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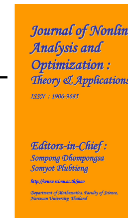

COMMON FIXED POINT OF PRESIĆ TYPE CONTRACTION MAPPINGS IN PARTIAL METRIC SPACES

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ABSTRACT. Presić (Publ. de L'Inst. Math. Belgrade, 5 (19), 75-78) introduced the concept of a kth-order Banach type contraction mapping and obtained fixed point of such mappings on metric spaces. Ćirić and Presić (Acta Math. Univ. Comenian. LXXVI (2) (2007), 143-147) extended the notion to kth-order Ciric type contraction mappings on a metric space. On the other hand, Matthews (Ann. New York Acad. Sci. 728 (1994), 183-197) introduced the concept of a partial metric as a part of the study of denotational semantics of dataflow networks. He gave a modified version of the Banach contraction principle, more suitable in this context. In this paper, we study the common fixed points of kth-order Ciric type contractions in the framework of partial metric spaces. We also present an example to validate our result.

KEYWORDS: Partial metric space; Common fixed point; Presić type contraction.
AMS Subject Classification: 54H25, 47H10, 15A24, 65H05.

1. INTRODUCTION AND PRELIMINARIES

Banach contraction principle [7] is a simple and powerful result with a wide range of applications, including iterative methods for solving linear, nonlinear, differential, integral, and difference equations. There are several generalizations and extensions of the Banach contraction principle in the existing literature.

Banach contraction principle reads as follows:

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Theorem 1.1. [7] *Let (X, d) be a complete metric space and mapping $f : X \to X$ satisfies*

$$d(fx, fy) \leq kd(x, y) \text{ for all } x, y \in X.$$

where $k \in [0, 1)$. Then, there exists a unique x in X such that $x = fx$. Moreover, for any $x_0 \in X$, the iterative sequence $x_{n+1} = fx_n$ converges to x.

Let $f : X^k \to X$, where $k \geq 1$ is a positive integer. A point $x^* \in X$ is called a fixed point of f if $x^* = f(x^*, ..., x^*)$. Consider the k-th order nonlinear difference equation

$$x_{n+1} = f(x_{n-k+1}, x_{n-k+2}, ..., x_n) \text{ for } n = k-1, k, k+1, ... \tag{1.1}$$

with the initial values $x_0, x_1, ..., x_{k-1} \in X$.

Equation (1.1) can be studied by means of fixed point theory in view of the fact that x in X is a solution of (1.1) if and only if x is a fixed point of f. One of the most important results in this direction is obtained by Presić [20] in the following way.

Theorem 1.2. [20] *Let (X, d) be a complete metric space, k a positive integer and $f : X^k \to X$. Suppose that*

$$d(f(x_1, ..., x_k), f(x_2, ..., x_{k+1})) \leq \sum_{i=1}^{k} q_i d(x_i, x_{i+1}) \tag{1.2}$$

holds for all $x_1, ..., x_{k+1}$ in X, where $q_i \geq 0$ and $\sum_{i=1}^{k} q_i \in [0, 1)$. Then f has a unique fixed point x^*. Moreover, for any arbitrary points $x_1, ..., x_{k+1}$ in X, sequence $\{x_n\}$ defined by $x_{n+k} = f(x_n, x_{n+1} ..., x_{n+k-1})$, for all $n \in \mathbb{N}$ converges to x^*.

It is easy to show that for $k = 1$, Theorem 1.1 reduces to the Banach contraction principle.

Ćirić and Presić [11] generalized above theorem as follows.

Theorem 1.3. [11] *Let (X, d) be a complete metric space, k a positive integer and $f : X^k \to X$. Suppose that*

$$d(f(x_1, ..., x_k), f(x_2, ..., x_{k+1})) \leq \lambda \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\}, \tag{1.3}$$

holds for all $x_1, ..., x_{k+1}$ in X, where $\lambda \in [0, 1)$. Then f has a fixed point $x^* \in X$. Moreover, for any arbitrary points $x_1, ..., x_{k+1} \in X$, the sequence $\{x_n\}$ defined by $x_{n+k} = f(x_n, x_{n+1} ..., x_{n+k-1})$, for all $n \in \mathbb{N}$ converges to x^*. Moreover, if

$$d(f(u, ..., u), f(v, ..., v)) < d(u, v),$$

holds for all $u, v \in X$, with $u \neq v$, then x^* is unique fixed point of f.

The applicability of the above result to the study of global asymptotic stability of the equilibrium for the nonlinear difference equation (1.1) can be found in [10]. For further work in this direction, we refer to [2, 16, 19, 23].

On the other hand, partial metric space is a generalized metric space in which each object does not necessarily have to have a zero distance from itself [17]. A motivation behind introducing the concept of a partial metric was to obtain appropriate mathematical models in the theory of computation [13, 17, 22, 24, etc]. Altun and Simsek [4], Oltra and Valero [18] and Valero [25] established potential generalizations of the results in [17]. Romaguera [21] proved a Caristi type fixed point theorem on partial metric spaces. Further results in this direction were proved in [1, 3, 5, 6, 8, 9, 14, 15].

Recently, Geroge et al. [12] proved generalized fixed point theorem of Presic type in cone metric spaces and gave its application to Markov process.

The aim of this paper is to study the common fixed point results for mappings satisfying Presić type contractive conditions in the setup of partial metric spaces.

In the sequel the letters $\mathbb{R}$, $\mathbb{R}^+$ and N will denote the set of all real numbers, the set of all nonnegative real numbers and the set of all positive integer numbers, respectively.

Consistent with [4] and [17], the following definitions and results will be needed in the sequel.

Definition 1.4. Let X be a nonempty set. A function $p : X \times X \to \mathbb{R}^+$ is said to be a partial metric on X if for any $x, y, z \in X$, the following conditions hold true:

- **(P_1):** $p(x, x) = p(y, y) = p(x, y)$ if and only if $x = y$;
- **(P_2):** $p(x, x) \leq p(x, y)$;
- **(P_3):** $p(x, y) = p(y, x)$;
- **(P_4):** $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

The pair (X, p) is then called a partial metric space.
If $p(x, y) = 0$, then (P_1) and (P_2) imply that $x = y$. But the converse does not hold always.

A trivial example of a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is defined as $p(x, y) = \max\{x, y\}$.

Example 1.5. [17] If $X = \{[a, b] : a, b \in \mathbb{R}, a \leq b\}$, then $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$ defines a partial metric p on X.

For some more examples of partial metric spaces, we refer to [4, 9, 21, 24].

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family open p-balls $\{B_p(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$,for all $x \in X$ and $\varepsilon > 0$.

Observe (see [17, p. 187]) that a sequence $\{x_n\}$ in a partial metric space (X, p) converges to a point $x \in X$, with respect to τ_p, if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

If p is a partial metric on X, then the function $p^S : X \times X \to \mathbb{R}^+$ given by $p^S(x, y) = 2p(x, y) - p(x, x) - p(y, y)$, defines a metric on X.

Furthermore, a sequence $\{x_n\}$ converges in (X, p^S) to a point $x \in X$ if and only if

$$\lim_{n,m\to\infty} p(x_n, x_m) = \lim_{n\to\infty} p(x_n, x) = p(x, x).$$

Definition 1.6. [17]. Let (X, p) be a partial metric space.

- **(a):** A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if $\lim\limits_{n,m\to\infty} p(x_n, x_m)$ exists and is finite.
- **(b):** (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x \in X$ such that $\lim\limits_{n\to\infty} p(x, x_n) = p(x, x)$. In this case, we say that the partial metric p is complete.

Lemma 1.7. [4, 17] *Let (X, p) be a partial metric space. Then:*

- **(a):** *A sequence $\{x_n\}$ in X is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in metric space (X, p^S).*
- **(b):** *A partial metric space (X, p) is complete if and only if the metric space (X, p^S) is complete.*

2. INTRODUCTION AND PRELIMINARIES

In this section, we obtain some common fixed point results for self maps satisfying Presić type contractions defined on a complete partial metric space. We begin with the following theorem.

Theorem 2.1. *Let (X, p) be a complete partial metric space. Suppose that $f, g : X^k \to X$ be two mappings satisfy*

$$p(f(x_1, ..., x_k), g(x_2, ..., x_{k+1})) \leq \lambda \max\{p(x_i, x_{i+1}) : 1 \leq i \leq k\}, \tag{2.1}$$

for all $x_1, ..., x_{k+1}$ in X, where $\lambda \in [0, 1)$, k a positive integer. Then f has a unique fixed point x^*. Moreover, for any arbitrary points $x_1, ..., x_{k+1}$ in X, sequence $\{x_n : n \in \mathbb{N}\}$ defined by $x_{n+k} = f(x_n, x_{n+1}..., x_{n+k-1})$ converges to x^*.

Proof. Let $x_1, ..., x_{k+1}$ be arbitrary k elements in X. Define

$$\begin{aligned} x_{2n+k} &= f(x_{2n}, x_{2n+1}, ..., x_{2n+k-1}) \text{ and} \\ x_{2n+1+k} &= g(x_{2n+1}, x_{2n+2}, ..., x_{2n+k}) \end{aligned}$$

for all $n = 1, 2, ...$ First, we prove that the following inequalities holds for each $n \in \mathbb{N}$:

$$p(x_{2n}, x_{2n+1}) \leq \lambda^{\frac{2n}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\}. \tag{2.2}$$

It is obvious to note that (2.2) is valid for $n = 1, 2, 3, ..., k$. Now let the following k inequalities:

$$\begin{aligned} p(x_n, x_{n+1}) &\leq \lambda^{\frac{2n}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\}, \\ p(x_{n+1}, x_{n+2}) &\leq \lambda^{\frac{2(n+1)}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\}, \\ &\ \ . . ., \\ p(x_{n+k-1}, x_{n+k}) &\leq \lambda^{\frac{2(n+k-1)}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\} \end{aligned}$$

by the induction hypotheses. Then we have

$$\begin{aligned} p(x_{2n+k}, x_{2n+k+1}) &= p\left(f(x_{2n}, ..., x_{2n+k-1}), g(x_{2n+1}, ..., x_{2n+k})\right) \\ &\leq \lambda \max\left\{p(x_i, x_{i+1}) : 2n \leq i \leq 2n+k-1\right\} \\ &\leq \lambda.\lambda^{\frac{2n}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\} \\ &= \lambda^{\frac{2n+k}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\} \end{aligned}$$

and the inductive proof of (2.2) is complete. In similar way, we obtain

$$p(x_{2n+k+1}, x_{2n+k+2}) \leq \lambda^{\frac{2n+k+1}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{2i}{k}}} : 1 \leq i \leq k\right\}.$$

Hence

$$p(x_{n+k}, x_{n+k+1}) \leq \lambda^{\frac{n+k}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{i}{k}}} : 1 \leq i \leq k\right\}$$

for all $n = 1, 2, ...$ Now we have

$$\begin{aligned} p_s(x_{n+k}, x_{n+k-1}) &= 2p(x_{n+k}, x_{n+k-1}) - p(x_{n+k}, x_{n+k}) - p(x_{n+k-1}, x_{n+k-1}) \\ &\leq 2p(x_{n+k}, x_{n+k-1}) + p(x_{n+k}, x_{n+k}) + p(x_{n+k-1}, x_{n+k-1}) \\ &\leq 4p(x_{n+k}, x_{n+k-1}) \\ &\leq 4\lambda^{\frac{n+k-1}{k}} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^{\frac{i}{k}}} : 1 \leq i \leq k\right\}. \end{aligned}$$

So we have

$$\begin{aligned} p_s(x_{n+k}, x_n) &\leq p_s(x_{n+k}, x_{n+k-1}) + ... + p_s(x_{n+1}, x_n) \\ &\leq 4\lambda^n[\lambda^{k-1} + ... + \lambda^1] \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^i} : 1 \leq i \leq k\right\} \\ &\leq \frac{4\lambda^n}{1-\lambda} \max\left\{\frac{p(x_i, x_{i+1})}{\lambda^i} : 1 \leq i \leq k\right\}. \end{aligned}$$

Hence $\{x_n\}$ is a Cauchy sequence in (X, p_s). By Lemma 1.7, $\{x_n\}$ is a Cauchy sequence in (X, p). Now, since (X, p) is complete, there exists u in X such that $x_n \to u$ as $n \to \infty$. So that

$$\lim_{n\to\infty} p(x_n, x_n) = \lim_{n\to\infty} p(x_n, u) = p(u, u).$$

Now, for any integer n we have

$$\begin{aligned} & p(u, g(u, u, ..., u)) \\ \leq\ & p(u, x_{2n+k}) + p(x_{2n+k}, g(u, u, ..., u)) - p(x_{2n+k}, x_{2n+k}) \\ =\ & p(u, x_{2n+k}) + p(f(x_{2n}, x_{2n+1} ..., x_{2n+k-1}), g(u, u, ..., u)) - p(x_{2n+k}, x_{2n+k}) \\ \leq\ & p(u, x_{2n+k}) + \lambda \max\{p(x_{2n}, u), p(x_{2n+1}, u), ..., p(x_{2n+k-1}, u)\} - p(x_{2n+k}, x_{2n+k}). \end{aligned}$$

On taking limit as $n \to \infty$, we obtain

$$p(u, g(u, u, ..., u)) \leq \lambda p(x_{2n}, u),$$

implies $u = g(u, u, ..., u)$. Again

$$\begin{aligned} & p(f(u, u, ..., u), u) \\ \leq\ & p(f(u, u, ..., u), x_{2n+k+1}) + p(x_{2n+k+1}, u) - p(x_{2n+k+1}, x_{2n+k+1}) \\ =\ & p(f(u, u, ..., u), g(x_{2n+1}, x_{2n+2} ..., x_{2n+k})) + p(u, x_{2n+k+1}) - p(x_{2n+k+1}, x_{2n+k+1}) \\ \leq\ & \lambda \max\{p(u, x_{2n+1}), p(u, x_{2n+2}), ..., p(u, x_{2n+k})\} + p(u, x_{2n+k+1}) - p(x_{2n+k+1}, x_{2n+k+1}) \end{aligned}$$

and on taking limit as $n \to \infty$, we get

$$p(f(u, u, ..., u), u) \leq \lambda p(u, u),$$

which implies $f(u, u, ..., u) = u$. Hence u is the common fixed point of f and g.

Now, to prove the uniqueness of u, let v be another point in X such that $v = f(v, v, ..., v) = g(v, v, ..., v)$. Then, we have

$$\begin{aligned} p(u, v) &= p(f(u, u, ..., u), g(v, v, ..., v)) \\ &\leq \lambda p(u, v), \end{aligned}$$

implies $u = v$. So, u is the unique common fixed point of f and g in X.

□

Example 2.2. Let $X = [0, 2]$. Let $p : X \times X \to \mathbb{R}^+$ defined by $p(x, y) = |x - y|$ if $x, y \in [0, 1)$, and $p(x, y) = \max\{x, y\}$ otherwise. It is easily seen that (X, p) is a complete partial metric space. For a positive integer k, we define $f, g : X^k \to X$ by

$$\begin{aligned} f(x_1, ..., x_k) &= \begin{cases} \dfrac{x_2 + x_3 + x_k}{6}, & \text{if } x_1, ..., x_k \in [0, 1) \\ 0, & \text{otherwise,} \end{cases} \\ g(x_1, ..., x_k) &= \begin{cases} \dfrac{x_1 + x_2 + x_k}{6}, & \text{if } x_1, ..., x_k \in [0, 1) \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

Now for all $x_1, x_2, ..., x_{k+1} \in [0, 1)$ and $\lambda = 1/2$, we have

$$\begin{aligned} p(f(x_1, ..., x_k), g(x_2, ..., x_{k+1})) &= \left| \frac{x_2 + x_3 + x_k}{6} - \frac{x_2 + x_3 + x_{k+1}}{6} \right| \\ &= \frac{1}{6} |x_k - x_{k+1}| \\ &\leq \frac{1}{2} \max\{p(x_i, x_{i+1}) : 1 \leq i \leq k\} \\ &= \lambda \max\{p(x_i, x_{i+1}) : 1 \leq i \leq k\}. \end{aligned}$$

If for $x_1, x_2, ..., x_k, \in [0, 1)$ and $x_{k+1} \in [1, 2]$, then

$$\begin{aligned} p(f(x_1, ..., x_k), g(x_2, ..., x_{k+1})) &= \frac{1}{6} (x_2 + x_3 + x_k) \\ &\leq \frac{1}{2} x_{k+1} = \lambda \max\{p(x_i, x_{i+1}) : 1 \leq i \leq k\}. \end{aligned}$$

When some $x_{j's} \in [1, 2]$, and $x_1, x_2, ..., x_{j-1}, x_{j+1}, ..., x_{k+1} \in [0, 1)$, then we obtain $p(f(x_1, ..., x_k), g(x_2, ..., x_{k+1})) = 0$ and (2.1) is satisfied obviously. Thus the conditions of Theorem 2.1 are satisfied and there exist a unique $u = 0$ in X such that $f(u, u, ..., u) = g(u, u, ..., u) = u$.

Corollary 2.3. *Let (X, p) be a complete partial metric space, k a positive integer and $f, g : X^k \to X$. Suppose that*

$$p(f(x_1, ..., x_k), g(x_2, ..., x_{k+1})) \leq \sum_{i=1}^{k} \lambda_i p(x_i, x_{i+1}), \tag{2.3}$$

holds for all $x_1, ..., x_{k+1}$ in X, where $\lambda_i \geq 0$ and $\sum_{i=1}^{k} \lambda_i \in [0, 1)$. Then f and g have a unique fixed point x^*.

In Theorem 2.1, take $f = g$ to obtain the following corollary which extends and generalizes the corresponding results of [20].

Corollary 2.4. *Let (X, p) be a complete partial metric space. Suppose that a mapping $f : X^k \to X$ satisfies*

$$p(f(x_1, ..., x_k), f(x_2, ..., x_{k+1})) \leq \lambda \max\{p(x_i, x_{i+1}) : 1 \leq i \leq k\}, \tag{2.4}$$

for all $x_1, ..., x_{k+1}$ in X, where $\lambda \in [0, 1)$, k a positive integer. Then f has a fixed point x^*. Moreover, for any arbitrary points $x_1, ..., x_{k+1}$ in X, sequence $\{x_n : n \in \mathbb{N}\}$ defined by $x_{n+k} = f(x_n, x_{n+1}..., x_{n+k-1})$ converges to x^*.

If we take $f = g$ in Theorem 2.1, then the following corollary is obtained which extends and generalizes the comparable results of [11].

Corollary 2.5. *Let (X, p) be a complete partial metric space, k a positive integer and $f : X^k \to X$. Suppose that*

$$p(f(x_1, ..., x_k), f(x_2, ..., x_{k+1})) \leq \sum_{i=1}^{k} \lambda_i p(x_i, x_{i+1}), \tag{2.5}$$

holds for all $x_1, ..., x_{k+1}$ in X, where $\lambda_i \geq 0$ and $\sum_{i=1}^{k} \lambda_i \in [0, 1)$. Then f has a unique fixed point x^*. Moreover, for any arbitrary points $x_1, ..., x_{k+1}$ in X, sequence $\{x_n\}$ defined by $x_{n+k} = f(x_n, x_{n+1}..., x_{n+k-1})$, for all $n \in \mathbb{N}$ converges to x^*.

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