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FIXED POINT THEOREMS FOR KANNAN TYPE CYCLIC WEAKLY CONTRACTIONS

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ABSTRACT. In this article, we introduce the notion of Kannan Type cyclic weakly contraction and derive the existence of fixed point theorems in the setup of complete metric spaces. Our main theorems extend and improve some fixed point theorems in the literature. Examples are given to support the usability of the results.

KEYWORDS : Fixed point theory; Cyclic φ-contraction; Kannan type mapping.
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1. INTRODUCTION AND PRELIMINARIES

Fixed point theory is one of the cornerstone in the development of nonlinear functional analysis. Besides mathematics, fixed point theory has been used effectively in many other discipline such as economics, chemistry, biology, computer science, engineering, and others. In particular, Banach's contraction mapping principle [2] has a significant role in fixed point theory and hence in nonlinear functional analysis.

Let (X, d) be a complete metric space and $T : X \longrightarrow X$ be a self-map. If there exists $k \in [0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y)$$

for all $x, y \in X$, then T has a unique fixed point.

Banach fixed point theorem not only guarantee the existence and uniqueness of a fixed point but also show how to get it. All things considered, Banach's contraction mapping principle differ from the origin and antecedents results. We also notice that a self mapping T, in Banach fixed point theorem, is necessarily continuous. Due to its importance, fixed point theory draw interest of many researcher (see, e.g.,[19, 11]).

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In 1968, Kannan [9] prove one of the interesting generalization of the Banach Contraction Principle.

Definition 1.1. [9, 10] A self-mapping $T : X \longrightarrow X$, on a metric space (X, d) is said to be Kannan type mapping if there exists $0 < k < 1$ such that, for all $x, y \in X$, the following inequality holds:

$$d(Tx, Ty) \leq \frac{k}{2}[d(x, Tx) + d(y, Ty)]].$$

In Kannan fixed point theorem, a self mapping T need not to be continuous. This is the most important gains of the Kannan's result.

On the other hand, Kirk et al.[17] in 2003 introduce the following notion of cyclic representation and characterize the Banach Contraction Principle in the context of cyclic mapping.

Definition 1.2. [17] Let X be a non-empty set and $T : X \longrightarrow X$ an operator. By definition, $X = \bigcup_{i=1}^{m} X_i$ is a cyclic representation of X with respect to T if

(a) X_i, $i = 1, \dots, m$ are non-empty sets,

(b) $T(X_1) \subset X_2, \cdots, T(X_{m-1}) \subset X_m, T(X_m) \subset X_1$.

After the distinguished notion and related fixed point result of Kirk et al.[17], a number of fixed point theorems are reported in the literature, for operators T defined on a complete metric space X with a cyclic representation of X with respect to T (see, e.g., [5] - [23]).

Very recently, Karapınar [11] characterize the notion of the cyclic weak φ-contraction and prove fixed point theorems for such types contractions in the context of cyclic mapping.

Definition 1.3. (See [11]) Let (X, d) be a metric space, $m \in \mathbb{N}$, $A_1, A_2, \dots, A_m$ closed nonempty subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. An operator $T : Y \longrightarrow Y$ is called a cyclic weak φ-contraction if

(1) $X = \bigcup_{i=1}^{m} A_i$ is a cyclic representation of X with respect to T;

(2) there exists a continuous, non-decreasing function $\varphi : [0, \infty) \longrightarrow [0, \infty)$ with $\varphi(t) > 0$ for $t \in (0, \infty)$ and $\varphi(0) = 0$ such that

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)),$$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \cdots, m$ where $A_{m+1} = A_1$.

Let $\mathbf{F}$ denote all the continuous functions $\varphi : [0, \infty) \longrightarrow [0, \infty)$ with $\varphi(t) > 0$ for $t \in (0, \infty)$ and $\varphi(0) = 0$.

Theorem 1.4. *(See [11]) Let (X, d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \dots, A_m$ nonempty subsets of X and $X = \bigcup_{i=1}^{m} A_i$. Suppose that T is a cyclic weak φ-contraction with $\varphi \in \mathbf{F}$. Then, T has a fixed point $z \in \bigcap_{i=1}^{n} A_i$.*

The aim of this paper is to use the concept of cyclic contraction and Kannan type mapping and introduce the notions of Kannan type cyclic weakly contractions and then derive fixed point theorems on it in the setup of complete metric spaces. Our results generalize fixed point theorems [11, 17] in the sense of metric spaces.

2. MAIN RESULTS

In this section, we introduce the notion of Kannan type cyclic weakly contraction in metric space. Before this, we introduce the following class of functions: Let $\mathbf{F_1}$ denote all the continuous functions $\psi : [0, \infty)^2 \longrightarrow [0, \infty)$ satisfying $\psi(x, y) = 0$ if and only if $x = y = 0$.

We introduce the notion of Kannan type cyclic weakly contraction in metric space, in the following way.

Definition 2.1. Let (X, d) be a metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ nonempty subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. An operator $T : Y \longrightarrow Y$ is called a Kannan type cyclic weakly contraction if

(1) $\bigcup_{i=1}^{m} A_i$ is a cyclic representation of Y with respect to T;

(2)

$$d(Tx, Ty) \quad \leq \quad \tfrac{1}{2}[d(x, Tx) + d(y, Ty)] - \psi(d(x, Tx), d(x, Ty)),$$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \cdots, m$ where $A_{m+1} = A_1$ and

$$\psi(d(x, Tx), d(y, Ty)) \in \mathbf{F_1}.$$

We state the main result of this section as follows:

Theorem 2.2. *Let (X, d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ nonempty closed subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. Suppose that T is a Kannan type cyclic weakly contraction. Then, T has a fixed point $z \in \bigcap_{i=1}^{n} A_i$.*

Proof. Take $x_0 \in X$. We construct a sequence in the following way:

$$x_{n+1} = Tx_n, \text{ for all } n = 0, 1, 2, \cdots.$$

If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0+1} = x_{n_0}$, then, the existence of the fixed point is proved. Indeed, we have $Tx_{n_0} = x_{n_0+1} = x_{n_0}$. On the occasion of that we assume $x_{n+1} \neq x_n$ for any $n = 0, 1, 2, \cdots$. Regarding that $X = \bigcup_{i=1}^{m} A_i$, for any $n > 0$ there exists $i_n \in \{1, 2, \cdots, m\}$ such that $x_{n-1} \in A_{i_n}$ and $x_n \in A_{i_{n+1}}$. Since T is a Kannan type cyclic weakly contraction, we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Tx_{n-1}, Tx_n) \\ &\leq \frac{1}{2}[d(x_{n-1}, Tx_{n-1}) + d(x_n, Tx_n)] - \psi(d(x_{n-1}, Tx_{n-1}), d(x_n, Tx_n)) \\ &= \frac{1}{2}[d(x_{n-1}, x_n) + d(x_n, x_{n+1})] - \psi(d(x_{n-1}, x_n), d(x_n, x_{n+1})) \quad (2.2) \\ &\leq \frac{1}{2}[d(x_{n-1}, x_n) + d(x_n, x_{n+1})]. \end{aligned} \tag{2.1}$$

Consequently,

$$d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n) \text{ for any } n \in \mathbb{N}.$$

By virtue of the fact that we conclude $\{d(x_n, x_{n+1})\}$ is a nondecreasing sequence of nonnegative real numbers. Therefore, there exists $\gamma \geq 0$ such that

$$\lim_{n \longrightarrow \infty} d(x_n, x_{n+1}) = \gamma. \tag{2.3}$$

Letting $n \longrightarrow \infty$ in (2.2) we derive that

$$\gamma \leq \lim_{n \longrightarrow \infty} \frac{1}{2}[2\gamma] \leq \gamma,$$

or

$$\lim_{n \longrightarrow \infty} d(x_{n-1}, x_{n+1}) = 2\gamma. \tag{2.4}$$

Setting $n \longrightarrow \infty$ in (2.2) and by using (2.3)and (2.4) we get

$$\begin{aligned} \gamma &\leq \frac{1}{2}(2\gamma) - \liminf_{n \longrightarrow \infty} \psi(d(x_{n-1}, x_n), d(x_n, x_{n+1})) \\ &\leq \gamma - \psi(\gamma, \gamma) \end{aligned}$$

or, $\psi(\gamma,\gamma) \leq 0$ by the continuity of ψ. This is a contradiction unless $\gamma = 0$, that is,

$$\lim_{n\longrightarrow\infty} d(x_n, x_{n+1}) = 0. \tag{2.5}$$

We shall show that the sequence $\{x_n\}$ is a Cauchy sequence. For this goal, we prove the following claim first:

(C) For every $\varepsilon > 0$ there exists $n \in \mathbb{N}$ such that if $r, q \geq n$ with $r - q \equiv 1(m)$, then $d(x_r, x_q) < \varepsilon$.

Assume the contrary of (C). Thus, there exists $\varepsilon > 0$ such that for any $n \in \mathbb{N}$ we can find $r_n > q_n \geq n$ with $r_n - q_n \equiv 1(m)$ satisfying

$$d(x_{q_n}, x_{r_n}) \geq \varepsilon. \tag{2.6}$$

Now, we take $n > 2m$. Then, corresponding to $q_n \geq n$ use can choose r_n in such a way that it is the smallest integer with $r_n > q_n$ satisfying $r_n - q_n \equiv 1(m)$ and $d(x_{q_n}, x_{r_n}) \geq \varepsilon$. Therefore, $d(x_{q_n}, x_{r_{n-m}}) \leq \varepsilon$. By using the triangular inequality

$$\begin{aligned}\varepsilon &\leq d(x_{q_n}, x_{r_n}) \leq d(x_{q_n}, x_{r_{n-m}}) + \sum_{i=1}^{m} d(x_{r_{n-i}}, x_{r_{n-i+1}})\\ &< \varepsilon + \sum_{i=1}^{m} d(x_{r_{n-i}}, x_{r_{n-i+1}}).\end{aligned}$$

Letting $n \longrightarrow \infty$ in the last inequality and taking into account that $\lim_{n\longrightarrow\infty} d(x_n, x_{n+1}) = 0$, we find

$$\lim_{n\longrightarrow\infty} d(x_{q_n}, x_{r_n}) = \varepsilon. \tag{2.7}$$

Again, by the triangular inequality

$$\begin{aligned}\varepsilon &\leq d(x_{q_n}, x_{r_n})\\ &\leq d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_{n+1}}, x_{r_{n+1}}) + d(x_{r_{n+1}}, x_{r_n})\\ &\leq d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_{n+1}}, x_{q_n}) + d(x_{q_n}, x_{r_n}) + d(x_{r_n}, x_{r_{n+1}}) + d(x_{r_{n+1}}, x_{r_n})\\ &= 2d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_n}, x_{r_n}) + 2d(x_{r_n}, x_{r_{n+1}}).\end{aligned} \tag{2.8}$$

Taking (2.5) and (2.7) into account, we obtain

$$\lim_{n\longrightarrow\infty} d(x_{q_{n+1}}, x_{r_{n+1}}) = \varepsilon \tag{2.9}$$

as $n \longrightarrow \infty$ in (2.7).

Due to the fact that x_{q_n} and x_{r_n} lie in different adjacently labeled sets A_i and A_{i+1} for certain $1 \leq i \leq m$, using the fact that T is a Kannan type cyclic weakly contraction, we have

$$\begin{aligned}d(x_{q_{n+1}}, x_{r_{n+1}}) &= d(Tx_{q_n}, Tx_{r_n})\\ &\leq \frac{1}{2}[d(x_{q_n}, Tx_{q_n}) + d(x_{r_n}, Tx_{r_n})] - \psi(d(x_{q_n}, Tx_{q_n}), d(x_{r_n}, Tx_{r_n}))\\ &\leq \frac{1}{2}[d(x_{q_n}, x_{q_{n+1}}) + d(x_{r_n}, x_{r_{n+1}})] - \psi(d(x_{q_n}, x_{q_{n+1}}), d(x_{r_n}, x_{r_{n+1}})).\end{aligned}$$

Regarding (2.5) and (2.7) and the continuity of ψ, letting $n \longrightarrow \infty$ in the last inequality, we conclude that $\varepsilon = 0$. This is a contradiction and hence (C) is proved.

By the help of (C), we shall show $\{x_n\}$ is a Cauchy sequence in Y. Fix $\varepsilon > 0$. By (C), we find $n_0 \in \mathbb{N}$ such that if $r, q \geq n_0$ with $r - q \equiv 1(m)$

$$d(x_r, x_q) \leq \frac{\varepsilon}{2}. \tag{2.10}$$

Since $\lim_{n\longrightarrow\infty} d(x_n, x_{n+1}) = 0$, we also find $n_1 \in \mathbb{N}$ such that

$$d(x_n, x_{n+1}) \leq \frac{\varepsilon}{2m} \tag{2.11}$$

for any $n \geq n_1$. Assume that $r, s \geq \max\{n_0, n_1\}$ and $s > r$. Then there exists $k \in \{1, 2, \cdots, m\}$ such that $s-r \equiv k(m)$. Hence, $s-r+\varphi \equiv 1(m)$ for $\varphi = m-k+1$. So, we have

$$d(x_r, x_s) \leq d(x_r, x_{s+j}) + d(x_{s+j}, x_{s+j-1}) + \cdots + d(x_{s+1}, x_s).$$

By (2.10) and (2.11) and from the last inequality, we get

$$d(x_r, x_s) \leq \frac{\varepsilon}{2} + j \times \frac{\varepsilon}{2m} \leq \frac{\varepsilon}{2} + m \times \frac{\varepsilon}{2m} = \varepsilon$$

By reason of the fact that (x_n) is a Cauchy sequence in Y. Since Y is closed in X, then Y is also complete and there exists $x \in Y$ such that $\lim_{n\longrightarrow\infty} x_n = x$. In what follows, we prove that x is a fixed point of T. In fact, since $\lim_{n\longrightarrow\infty} x_n = x$ and, as $Y = \bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T, the sequence (x_n) has infinite terms in each A_i for $i \in \{1, 2, \cdots, m\}$. Suppose that $x \in A_i$, $Tx \in A_{i+1}$ and we take a subsequence x_{n_k} of (x_n) with $x_{n_k} \in A_{i-1}$ (the existence of this subsequence is guaranteed by the above-mentioned comment). By using the contractive condition, we can obtain

$$\begin{aligned} d(x_{n_{k+1}}, Tx) &= d(Tx_{n_k}, Tx) \\ &\leq \frac{1}{2}[d(x_{n_k}, Tx_{n_k}) + d(x, Tx)] - \psi(d(x_{n_k}, Tx_{n_k}), d(x, Tx)) \\ &= \frac{1}{2}[d(x_{n_k}, x_{n_{k+1}}) + d(x, Tx)] - \psi(d(x_{n_k}, x_{n_{k+1}}), d(x, Tx)). \end{aligned}$$

Setting $n \longrightarrow \infty$ and using $x_{n_k} \longrightarrow x$, continuity of ψ, we have

$$d(x, Tx) \leq \frac{1}{2}d(x, Tx) - \psi(0, d(x, Tx)) \leq \frac{1}{2}d(x, Tx)$$

which is a contradiction unless $d(x, Tx) = 0$.Consequently, x is a fixed point of T.

We shall prove that x is a unique fixed point of T. Suppose, to the contrary that, there exists $z \in X$ with $x \neq z$ and $Tz = z$. By using the contractive condition we obtain

$$\begin{aligned} d(x, z) = d(Tx, Tz) &\leq \frac{1}{2}[d(x, Tx) + d(z, Tz)] \\ &\quad -\psi(d(x, Tx), d(z, Tz)) = 0. \end{aligned}$$

which is a contradiction. □

Corollary 2.3. *Let (X, d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ non-empty closed subsets of X and $Y = \bigcup_{i=1}^m A_i$. Suppose that $T : Y \longrightarrow Y$ be an operator such that*

(1) $\bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;

(ii) there exists $\alpha \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq \alpha[d(x, Tx) + d(y, Ty)] \tag{2.12}$$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \cdots, m$ where $A_{m+1} = A_1$. Then, T has a fixed point $z \in \bigcap_{i=1}^n A_i$.

Proof. Let $\alpha \in [0, \frac{1}{2})$. Here, it suffices to take the function $\psi : [0, +\infty)^2 \longrightarrow [0, +\infty)$ as $\psi(a, b) = (\frac{1}{2} - \alpha)(a + b)$. It is clear that ψ satisfies the conditions:

(i) $\psi(a, b) = 0$ if and only if $a = b = 0$, and

(ii) $\psi(x, y) = (\frac{1}{2} - \alpha)(x + y) = \psi(x + y, 0)$.

Hence, we apply Theorem 2.2 and get the desired result. □

The following corollary gives us a fixed point theorem with a contractive condition of integral type for cyclic contractions.

Corollary 2.4. *Let (X,d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ nonempty closed subsets of X and $Y = \bigcup_{i=1}^m A_i$. Suppose that $T : Y \longrightarrow Y$ be an operator such that*

(i) $\bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;

(ii) there exists $\alpha \in [0, \frac{1}{2})$ such that

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq \alpha \int_0^{d(x,Tx)+d(y,Ty)} \rho(t)dt$$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \cdots, m$ where $A_{m+1} = A_1$, and $\rho : [0,\infty) \longrightarrow [0,\infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)dt$ for $\varepsilon > 0$. Then T has unique fixed point $z \in \bigcap_{i=1}^m A_i$.

Proof. It is easily proved that the function $\varphi : [0,\infty) \longrightarrow [0,\infty)$ given by $\varphi(t) = \int_0^t \rho(s)ds$ satisfies that $\varphi \in \mathbf{F_1}$. Therefore, Corollary 2.3 is obtained from Theorem 2.2, taking as φ the above-defined function and as ψ the function $\psi(x,y) = (\frac{1}{2} - \alpha)(x+y) = \varphi(x+y, 0)$. □

If in Corollary 2.4, we take $A_i = X$ for $i = 1, 2, \cdots, m$, we obtain the following result.

Corollary 2.5. *Let (X,d) be a complete metric space and $T : X \longrightarrow X$ a mapping such that for any $x, y \in X$,*

$$\int_0^{d(Tx,Ty)} \rho(t)dt \leq \alpha \int_0^{d(x,Tx)+d(y,Ty)} \rho(t)dt$$

where $\rho : [0,\infty) \longrightarrow [0,\infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)dt$ for $\varepsilon > 0$ and the constant $\alpha \in [0, \frac{1}{2})$. Then T has unique fixed point.

If in Theorem 2.2 we put $A_i = X$ for $i = 1, 2, \cdots, m$ we get the generalized result of [9, 10].

Corollary 2.6. *Let (X,d) be a complete metric space and $T : X \longrightarrow X$ a mapping such that for any $x, y \in X$,*

$$d(Tx,Ty) \leq \frac{1}{2}[d(x,Tx) + d(y,Ty)] - \psi(d(x,Tx) + d(y,Ty)),$$

where $\psi \in \mathbf{F_1}$. Then T has unique fixed point.

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