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COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE UNDER $(S-B)$ PROPERTY

PRAVEEN KUMAR SHARMA[1,*] AND SUSHIL SHARMA[2]

[1] Department of Mathematics, IES, IPS Academy, Rajendra Nagar, A.B. Road, Indore - 452012 (M.P.), India

[2] Professor and Head Department of Mathematics, Govt. Madhav Science College, Ujjain (M.P.), India

ABSTRACT. The main purpose of this paper is to give common fixed point theorem in intuitionistic fuzzy metric space under strict contractive conditions for mappings satisfying $(S-B)$ property.

KEYWORDS : Intuitionistic fuzzy metric space; Common fixed point; Compatible maps; Weakly compatible maps; $(S-B)$ property.
AMS Subject Classification: 47H10 54H25

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [37] in 1965. Since, then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Atanassov [5] Introduced and studied the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. Coker [7] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [3] proved the well- known fixed point theorems of Banach [6] in the setting of intuitionisitc fuzzy metric spaces. Later on, Turkoglu et al. [35] Proved Jungcks [12] common fixed point theorem in the setting of intuitionisitc fuzzy metric space. Turkoglu et al. [35] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of pants theorem [20]. Gregori et al. [10], Saadati and Park [26] studied the concept of intuitionistic fuzzy metric space and its applications. Recently, many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric space (See [9, 11, 23, 24, 31, 32, 33, 34, 36].

* *Corresponding author.*
Email address : praveen_jan1980@rediffmail.com(P.K. Sharma) and sksharma2005@yahoo.com(S. Sharma).
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The study of common fixed points of non compatible mappings is also very interesting. Work along these lines has recently been initiated by pant [21, 22]. Sharma and Bamboria [30] defined a property $(S-B)$ for self maps and obtained some common fixed point theorems for such mappings under strict contractive conditions. The class of $(S-B)$ maps contains the class of non compatible maps. Kamran [15] obtained some coincidence and fixed point theorems for hybrid strict contractions.

Definition 1.1. [27] A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-norm if $*$ is satisfying the following condition;
(i) * is commutative and associative;
(ii) * is continuous
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 1.2. [27] A binary operation $\diamond : [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous t-conorm if $\diamond$ is satisfying the following conditions;
(i) $\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous
(iii) $a \diamond 0 = a$ for all $a \in [0,1]$;
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 1.3. A 5 tuple$(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\diamond$ is a continuous t-conorm X^2 and $M, N,$ are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions;
(i) $M(x,y,t) + N(x,y,t) \leq 1$ for all $x, y \leq X$ and $t > 0$;
(ii) $M(x,y,0) = 0$ for all $x, y \in X$;
(iii) $M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x,y,t) = M(y,x,t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y \in X, M(x,y,\cdot)$ is left continuous;
(vii) $\lim_{t \longrightarrow \infty}$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x,y,0) = 1$ for all $x, y \in X$;
(ix) $N(x,y,t) = 0$ for all $x, y \in X$ and $t > 0$ iff $x = y$;
(x) $N(x,y,t) = N(y,x,t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) for all $x, y \in X, N(x,y,\cdot) : [0,\infty) \longrightarrow [0,1]$ is right continuous;
(xiii) $\lim_{t \longrightarrow \infty} N(x,y,t) = 0$ for all x, y in X.
(M, N) is called an intuitionistic fuzzy metric on X.
The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of nonnearness between x and y with respect to t respectively.

Remark 1.4. An intuitionistic fuzzy metric spaces with continous t-norm * and continous t-conorm $\diamond$ defined by $a * a \geq a$ and $(1-a) \diamond (1-a) \leq (1-a)$ for all $a \in [0,1]$. Then for all $x, y \in X, M(x,y,*)$ is non- decreasing and $N(x,y,\diamond)$ is nonincreasing.

Alaca, Turkoglu and Yildiz [3] introduced the following notions;

Definition 1.5. Let $(X, M, N*, \diamond)$ be an intuitionistic fuzzy metric space. Then (a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $lim_{n \longrightarrow \infty} M(x_{n+p}, x_n, t) = 1, lim_{n \longrightarrow \infty} N(x_{n+p}, x_n, t) = 0$. (b) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if , for all $t > 0$, $lim_{n \longrightarrow \infty} M(x_n, x, t) = 1, lim_{n \longrightarrow \infty} N(x_n, x, t) = 0$;

Since * and $\diamond$ are continuous, the limit is uniquely determined from (v) and (xi) of definition 1.3, respectively. An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 1.6. [32] A pair of self mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be compatible if $\lim_{n\longrightarrow\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\longrightarrow\infty} N(fgx_n, gfx_n, t) = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\longrightarrow\infty} fx_n = \lim_{n\longrightarrow\infty} gx_n = z$ for some $z \in X$.

Definition 1.7. A pair of self mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be non-compatible $\lim_{n\longrightarrow\infty} M(fgx_n, gfx_n, t) \neq 1$ or non existent and $\lim_{n\longrightarrow\infty} N(fgx_n, gfx_n, t) \neq 0$ or non existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\longrightarrow\infty} fx_n = \lim_{n\longrightarrow\infty} gx_n = z$ for some $z \in X$.

In 1998, Jungck and Rhodes [14] introduced the concept of weakly compatible maps as follows;

Definition 1.8. Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Aamri and Moutawakil [1] generalized the concept of non compatibility in metric spaces by defining the notion of E.A. Property and proved common fixed point theorems using this property.

Sharma and Bamboria [30] defined the $(S - B)$ property and proved common fixed point theorems in fuzzy metric spaces using this property.

Definition 1.9. A pair of self mappings (S, T) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to satisfy $(S - B)$ property if there exists a sequence $\{x_n\}$ is X such that

$$\lim_{n\longrightarrow\infty} M(Sx_n, Tx_n, t) = 1, \lim_{n\longrightarrow\infty} N(Sx_n, Tx_n, t) = 0$$

Example 1.10. Let $X = [0, \infty)$ consider $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, where M and N are two fuzzy sets defined by $M(x, y, t) = t/[t + d(x, y)]$ and $N(x, y, t) = d(x, y)/[t + d(x, y)]$ where d is usual metric. Define $T, S : X \longrightarrow [0, \infty)$ by $Tx = x/5$ and $Sx = 2x/5$ for all x in X. Consider $x_n = 1/n$. Now, $\lim_{n\longrightarrow\infty} M(Sxn, Txn, t) = 1$ and $\lim_{n\longrightarrow\infty} N(Sxn, Txn, t) = 0$. Therefore S and T satisfy property $(S - B)$.

Lemma 1.11. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1)$, $M(x, y, kt).M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ then $x = y$.*

Lemma 1.12. [2] *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exist is a number $k \in (0, 1)$ such that;*

$$M(y_{n+2}, y_{n+1}, kt).M(y_{n+1}, y_n, t)$$

and

$$N(y_{n+2}, y_{n+1}, kt).N(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, \ldots$ Then $\{y_n\}$ is a Cauchy sequence in X.

2. MAIN RESULTS

Theorem 2.1. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with $a*a \geq a$ and $(1-a) \diamond (1-a) \leq (1-a)$ for all $a \in [0, 1)$, let A, B, S, and T be self mappings of X into itself such that,*
(1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,
(1.2) (A, S) or (B, T) satisfies the property $(S-B)$.
(1.3) there exists a number $k \in (0, 1)$ such that

$$[1 + PM(Sx, Ty, kt)] * M(Ax, By, kt)$$
$$\geq \acute{\phi}[PM(Ax, Sx, kt) * M(By, Ty, kt) + M(Ax, Ty, kt) * M(By, Sx, kt) + M(Sx, Ty, t)]$$
$$*M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, t) * M(Ax, Ty, (2-\alpha)t)$$

and

$$[1 + PN(Sx, Ty, kt)] \diamond N(Ax, By, kt)$$
$$\leq \psi[PN(Ax, Sx, kt) \diamond N(By, Ty, kt) + N(Ax, Ty, kt) \diamond N(By, Sx, kt) + N(Sx, Ty, t)$$
$$\diamond(N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, t) \diamond N(Ax, Ty, (2-a)t)]$$

for all $x, y \in X, P \geq 0$, $\alpha \in (0, 2)$ and $t > 0$. Where $p\acute{h}i, \psi : [0, 1] \longrightarrow [0, 1]$ is continuous function such that $\acute{\phi}(S) > S$ and $\psi(S) < S$ for each $0 < S < 1$ with $M(x, y, t) > 0$.
(1.4) the pairs (A, S) and (B, T) are weakly compatible,
(1.5) one of A(X), B(X), S(X) or T(X) is a closed subset of X. Then A, B, S and T have a unique common fixed point in X.

Proof *Suppose that (B, T) satisfies the $(S-B)$ property, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Tx_n = z$ for some $z \in X$. Since $B(X) \subset S(X)$ there exists a sequence $\{y_n\} \in X$ such that $\lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Sy_n = z$. Now we shall show that $\lim_{n\longrightarrow\infty} Ay_n = z$. From (1.3) for $f\alpha = 1-q, q \in (0, 1)$ we have;*

$$[1 + PM(Sy_n, Tx_n, kt)] * M(Ay_n, Bx_n, kt)$$
$$\acute{\phi}[PM(Ay_n, Sy_n, kt) * M(Bx_n, Tx_n, kt) + M(Ay_n, Tx_n, kt) * M(Bx_n, Sy_n, kt)$$
$$+M(Sy_n, Tx_n, t) * M(Ay_n, Sy_n, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Sy_n, t)$$
$$*M(Ay_n, Tx_n(2-\alpha)t)]$$

and

$$[1 + PN(Sy_n, Tx_n, kt)] * N(Ay_n, Bx_n, kt)$$
$$\leq \psi[PN(Ay_n, Sy_n, kt) \diamond N(Bx_n, Tx_n, kt) + N(Ay_n, Tx_n, kt) \diamond N(Bx_n, Sy_n, kt)$$
$$+N(Sy_n, Tx_n, t) \diamond N(Ay_n, Sy_n, t) \diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Sy_n, t)$$
$$\diamond N(Ay_n, Tx_n, (2-\alpha)t)]$$

$$M(Ay_n, Bx_n, kt) + P[M(Sy_n, Tx_n, kt) * M(Ay_n, Bx_n, kt)]$$
$$\geq \acute{\phi}[PM(Ay_n, Sy_n, kt) * M(Bx_n, Tx_n, kt) + M(Ay_n, Tx_n, kt) * M(Bx_n, Sy_n, kt)$$
$$+M(Sy_n, Tx_n, t) * M(Ay_n, Sy_n, t) * M(Bx_n, Tx_n, t)$$
$$*M(Bx_n, Sy_n, t) * M(Ay_n, Tx_n, (1+q)t)]$$

and

$$N(Ay_n, Bx_n, kt) + P[N(Sy_n, Tx_n, kt) \diamond N(Ay_n, Bx_n, kt)]$$
$$\leq \psi[PN(Ay_n, Sy_n, kt) \diamond N(Bx_n, Tx_n, kt) + N(Ay_n, Tx_n, kt) \diamond N(Bx+n, Sy_n, kt)$$
$$+N(Sy_n, Tx_n, t) \diamond N(Ay_n, Sy_n, t)$$

$\diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Sy_n, t) \diamond N(Ay_n, Tx_n, (1+q)t)]$

$$M(Ay_n, Bx_n, kt) + P[M(Bx_n, Tx_n, kt) * M(Ay_n, Bx_n, kt)]$$
$$\geq \phi[PM(Ay_n, Bx_n, kt) * M(Bx_n, Tx_n, kt) + M(Ay_n, Tx_n, kt) * M(Bx_n, Bx_n, kt)$$
$$+M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, t)$$
$$*M(Bx_n, Bx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, qt)]$$

and

$$N(Ay)n, Bx_n, kt) + P[N(Bx_n, Tx_n, kt) \diamond N(Ay_n, Bx_n, kt)]$$
$$\leq \psi[PN(Ay_n, Bx_n, kt) \diamond N(Bx_n, Tx_n, kt) + N(Ay_n, Tx_n, kt) \diamond N(Bx_n, Bx_n, kt)$$
$$+N(Bx_n, Tx_n, t) \diamond N(Ay_n, Bx_n, t) \diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Bx_n, t)$$
$$\diamond N(Ay_n, Bx_n, t) \diamond N(Bx_n, Tx_n, qt)]$$

Thus it follows that,

$$M(Ay_n, Bx_n, kt) \geq \phi[M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t) * M(Bx_n, Tx_n, qt)]$$

and

$$N(Ay_n, Bx_n, kt) \leq \psi[N(Bx_n, Tx_n, t) \diamond N(Ay_n, Bx_n, t)) \diamond N(Bx_n, Tx_n, qt)]$$

*Since the t-norm * and t-conorm $\diamond$ is continuous and $M(x, y, \cdot)$ and $N(x, y, \cdot)$ is continuous, letting $q \longrightarrow 1$ we have,*

$$M(Ay_n, Bx_n, kt) \geq \phi[M(Bx_n, Tx_n, t) * M(Ay_n, Bx_n, t)]$$

and

$$N(Ay_n, Bx_n, kt) \leq \psi[N(Bx_n, Tx_n, t) \diamond N(Ay_n, Bx_n, t)]$$

It follows that

$$\begin{aligned} \lim_{n\longrightarrow\infty} M(Ay_n, Bx_n, kt) &\geq \phi[\lim_{n\longrightarrow\infty} M(Ay_n, Bx_n, t), M(\lim_{n\longrightarrow\infty} Ay_n, z, kt)] \\ &> M(\lim_{n\longrightarrow\infty} Ay_n, z, t) \end{aligned}$$

and

$$\begin{aligned} \lim_{n\longrightarrow\infty} N(Ay_n, Bx_n, kt) &\leq \psi[\lim_{n\longrightarrow\infty} N(Ay_n, Bx_n, t), N(\lim_{n\longrightarrow\infty} Ay_n, z, kt)] \\ &< N(\lim_{n\longrightarrow\infty} Ay_n, z, t) \end{aligned}$$

and we deduce that $\lim_{n\longrightarrow\infty} Ay_n = z$. Suppose $S(X)$ is a closed subset of X. Then $z = Su$ for some $u \in X$. Subsequently we have,

$$\lim_{n\longrightarrow\infty} Ay_n = \lim_{n\longrightarrow\infty} Bx_n = \lim_{n\longrightarrow\infty} Tx_n = \lim_{n\longrightarrow\infty} Sy_n = Su.$$

By (1.3) with $f\alpha = 1$, we have;

$$[1 + PM(Su, Tx_n, kt)] * M(Au, Bx_n, kt)$$
$$\geq \phi[PM(Au, Su, kt) * M(Bx_n, Tx_n, kt) + M(Bx_n, Su, t) * M(Au, Tx_n, t)$$
$$+M(Su, Tx_n, t) * M(Au, Su, t) * M(Bx_n, Tx_n, t) * M(Bx_n, Su, t) * M(Au, Tx_n, t)]$$

and

$$[1 + PN(Su, Tx_n, kt)] \diamond N(Au, Bx_n, kt)$$
$$\leq \psi[PN(Au, Su, kt) \diamond N(Bx_n, Tx_n, kt) + N(Bx_n, Su, t) \diamond N(Au, Tx_n, t) +$$
$$N(Su, Tx_n, t) \diamond N(Au, Su, t) \diamond N(Bx_n, Tx_n, t) \diamond N(Bx_n, Su, t) \diamond N(Au, Tx_n, t)]$$

$$M(Au, Bx_n, kt) + P[M(Su, Tx_n, kt) * M(Au, Bx_n, kt)]$$
$$\geq \phi[P[M(Au, Su, kt) * M(Bx_n, Tx_n, kt) + M(Au, Tx_n, kt) * M(Bx_n, Su, kt)]$$

$$+M(Su,Tx_n,kt)*M(Au,Su,t)*M(Bx_n,Tx_n,t)*M(Bx_n,Su,t)*M(Au,Tx_n,t)]$$

and

$$N(Au,Bx_n,kt)+P[N(Su,Tx_n,kt))\diamond N(Au,Bx_n,kt)]$$
$$\leq \psi[PN(Au,Su,kt)\diamond N(Bx_n,Tx_n,kt)+N(Au,Tx_n,kt)\diamond N(Bx_n,Su,kt)$$
$$+[N(Su,Tx_n,kt)\diamond N(Au,Su,t)\diamond N(Bx_n,Tx_n,t)\diamond N(Bx_n,Su,t))\diamond N(Au,Tx_n,t)]$$

Taking the $\lim_{n\longrightarrow\infty}$ *we have;*

$$M(Au,Su,kt)\geq \phi[PM(Au,Su,kt)*M(Su,Su,kt)+M(Su,Su,t)*$$
$$M(Au,Su,t)*M(Su,Su,t)*M(Su,Su,t)*M(Au,Su,t)]$$

and

$$N(Au,Su,kt) \leq \psi[PN(Au,Su,kt)\diamond N(Su,Su,kt)+N(Su,Su,t)\diamond$$
$$N(Au,Su,t)\Diamond N(Su,Su,t)\Diamond N(Su,Su,t)\Diamond N(Au,Su,t)]$$

This gives

$$M(Au,Su,kt)>M(Au,Su,t)$$

and

$$N(Au,Su,kt)<N(Au,Su,t)$$

Therefore by lemma 1, we have $Au=Su$. *I.e.* A *and* S *have a coincidence point. The weak compatibility of* A *and* S *implies that* $ASu=SAu$ *and then*

$$AAu=ASu=SAu=SSu.$$

On the other hand, since $A(X)\subset T(X)Y$, *there exists a point* $v\in X$ *such that* $Au=Tv$. *We claim that* $Tv=Bv$ *using (1.3) with* $\alpha=1$, *we have,*

$$[1+PM(Su,Tv,kt)]*M(Au,Bv,kt)$$
$$\geq \acute{\phi}[PM(Au,Su,kt)*M(Bv,Tv,kt)+M(Au,Tv,kt)*M(Bv,Su,kt)+$$
$$M(Su,Tv,t)*M(Au,Su,t)*M(Bv,Tv,t)*M(Bv,Su,t)*M(Au,Tv,t)]$$

and

$$[1+PN(Su,Tv,kt)]\Diamond N(Au,Bv,kt)$$
$$\leq \Psi[PN(Au,Su,kt)\Diamond N(Bv,Tv,kt)+N(Au,Tv,kt)\Diamond N(Bv,Su,kt)+N(Su,Tv,t)\Diamond$$
$$N(Au,Su,t)\Diamond N(Bv,Tv,t)\Diamond N(Bv,Su,t)\Diamond N(Au,Tv,t)]$$

$$M(Au,Bv,kt)+P[M(Su,Tv,kt)*M(Au,Bv,kt)]$$
$$\geq \acute{\phi}[PM(Au,Su,kt)*M(Bv,Tv,kt)+M(Au,Tv,kt)*M(Bv,Su,kt)+$$
$$M(Su,Tv,t)*M(Au,Su,t)*M(Bv,Tv,t)*M(Bv,Su,t)*M(Au,Tv,t)]$$

And

$$N(Au,Bv,kt)+P[N(Su,Tv,kt)N(Au,Bv,kt)]$$
$$\leq \Psi[PN(Au,Su,kt)\Diamond N(Bv,Tv,kt)+N(Au,Tv,kt).N(Bv,Su,kt)+$$
$$N(Su,Tv,t).N(Au,Su,t)\Diamond N(Bv,Tv,t)\Diamond N(Bv,Su,t)\Diamond N(Au,Tv,t)]$$

Thus it follows that, $M(Au,Bv,kt)>M(Au,Bv,t)$ *and*

$$N(Au,Bv,kt)<N(Au,Bv,t)$$

$$M(AAu,Bv,kt)+P[M(SAu,Tv,kt)*M(AAu,Bv,kt)]$$
$$\geq \acute{\phi}[PM(AAu,SAu,kt)*M(Bv,Tv,kt)+M(AAu,Tv,kt)*M(Bv,SAu,kt)$$
$$+M(SAu,Tv,t)*M(AAu,SAu,t)*M(Bv,Tv,t)*M(Bv,SAu,t)*M(AAu,Tv,t)]$$

and

$$\begin{aligned}&N(AAu,Bv,kt)+P[N(SAu,Tv,kt)\diamondsuit N(AAu,Bv,kt)]\\&\leq \Psi[PN(AAu,SAu,kt)\diamondsuit N(Bv,Tv,kt)+N(AAu,Tv,kt)\diamondsuit N(Bv,SAu,kt)\\&+N(SAu,Tv,t)\diamondsuit N(AAu,SAu,t)\diamondsuit N(Bv,Tv,t)\diamondsuit N(Bv,SAu,t)\diamondsuit N(AAu,Tv,t)]\end{aligned}$$

$$\begin{aligned}&M(AAu,Au,kt)+P[M(AAu,Au,kt)*M(AAu,Au,kt)]\\&\geq \acute{\phi}[PM(AAu,AAu,kt)*M(Au,Au,kt)+M(AAu,Au,kt)*M(Au,AAu,kt)\\&+M(AAu,Au,t)*M(AAu,AAu,t)*M(Au,Au,t)\\&*M(Au,Au,t)*M(AAu,Au,t)]\end{aligned}$$

and

$$\begin{aligned}&N(AAu,Au,kt)+P[N(AAu,Au,kt)\diamondsuit N(AAu,Au,kt)]\\&\leq \Psi[PN(AAu,AAu,kt)\diamondsuit N(Au,Au,kt)+N(AAu,Au,kt)\diamondsuit N(Au,AAu,kt)\\&+N(AAu,Au,t)\diamondsuit N(AAu,AAu,t)\diamondsuit N(Au,Au,t)\\&\diamondsuit N(Au,AAu,t)\diamondsuit N(AAu,Au,t)]\end{aligned}$$

Thus it follows that

$$M(AAu,Au,kt)>M(AAu,Au,t)$$

and

$$N(AAu,Au,kt)<N(AAu,Au,t)$$

Therefore by lemma 1, we have $Au = AAu = SAu$. I.e. Au is a common fixed point of A and S. Similarly, we prove that Bv is a common fixed point of B and T. Since $Au = Bv$, we conclude that Au is a common fixed point of A, B, S and T. If $Au = Bu = Su = Tu = u$ and $Av = Bv = Sv = Tv = v$, then by (1.3) with $\alpha = 1$, we have;

$$\begin{aligned}&[1+PM(Su,Tv,kt)]*(Au,Bv,kt)\\&\geq \acute{\phi}[PM(Au,Su,kt)*M(Bv,Tv,kt)+M(Au,Tv,kt)*M(Bv,Su,kt)+\\&M(Su,Tv,t)*M(Au,Su,t)*M(Bv,Tv,t)*M(Bv,Su,t)*M(Au,Tv,t)]\end{aligned}$$

$$\begin{aligned}&[1+PN(Su,Tv,kt)]\diamondsuit(Au,Bv,kt)\\&\leq \Psi[PN(Au,Su,kt)\diamondsuit N(Bv,Tv,kt)+N(Au,Tv,kt)\diamondsuit N(Bv,Su,kt)+\\&N(Su,Tv,t).N(Au,Su,t)\diamondsuit N(Bv,Tv,t)\diamondsuit N(Bv,Su,t)\diamondsuit N(Au,Tv,t)]\end{aligned}$$

$$\begin{aligned}&M(u,v,kt)+P[M(u,v,kt)*M(u,v,kt)]\\&\geq \acute{\phi}[PM(u,v,kt)*M(v,v,kt)+M(u,v,kt)*M(v,u,kt)+M(u,v,t)*\\&M(u,u,t)*M(v,v,t)*M(v,u,t)*M(u,v,t)]\end{aligned}$$

and

$$\begin{aligned}&N(u,v,kt)+P[N(u,v,kt).N(u,v,kt)]\\&\leq \Psi[PN(u,u,kt)\diamondsuit N(v,v,kt)+N(u,v,kt)\diamondsuit N(v,u,kt)+N(u,v,t)\\&\diamondsuit N(u,u,t)\diamondsuit N(v,v,t)\diamondsuit N(v,u,t)\diamondsuit N(u,v,t)]\end{aligned}$$

$$M(u,v,kt)>M(u,v,t)$$

and

$$N(u,v,kt)<N(u,v,t)$$

By lemma 1, we have $u = v$. Hence the common fixed point is a unique. This completes the proof of the theorem.

Corollary 2.2. *Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space with $a*a \geq a$ and $(1-a)\diamondsuit(1-a) \leq (1-a)$ for all $a \in (0,1)$, let A, B, S, and T be self mappings of X into itself such that;*

(1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$

(1.2) (A, S) or (B, T) satisfies the property $(S-B)$.

(1.3) there exists a number $k \in (0,1)$ such that

$$M(Ax, By, kt) \geq \acute{\phi}[M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, t) * M(Ax, Ty, (2-\alpha)t)]$$

and

$$N(Ax, By, kt) \leq \Psi[N(Sx, Ty, t)\diamondsuit N(Ax, Sx, t)\diamondsuit N(By, Ty, t)\diamondsuit N(By, Sx, t)\diamondsuit N(Ax, Ty, (2-\alpha)t)]$$

for all $x, y \in X$, $P \geq 0, \alpha \in (0,2)$ and $t > 0$. Where $\acute{\phi}, \Psi : [0,1] \to [0,1]$ is continuous function such that $\acute{\phi}(S) > S$ and $f\Psi(S) < S$ for each $0 < S < 1$ with $M(x, y, t) > 0$.

(1.4) the pairs $\{A, S\}$ and (B, T) are weakly compatible;

(1.5) one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X. Then A, B, S, and T have a unique common fixed point in X.

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