demand of the products at jk^{th} (j=1 2 3 ...n_k ,k=1 2 3 ...n) retailer's fag end is as

$$D^{rc}_{jk}=a_{jk}e^{-\alpha T}-\frac{c(\eta+\beta)}{2}-\frac{a_{jk}(1-e^{-\alpha T})}{2\alpha T}+\frac{\beta p^m}{2}+\frac{(\eta+\beta)Th}{4} \tag{3.38}$$

Proposition 3.6. *In the centralized scenario the optimal selling price of jk^{th} retailer associated with manufacturing cost is p^{rc*}_{jk}, where*

$$p^{rc*}_{jk}=\frac{c(\eta+\beta)T+\frac{a_{jk}}{\alpha}(1-e^{-\alpha T})+\beta p^m T-(\eta+\beta)\frac{T^2}{2}}{2(\eta+\beta)Th} \tag{3.39}$$

Proof. Partial differentiation of equation (3.37) gives

$$\begin{aligned} \frac{\partial NP^c}{\partial p^{r*}_{jk}} = &\sum_{j=1}^{n_k}\sum_{k=1}^{n}\left[-\left(p^r_{jk}-c\right)(\eta+\beta)T+\left(\frac{a_{jk}}{\alpha}(1-e^{-\alpha T})-(\eta+\beta)p^r_{jk}T+\beta p^m T\right)\right] \\ &-\sum_{j=1}^{n_k}\sum_{k=1}^{n}(\eta+\beta)\frac{T^2}{2}h \end{aligned} \tag{3.40}$$

If p^{rc*}_{jk} is an optimal value of p^{rc}_{jk} then $\frac{\partial NP^c}{\partial p^{r*}_{jk}}=0$
i.e.

$$\begin{aligned} &\left(p^r_{jk}-c\right)(\eta+\beta)T-\left(\frac{a_{jk}}{\alpha}(1-e^{-\alpha T})-(\eta+\beta)p^r_{jk}T+\beta p^m T\right) \\ &+(\eta+\beta)\frac{T^2}{2}h=0 \end{aligned} \tag{3.41}$$

solution of equation (3.41) gives

$$p^{rc*}_{jk}=\frac{c(\eta+\beta)T+\frac{a_{jk}}{\alpha}(1-e^{-\alpha T})+\beta p^m T-(\eta+\beta)\frac{T^2}{2}h}{2(\eta+\beta)T} \tag{3.42}$$

for optimality of NP^c at point p^{rc}_{jk}=p^{rc*}_{jk}, we have
$-2n^r(\eta+\beta)T$, for $\eta>0$ and $\beta>0$,
Hence optimum value of NP^c exists at p^{rc*}_{jk}. □

3.3. **Numerical example.** For numerically illustration of this supply chain model we have assumed that the supply chain is formed by a manufacturer M, two distributors (D_1, D_2) and four retailers (R_{11},R_{12}, R_{21} and R_{22}). According to the Figure (1), each retailer is associated with certain distributors. A manufacturer has to provide certain quantity of product and distributors have to provide certain quantity of product to respective retailers. We consider the following data set, the demand scale parameters at each retailer's end are a_{11} =75, a_{12} = 73, a_{21} = 74, a_{22} = 76 units, manufacturer determined maximum retail price is p^m=275 price coefficient parameter is $\eta = 0.1$, difference coefficient of retail price and suggested price is $\beta = 1.5$, production cost is c =150, shape parameter is $\alpha = 0.002$ and random time is T=1.05. The model outputs are given in the following table:

Table 1: Decentralized Policy

Optimal	R_{11}	R_{12}	R_{21}	R_{22}	D_1	D_2	M
Price	285.11	284.48	284.80	285.80	265.58	265.89	227.15
Demand	31	30	30	31	61	61	122
EOL	32	31	34	33	-	-	-
Profit	640.40	600.10	610	650.68	2480.54	2522.15	10002
Total profit	17515.87						

Table 2: Centralized Policy

Optimal	R_{11}	R_{12}	R_{21}	R_{22}	D_1	D_2	M
Price	227.31	226.69	227	227.62	-	-	-
Demand	-	-	-	-	-	-	493
Profit	-	-	-	-	-	-	40050.14
Total profit	40050.14						

3.4. **Sensitivity Analysis.** Through the analysis of table 1 and 2 shows that, in the decentralized policy retail price of product is comparatively higher than the centralized policy but due to less demand of products, total profit of whole supply chain is more less than the centralized policy.

Proposition 3.7. *All profits are as follows with respect to basic demand of product* $\frac{\partial NP^r_{jk}}{\partial a_{jk}} > 0$, $\frac{\partial NP^d_k}{\partial a_k} > 0$, $\frac{\partial NP^m}{\partial a} > 0$, *and* $\frac{\partial NP^c}{\partial a_{jk}} > 0$,

Intuitively, all supply chain member's profit in both policy shows incremental property with respect to basic demand when retailing price and suggested retail price are constant. It is shown in the table 3.

Table 3: Sensitive analysis with base demand parameter

	changes	NP^r_{11}	NP^r_{12}	NP^r_{21}	NP^r_{22}	NP^d_1	NP^d_2	NP^m	NP^c
	-15%	585.3	546.8	556.3	595.2	2263.8	2303.6	8697.2	36566
a	-5%	621.0	580.4	590.2	630.2	2400.6	2441.5	9220.4	38765
	5%	661.1	620.1	630.2	621.4	2561.8	2604.0	9837.1	41356
	15%	698.0	655.9	666.3	708.6	2707.2	2750.6	10393.1	43692

Proposition 3.8. *Behavior of each profit function with respect to* α *are as follow* $\frac{\partial NP^r_{jk}}{\partial \alpha} < 0$, $\frac{\partial NP^d_k}{\partial \alpha} < 0$, $\frac{\partial NP^m}{\partial \alpha} < 0$, *and* $\frac{\partial NP^c}{\partial \alpha} < 0$,

Proposition 3.8 states the impact of shape parameter on each supply chain members, when selling price and suggested retail price are constant, then profit of each supply chain member decreases, as α increases.

Table 4: Sensitive analysis with base scale parameter

	changes	NP_{11}^r	NP_{12}^r	NP_{21}^r	NP_{22}^r	NP_1^d	NP_2^d	NP^m	NP^c
α	-15%	640.4	600.1	610.0	650.7	2480.6	2522.2	9526.6	40051
	-5%	640.5	600.2	610.0	650.8	2480.8	2522.4	9527.2	40053
	5%	640.4	600.1	610.0	650.7	2480.5	2522.0	9526.0	40048
	15%	640.3	600.1	610.0	650.7	2480.3	2521.2	9526.4	40046

Proposition 3.9. *Partial derivative all profits with respect to η are as follow* $\frac{\partial NP_{jk}^r}{\partial \eta} < 0$, $\frac{\partial NP_k^d}{\partial \eta} < 0$, $\frac{\partial NP^m}{\partial \eta} < 0$, *and* $\frac{\partial NP^c}{\partial \eta} < 0$,

Proposition 3.9 states the influence of the parameter η, which measure the sensitivity of consumers to the retailing price of product, profit of each supply chain member is decreases as η increases.

Table 5: Sensitive analysis with η

	changes	NP_{11}^r	NP_{12}^r	NP_{21}^r	NP_{22}^r	NP_1^d	NP_2^d	NP^m	NP^c
η	-15%	658.2	617.1	627.2	668.6	2550.0	2592.4	9792.5	41168
	-5%	646.3	605.8	615.8	656.6	2503.5	2545.4	9614.3	40420
	5%	634.5	594.5	604.4	644.8	2457.7	2499.1	9439.0	39683
	15%	547.6	511.4	520.3	556.9	2117.5	2155.1	8135.6	34205

Proposition 3.10. *Partial derivatives of all profits with respect to β are as follow* $\frac{\partial NP_{jk}^r}{\partial \beta} > 0$, $\frac{\partial NP_k^d}{\partial \beta} > 0$, $\frac{\partial NP^m}{\partial \beta} > 0$, *and* $\frac{\partial NP^c}{\partial \beta} > 0$,

Proposition 3.10 shows the influence of coefficient of difference between manufacturer determined retail price and actual selling price β of the product. Increment of β increases the profit of all supply chain members.

Table 6: Sensitive analysis with β

	changes	NP_{11}^r	NP_{12}^r	NP_{21}^r	NP_{22}^r	NP_1^d	NP_2^d	NP^m	NP^c
β	-15%	586.9	545.4	555.6	597.5	2216.5	2259.5	8702.7	36596
	-5%	622.4	581.8	591.8	632.8	2407.9	2449.8	9256.2	38891
	5%	658.4	618.5	628.4	668.7	2553.6	2594.9	9803.8	41215
	15%	694.9	655.5	665.3	704.9	2700.6	2741.4	10362.7	43562

Proposition 3.11. *Behavior of each profit function with respect to p^m are as follow* $\frac{\partial NP_{jk}^r}{\partial p^m} > 0$, $\frac{\partial NP_k^d}{\partial p^m} > 0$, $\frac{\partial NP^m}{\partial p^m} > 0$, *and* $\frac{\partial NP^c}{\partial p^m} > 0$,

For certain data set proposition shows that profits of all supply chain member increases as manufacturer determined selling price p^m increases which is shown in the following table.

Table 7: Sensitive analysis with p^m

	changes	NP_{11}^r	NP_{12}^r	NP_{21}^r	NP_{22}^r	NP_1^d	NP_2^d	NP^m	NP^c
p^m	275	640.4	600.1	610.0	650.6	2480.54	2522.15	10002	40050
	280	679.4	637.9	648.1	690.0	2634.15	2677.00	10619	42519
	285	719.6	676.8	687.3	730.4	2792.37	2836.47	11254	45061
	290	760.9	716.9	727.7	772.1	2955.21	3000.56	11908	47678

4. Conclusion

We have developed an integrated multi-channel and multi-echelon supply chain coordination policy for two different scenarios, in which first one is decentralized and second one is centralized scenario. The model follows the exponential time declining, price sensitive and manufacturer determined retail price dependent demand, incorporating sharing holding cost among retailers and distributors. Particularly, the manufacturer who act as stackelberg leader of whole chain, decides wholesale and suggested retail price of product, according to their goal and expenditure. On the basis of manufacturer's decision we optimized the retail price, wholesale price of distributors, initial order quantity for retailers, distributors and manufacturer, optimal profits of each supply members in certain finite time horizon. Model may be applicable on those products which are well established in the market and have high holding cost as long as non fluctuated demand with time.

Management should follow the following suggestions for beneficial purposes (i) Keep balance between retail price and suggested retail price, because profits of all supply chain members show positive behavior with suggested retail price. But increment of suggested retail price may causes reducing demand. (ii) Keep always $p^m > p^r_{jk}$ i.e $\beta > 0$, because profits of all supply chain members show positive behavior with respect to β. (iii) Proposition 3.8 shows the profit of all supply chain members are sensitive with retailing price, therefore management should make better strategies before making the changes in retail price. (iv) Managerial insights of study is that firstly management should collect all information about demand of product with the help of retailers and then announce the whole sale price and manufacturer determined retail price. Observation of model outputs shows that management should make a contractual policy for better coordination among all supply chain members because in the centralized scenario model outputs are better than the decentralized scenario. One can be extended this model by incorporating stockout situation at retailers end. One can be extend this model by incorporating probabilistic demand or discrete demand and also one can extend this model by incorporating variable holding cost. One can be extended this model by incorporating setup cost dependent suggested retail price by manufacturer.

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