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FIXED POINT THEOREMS IN NON-ARCHIMEDEAN MENGER PM-SPACES

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ABSTRACT. Recently, Sintunavarat and Kumam [Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces, J. Appl. Math. vol. 2011, Article ID 637958, 14 pages, 2011] defined the notion of $(CLRg)$ property which is more general than property (E.A). In the present paper, we prove a common fixed point theorem for a pair of weakly compatible mappings in Non-Archimedean Menger probabilistic metric spaces by using $(CLRg)$ property. As an application to our main result, we present a common fixed point theorem for two finite families of self mappings. Our results improve and extend several known results existing in the literature.

KEYWORDS : t-Norm; Non-Archimedean Menger probabilistic metric space; Weakly compatible mappings, Property (E.A); $(CLRg)$ property; Fixed point.
AMS Subject Classification: 47H10, 54H25

1. INTRODUCTION

In 1974, Istrătescu and Crivăt [10] introduced the concept of Non-Archimedean probabilistic metric spaces (shortly, N.A. PM-spaces) (see [9, 11]). Some fixed point theorems on N.A. Menger PM-spaces have been established by Istrătescu [7, 8] as a generalization of the results of Sehgal and Bharucha-Reid [20]. Further, Hadžić [5] studied the results of Istrătescu [7, 8].

In 1987, Singh and Pant [26] introduced the notion of weakly commuting mappings in N.A. Menger PM-spaces and proved some common fixed point theorems. Afterwards, Dimri and Pant [4] studied the application of N.A. Menger PM-spaces to product spaces. Jungck and Rhoades [12, 13] weakened the notion of compatible mappings by introducing the notion of weak compatibility and proved fixed point theorems without any requirement of continuity of the involved mappings. Many mathematicians proved common fixed point theorems in N.A. Menger PM-spaces

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using different contractive conditions (see [2, 3, 4, 14, 15, 16, 21, 22, 24, 25, 27]). In 2002, Aamri and El Moutawakil [1] defined the notion of property (E.A) which contained the class of non-compatible mappings. It is observed that property (E.A) requires the completeness (or closedness) of the underlying space (or subspaces) for the existence of the fixed points. Recently, Sintunavarat and Kumam [28] defined the notion of "common limit in the range property" with respect to mapping g (briefly, $(CLRg)$ property) in fuzzy metric spaces. They showed that $(CLRg)$ property never requires the closedness of the subspace (also see [29]).

The aim of this paper is to prove a common fixed point theorem for a pair of weakly compatible mappings in N.A. Menger PM-spaces employing $(CLRg)$ property. We give an example to support the useability of our main result. As an application to our main result, we present a common fixed point theorem for two finite families of self mappings.

2. PRELIMINARIES

Definition 2.1. [19] A triangular norm (shortly, t-norm) $\mathcal{T}$ is a binary operation on the unit interval [0,1] such that for all $a, b, c, d \in [0, 1]$ and the following conditions are satisfied:

(i) $\mathcal{T}(a, 1) = a$;
(ii) $\mathcal{T}(a, b) = \mathcal{T}(b, a)$;
(iii) $\mathcal{T}(a, b) \leq \mathcal{T}(c, d)$, whenever $a \leq c$ and $b \leq d$;
(iv) $\mathcal{T}(a, \mathcal{T}(b, c)) = \mathcal{T}(\mathcal{T}(a, b), c)$.

Definition 2.2. [19] A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is said to be a distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$. We shall denote $\Im$ by the set of all distribution functions.

If X is a non-empty set, $\mathcal{F} : X \times X \to \Im$ is called a probabilistic distance on X and $F(x, y)$ is usually denoted by $F_{x,y}$ for all $x, y \in X$.

Definition 2.3. [8, 10] The ordered pair $(X, \mathcal{F})$ is said to be non-Archimedean probabilistic metric space (shortly N.A. PM-space) if X is a non-empty set and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, t_1, t_2 > 0$,

(i) $F_{x,y}(t) = 1 \Leftrightarrow x = y$;
(ii) $F_{x,y}(t) = F_{y,x}(t)$;
(iii) $F_{x,y}(0) = 0$;
(iv) If $F_{x,y}(t_1) = 1$ and $F_{y,z}(t_2) = 1$ then $F_{x,z}(\max\{t_1, t_2\}) = 1$.

The ordered triplet $(X, \mathcal{F}, \mathcal{T})$ is called a N.A. Menger PM-space if $(X, \mathcal{F})$ is a N.A. PM-space, $\mathcal{T}$ is a t-norm and the following inequality holds:

$$F_{x,z}(\max\{t_1, t_2\}) \geq \mathcal{T}(F_{x,y}(t_1), F_{y,z}(t_2)),$$

for all $x, y, z \in X$ and $t_1, t_2 > 0$.

Example 2.4. Let X be any set with at least two elements. If we define $F_{x,x}(t) = 1$ for all $x \in X, t > 0$ and

$$F_{x,y}(t) = \begin{cases} 0, & \text{if } t \leq 1; \\ 1, & \text{if } t > 1, \end{cases}$$

where $x, y \in X, x \neq y$, then $(X, \mathcal{F}, \mathcal{T})$ is a N.A. Menger PM-space with $\mathcal{T}(a, b) = \min\{a, b\}$ or (ab) for all $a, b \in [0, 1]$.

Example 2.5. Let $X = \mathbb{R}$ be the set of real numbers equipped with metric defined by $d(x, y) = |x - y|$ and

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

Then $(X, \mathcal{F}, \mathcal{T})$ is a N.A. Menger PM-space with $\mathcal{T}$ as continuous t-norm satisfying $\mathcal{T}(a, b) = \min\{a, b\}$ or (ab) for all $a, b \in [0, 1]$.

Definition 2.6. [3] A N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ is said to be of type $(C)_{\mathfrak{g}}$ if there exists a $\mathfrak{g} \in \Omega$ such that

$$\mathfrak{g}(F_{x,z}(t)) \le \mathfrak{g}(F_{x,y}(t)) + \mathfrak{g}(F_{y,z}(t)),$$

for all $x, y, z \in X, t \ge 0$, where $\Omega = \{\mathfrak{g} \mid \mathfrak{g} : [0, 1] \to [0, \infty)$ is continuous, strictly decreasing with $\mathfrak{g}(1) = 0$ and $\mathfrak{g}(0) < \infty\}$.

Definition 2.7. [3] A N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ is said to be of type $(D)_{\mathfrak{g}}$ if there exists a $\mathfrak{g} \in \Omega$ such that

$$\mathfrak{g}(\mathcal{T}(t_1, t_2)) \le \mathfrak{g}(t_1) + \mathfrak{g}(t_2),$$

for all $t_1, t_2 \in [0, 1]$.

Remark 2.8. [3]

(i) If a N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ is of type $(D)_{\mathfrak{g}}$ then it is of type $(C)_{\mathfrak{g}}$.
(ii) If a N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ is of type $(D)_{\mathfrak{g}}$, then it is metrizable, where the metric d on X is defined by

$$d(x, y) = \int_0^1 \mathfrak{g}(F_{x,y}(t))dt,$$

for all $x, y \in X$.

Throughout this paper $(X, \mathcal{F}, \mathcal{T})$ is a N.A. Menger PM-space with a continuous strictly increasing t-norm $\mathcal{T}$.

Let $\phi : [0, \infty) \to [0, \infty)$ be a function satisfying the condition (Φ): ϕ is upper semi-continuous from the right and $\phi(t) < t$ for $t > 0$.

Lemma 2.9. [3] *If a function $\phi : [0, \infty) \to [0, \infty)$ satisfies the condition (Φ) then we have:*

(i) *for all $t \ge 0, \lim_{n\to\infty} \phi^n(t) = 0$, where $\phi^n(t)$ is the n^{th} iteration of $\phi(t)$.*
(ii) *If $\{t_n\}$ is a non-decreasing sequence of real numbers and $t_{n+1} \le \phi(t_n)$ where $n = 1, 2, \ldots$ then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \le \phi(t)$, for each $t \ge 0$ then $t = 0$.*

Definition 2.10. A pair (f, g) of self mappings of a N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ is said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z,$$

for some $z \in X$.

Definition 2.11. [18] A pair (f, g) of self mappings of a non-empty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is, if $fz = gz$ for some $z \in X$, then $fgz = gfz$.

It is known that a pair (f, g) of compatible mappings is weakly compatible but converse is not true in general.

Remark 2.12. It is noticed that the concepts of weak compatibility and property (E.A) are independent to each other (see [17, Example 2.2]).

Inspired by Sintunavarat and Kumam [28], we define the "common limit in the range property" with respect to mapping g in N.A. Menger PM-space as follows:

Definition 2.13. A pair (f, g) of self mappings of a N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ is said to satisfy the $(CLRg)$ property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$.

Now, we show examples of self mappings f and g which are satisfying the $(CLRg)$ property.

Example 2.14. Let $(X, \mathcal{F}, \mathcal{T})$ be a N.A. Menger PM-space, where $X = [1, \infty)$ and metric d is defined as condition (2) of Remark 2.8. Define self mappings f and g on X by $f(x) = x + 2$ and $g(x) = 3x$ for all $x \in X$. Let a sequence $\{x_n\} = \{1 + \frac{1}{n}\}_{n\in\mathbb{N}}$ in X, we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 3 = g(1) \in X,$$

which shows that f and g satisfy the $(CLRg)$ property.

Example 2.15. Let $(X, \mathcal{F}, \mathcal{T})$ be a N.A. Menger PM-space, where $X = [0, \infty)$ and metric d is defined as condition (2) of Remark 2.8. Define self mappings f and g on X by $f(x) = \frac{x}{2}$ and $g(x) = \frac{2x}{3}$ for all $x \in X$. Let a sequence $\{x_n\} = \{\frac{1}{n}\}_{n\in\mathbb{N}}$ in X. Since

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 0 = g(0) \in X,$$

therefore f and g satisfy the $(CLRg)$ property.

Definition 2.16. [6] Two families of self mappings $\{f_i\}$ and $\{g_j\}$ are said to be commuting pairwise if:

(i) $f_i f_j = f_j f_i$, $i, j \in \{1, 2, \ldots, m\}$,
(ii) $g_k g_l = g_l g_k$, $k, l \in \{1, 2, \ldots, n\}$,
(iii) $f_i g_k = g_k f_i$, $i \in \{1, 2, \ldots, m\}$, $k \in \{1, 2, \ldots, n\}$.

3. RESULTS

Theorem 3.1. *Let $(X, \mathcal{F}, \mathcal{T})$ be a N.A. Menger PM-space and the pair (f, g) of self mappings is weakly compatible such that*

$$\mathfrak{g}(F_{fx,fy}(t)) \leq \phi\left(\max\left\{\begin{array}{c}\mathfrak{g}(F_{gx,gy}(t)), \mathfrak{g}(F_{fx,gx}(t)), \mathfrak{g}(F_{fy,gy}(t)),\\ \frac{1}{2}\left(\mathfrak{g}(F_{gx,fy}(t)) + \mathfrak{g}(F_{fx,gy}(t))\right)\end{array}\right\}\right), \tag{3.1}$$

holds for all $x, y \in X$, $t > 0$, where $\mathfrak{g} \in \Omega$ and ϕ satisfies the condition (Φ). If f and g satisfy the $(CLRg)$ property, then f and g have a unique common fixed point.

Proof. Since the pair (f, g) satisfies the $(CLRg)$ property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$. We assert that $fu = gu$. On using inequality (3.1) with $x = x_n$, $y = u$, we get

$$\mathfrak{g}(F_{fx_n,fu}(t)) \leq \phi\left(\max\left\{\begin{array}{c}\mathfrak{g}(F_{gx_n,gu}(t)), \mathfrak{g}(F_{fx_n,gx_n}(t)), \mathfrak{g}(F_{fu,gu}(t)),\\ \frac{1}{2}\left(\mathfrak{g}(F_{gx_n,fu}(t)) + \mathfrak{g}(F_{fx_n,gu}(t))\right)\end{array}\right\}\right),$$

passing to limit as $n \to \infty$, we have

$$\mathfrak{g}(F_{gu,fu}(t)) \quad \leq \quad \phi\left(\max\left\{\begin{array}{c}\mathfrak{g}(F_{gu,gu}(t)), \mathfrak{g}(F_{gu,gu}(t)), \mathfrak{g}(F_{fu,gu}(t)),\\ \frac{1}{2}\left(\mathfrak{g}(F_{gu,fu}(t)) + \mathfrak{g}(F_{gu,gu}(t))\right)\end{array}\right\}\right)$$

$$\begin{aligned} &= \phi\left(\max\left\{\ \mathfrak{g}(1), \mathfrak{g}(1), \mathfrak{g}(F_{fu,gu}(t)), \tfrac{1}{2}\left(\mathfrak{g}(F_{gu,fu}(t)) + \mathfrak{g}(1)\right)\ \right\}\right) \\ &= \phi\left(\max\left\{0, 0, \mathfrak{g}(F_{fu,gu}(t)), \frac{1}{2}\left(\mathfrak{g}(F_{gu,fu}(t))\right)\right\}\right) \\ &= \phi(\mathfrak{g}(F_{fu,gu}(t))), \end{aligned}$$

for all $t > 0$, which implies that $\mathfrak{g}(F_{fu,gu}(t)) = 0$. By Lemma 2.9, we get $fu = gu$.

Next, we let $z = fu = gu$. Since the pair (f, g) is weakly compatible, $fgu = gfu$ which implies that $fz = fgu = gfu = gz$. Now we show that $z = fz$. On using inequality (3.1) with $x = z$, $y = u$, we get

$$\mathfrak{g}(F_{fz,fu}(t)) \leq \phi\left(\max\left\{\begin{array}{c} \mathfrak{g}(F_{gz,gu}(t)), \mathfrak{g}(F_{fz,gz}(t)), \mathfrak{g}(F_{fu,gu}(t)), \\ \frac{1}{2}\left(\mathfrak{g}(F_{gz,fu}(t)) + \mathfrak{g}(F_{fz,gu}(t))\right) \end{array}\right\}\right),$$

and so

$$\begin{aligned} \mathfrak{g}(F_{fz,z}(t)) &\leq \phi\left(\max\left\{\begin{array}{c} \mathfrak{g}(F_{fz,z}(t)), \mathfrak{g}(F_{fz,fz}(t)), \mathfrak{g}(F_{z,z}(t)), \\ \frac{1}{2}\left(\mathfrak{g}(F_{fz,z}(t)) + \mathfrak{g}(F_{fz,z}(t))\right) \end{array}\right\}\right) \\ &= \phi\left(\max\left\{\mathfrak{g}(F_{fz,z}(t)), \mathfrak{g}(1), \mathfrak{g}(1), \frac{1}{2}\left(\mathfrak{g}(F_{fz,z}(t)) + \mathfrak{g}(F_{fz,z}(t))\right)\right\}\right) \\ &= \phi\left(\max\left\{\mathfrak{g}(F_{fz,z}(t)), 0, 0, \mathfrak{g}(F_{fz,z}(t))\right\}\right) \\ &= \phi(\mathfrak{g}(F_{fz,z}(t))), \end{aligned}$$

for all $t > 0$, which implies that $\mathfrak{g}(F_{fz,z}(t)) = 0$. By Lemma 2.9, we have $fz = z$ and so $z = fz = gz$. Therefore z is a common fixed point of f and g.

Uniqueness: Let $w (\neq z)$ be another common fixed point of f and g. On using inequality (3.1) with $x = z$, $y = w$, we have

$$\mathfrak{g}(F_{fz,fw}(t)) \leq \phi\left(\max\left\{\begin{array}{c} \mathfrak{g}(F_{gz,gw}(t)), \mathfrak{g}(F_{fz,gz}(t)), \mathfrak{g}(F_{fw,gw}(t)), \\ \frac{1}{2}\left(\mathfrak{g}(F_{gz,fw}(t)) + \mathfrak{g}(F_{fz,gw}(t))\right) \end{array}\right\}\right),$$

or

$$\begin{aligned} \mathfrak{g}(F_{z,w}(t)) &\leq \phi\left(\max\left\{\begin{array}{c} \mathfrak{g}(F_{z,w}(t)), \mathfrak{g}(F_{z,z}(t)), \mathfrak{g}(F_{w,w}(t)), \\ \frac{1}{2}\left(\mathfrak{g}(F_{z,w}(t)) + \mathfrak{g}(F_{z,w}(t))\right) \end{array}\right\}\right) \\ &= \phi\left(\max\left\{\mathfrak{g}(F_{z,w}(t)), \mathfrak{g}(1), \mathfrak{g}(1), \frac{1}{2}\left(\mathfrak{g}(F_{z,w}(t)) + \mathfrak{g}(F_{z,w}(t))\right)\right\}\right) \\ &= \phi\left(\max\left\{\mathfrak{g}(F_{z,w}(t)), 0, 0, \mathfrak{g}(F_{z,w}(t))\right\}\right) \\ &= \phi(\mathfrak{g}(F_{z,w}(t))), \end{aligned}$$

for all $t > 0$, which implies that $\mathfrak{g}(F_{z,w}(t)) = 0$. By Lemma 2.9, we get $z = w$. Therefore f and g have a unique a common fixed point. □

Remark 3.2. From the result, it is asserted that $(CLRg)$ property never requires any condition on closedness of the subspace, continuity of one or more mappings and containment of ranges of the involved mappings.

Our next theorem is proved for a pair of weakly compatible mappings in N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ using property (E.A).

Theorem 3.3. *Let $(X, \mathcal{F}, \mathcal{T})$ be a N.A. Menger PM-space and the pair of self mappings (f, g) is weakly compatible satisfying inequality (3.1) of Theorem 3.1. If f and g satisfy the property (E.A) and the range of g is a closed subspace of X, then f and g have a unique common fixed point.*

Proof. Since the pair (f, g) satisfies the (E.A) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z,$$

for some $z \in X$. It follows from $g(X)$ being a closed subspace of X that there exists $u \in X$ such that $z = gu$. Therefore f and g satisfy the $(CLRg)$ property. It follows from Theorem 3.1 that there exists a unique common fixed point of f and g. □

Remark 3.4. Theorem 3.1 improves the results of Cho et al. [3], Singh et al. [24, Theorem 3.1, Corollary 3.3], Singh et al. [23, Theorem 3.1, Theorem 3.2], Singh et al. [25, Theorem 3.1, Corollary 3.1] and Singh and Dimri [22, Corollary 3.1] without any requirement of completeness (or closedness) of the underlying space (or subspaces), continuity of the mappings and containment of ranges of the involved mappings. Theorem 3.1 also generalize the results of Rao and Ramudu [18, Theorem 14].

The following example illustrates Theorem 3.1.

Example 3.5. Let $(X, \mathcal{F}, \mathcal{T})$ be a N.A. Menger PM-space, where $X = [1, 15)$ and metric d is defined as condition (2) of Remark 2.8. Define the self mappings f and g by

$$f(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3, 15); \\ 8, & \text{if } x \in (1, 3]. \end{cases} \quad g(x) = \begin{cases} 1, & \text{if } x = 1; \\ 7, & \text{if } x \in (1, 3]; \\ \frac{x+1}{4}, & \text{if } x \in (3, 15). \end{cases}$$

Taking $\{x_n\} = \left\{3 + \frac{1}{n}\right\}_{n \in \mathbb{N}}$ or $\{x_n\} = \{1\}$, it is clear that the pair (f, g) satisfies the $(CLRg)$ property.

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 1 = g(1) \in X.$$

It is noted that $f(X) = \{1, 8\} \nsubseteq [1, 4) \cup \{7\} = g(X)$. Thus, all the conditions of Theorem 3.1 are satisfied and 1 is a unique common fixed point of the pair (f, g). Also, all the involved mappings are even discontinuous at their unique common fixed point 1. Here, it is pointed out that $g(X)$ is not a closed subspace of X.

Now we utilize the notion of commuting pairwise and extend Theorem 3.1 to two finite families of self mappings in N.A. Menger PM-space.

Corollary 3.6. *Let $\{f_1, f_2, \ldots, f_p\}$ and $\{g_1, g_2, \ldots, g_q\}$ be two finite families of self mappings of a N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$ such that $f = f_1 f_2 \ldots f_p$ and $g = g_1 g_2 \ldots g_q$ which also satisfy inequality (3.1) of Theorem 3.1. Suppose that the pair (f, g) satisfies the $(CLRg)$ property.*

Moreover, if the family $\{f_i\}_{i=1}^{p}$ commutes pairwise with the family $\{g_i\}_{j=1}^{q}$, then (for all $i \in \{1, 2, \ldots, p\}$ and $j \in \{1, 2, \ldots, q\}$) f_i and g_j have a unique common fixed point.

Remark 3.7. Corollary 3.6 improves and extends the result of Singh and Dimri [22, Theorem 3.1].

By setting $f_1 = f_2 = \ldots = f_p = f$ and $g_1 = g_2 = \ldots = g_q = g$ in Corollary 3.6, we deduce the following:

Corollary 3.8. *Let f and g be self mappings of a N.A. Menger PM-space $(X, \mathcal{F}, \mathcal{T})$. Suppose that the pair (f^p, g^q) satisfies the $(CLRg)$ property such that*

$$\mathfrak{g}(F_{f^p x, f^p y}(t)) \le \phi\left(\max\left\{\begin{array}{c} \mathfrak{g}(F_{g^q x, g^q y}(t)), \mathfrak{g}(F_{f^p x, g^q x}(t)), \mathfrak{g}(F_{f^p y, g^p y}(t)), \\ \frac{1}{2}\left(\mathfrak{g}(F_{g^q x, f^p y}(t)) + \mathfrak{g}(F_{f^p x, g^q y}(t))\right) \end{array}\right\}\right), \quad (3.2)$$

holds for all $x, y \in X$, $t > 0$, $\mathfrak{g} \in \Omega$ where ϕ satisfies the condition (Φ) and p, q are fixed positive integers. Then f and g have a unique common fixed point provided $fg = gf$.

Remark 3.9. The conclusion of Theorem 3.1 remains true if we replace inequality (3.1) by one of the following:

$$\mathfrak{g}(F_{fx,fy}(t)) \leq \phi\left(\max\left\{\mathfrak{g}(F_{gx,gy}(t)), \mathfrak{g}(F_{fx,gx}(t)), \mathfrak{g}(F_{fy,gy}(t)), \mathfrak{g}(F_{gx,fy}(t))\right\}\right), \tag{3.3}$$

for all $x, y \in X$, $t > 0$, where $\mathfrak{g} \in \Omega$ and ϕ satisfies the condition (Φ).
And

$$\mathfrak{g}(F_{fx,fy}(t)) \leq \phi\left(\max\left\{\mathfrak{g}(F_{gx,gy}(t)), \mathfrak{g}(F_{fx,gx}(t)), \mathfrak{g}(F_{fy,gy}(t))\right\}\right), \tag{3.4}$$

for all $x, y \in X$, $t > 0$, where $\mathfrak{g} \in \Omega$ and ϕ satisfies the condition (Φ).

Remark 3.10. Notice that results similar to Corollary 3.6 and Corollary 3.8 can also be outlined in respect of Remark 3.9 but we omit the details with a view to avoid any repetition.

Remark 3.11. The results (in view of Remark 3.9) improve the results of Khan and Sumitra [15, Theorem 2, Corollary 1], Singh et al. [23, Corollary 3.3, Corollary 3.4] and Singh et al. [21, Theorem 3.1].

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