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SOME NOTES ON (α, β)-GENERALIZED HYBRID MAPPINGS

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ABSTRACT. In 1973, Bruck generalized the notion nonexpansive mappings by introducing firmly nonexpansive mappings. Kohsaka and Takahashi introduced nonspreading mappings in 2008 and Takahashi introduced hybrid mappings in 2010. It is worth noting that each nonexpansive mapping is a 1-hybrid mapping and each nonspreading mapping is a 0-hybrid mapping. Thus, the notion of λ-hybrid mappings is a generalization of the notions of firmly nonexpansive mappings and nonspreading mappings. In 2011, Takahashi introduced generalized hybrid mappings and Aoyama and Kohsaka defined α-nonexpansive mappings on Banach spaces. Kocourek, Takahashi and Yao gave the notions of $(\alpha, \alpha - 1)$-generalized hybrid mappings and (α, β, γ)-super hybrid mappings. In this paper, we discuss (α, β)-generalized hybrid mappings. By using and combining ideas of some recent papers, we generalize the notion of α-nonexpansivity to (α, β)-nonexpansivity and give some results on the subject.

KEYWORDS : Ideal; Filter; Sequence of moduli; Lipschitz function; I-convergence field; I-convergent; Monotone; Solid spaces

1. INTRODUCTION

Let $(H, < ., . >)$ be a Hilbert space and C a nonempty subset of H. In 1973, Bruck generalized the notion nonexpansive mappings by introducing firmly nonexpansive mappings ([4]). We say that $T : C \longrightarrow H$ is a firmly nonexpansive mapping whenever $\|Tx - Ty\| \leq \|r(x - y) + (1 - r)(Tx - Ty)\|$ for all $r > 0$ and $x, y \in C$. A mapping $T : C \longrightarrow H$ is said to be quasi-nonexpansive whenever $F(T)$ is a nonempty set and $\|Tx - z\| \leq \|x - z\|$ for all $x \in C$ and $z \in F(T)$. In 2008, Kohsaka and Takahashi introduced nonspreading mappings ([8]). In 2010, Kurokawa and Takahashi proved some weak and strong convergence theorems for nonspreading mappings in Hilbert spaces ([9]). Later, Aoyama and Kohsaka generalized some of their results in 2011 ([2]). On the other hand, Aoyama, Iemoto, Kohsaka and Takahashi proved some fixed point results about λ-hybrid

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mappings ($\lambda \in \mathbb{R}$) ([1]). A mapping $T : C \longrightarrow H$ is said to be λ-hybrid if $2\|Tx - Ty\|^2 \leq \|x - Ty\|^2 + \|y - Tx\|^2 - 2\lambda Re < x - Tx, y - Ty >$ or equivalently $\|Tx - Ty\|^2 \leq \|x - y\|^2 + 2(1 - \lambda)Re < x - Tx, y - Ty >$ for all $x, y \in C$. In fact, each nonexpansive mapping is a 1-hybrid mapping ([1]) and each nonspreading mapping is a 0-hybrid mapping ([2]). Also, T is $\frac{1}{2}$-hybrid if and only if T is a hybrid mapping in the sense of [13] (see for example, [2]). Let $\kappa \in [0, 1)$. A mapping $T : C \longrightarrow H$ is said to be κ-strictly pseudononspreading if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + 2Re < x - Tx, y - Ty > +\kappa\|x - Tx - (y - Ty)\|^2$$

for all $x, y \in C$ ([11]). Let $0 \leq \kappa \leq \beta < 1$ and T be a κ-strictly pseudononspreading mapping. Then, $T_\beta = \beta I + (1 - \beta)T$ is a $\frac{-\beta}{1-\beta}$-hybrid mapping ([2] and [11]). Recently, Takahashi introduced generalized hybrid mappings and proved some weak convergence theorems for generalized hybrid mappings in Banach spaces ([14]). In 2010, Klin-eam and Suantai, by using a multiindex $\alpha = (\alpha_1, \cdots, \alpha_n) \in \mathbb{R}^n$ satisfying $\alpha_i \geq 0$ ($i = i, 2, \cdots, n$) and $\sum_{i=1}^n \alpha_i = 1$, introduced α-nonexpansive mappings and proved some fixed point results for the mappings ([6]). In 2011, Aoyama and Kohsaka introduced α-nonexpansive mappings on Banach spaces in a different form and provided some fixed point theorems for α-nonexpansive mappings ([3]). Let E be a Banach space, C a nonempty subset of E and α a real number such That $\alpha < 1$. A mapping $T : C \longrightarrow E$ is said to be α-nonexpansive if $\|Tx - Ty\|^2 \leq \alpha\|Tx - y\|^2 + \alpha\|x - Ty\|^2 + (1 - 2\alpha)\|x - y\|^2$ for all $x, y \in C$. Aoyama and Kohsaka proved that for $\lambda < 2$, T is a λ-hybrid mapping if and only if T is a $\frac{1-\alpha}{2-\alpha}$-nonexpansive mapping (see Proposition 2.2 in [3]). Let l^∞ be the Banach space of bounded real sequences with the supremum norm. It is known that there exists a bounded linear functional μ on l^∞ such that $\mu(\{t_n\}) \geq 0$ for all $\{t_n\} \in l^\infty$ with $t_n \geq 0$ ($n \geq 1$), $\mu(\{t_n\}) = 1$ for all $\{t_n\} \in l^\infty$ with $t_n = 1$ ($n \geq 1$) and $\mu(\{t_{n+1}\}) = \mu(\{t_n\})$ for all $\{t_n\} \in l^\infty$. The functional μ is called Banach limit and the value of μ at $\{t_n\} \in l^\infty$ is denoted by $\mu_n t_n$ ([3] and [12]). In this paper, we give some results on (α, β)-generalized hybrid mapping. Also, by using and combining ideas of [1], [2], [3], [13] and [14], we generalize the notion of α-nonexpansivity to (α, β)-nonexpansivity and give some results about the subject. Finally, we appeal the following result which has been proved in [3].

Lemma 1.1. *Let C be a nonempty, closed and convex subset of a uniformly convex Banach space E and T a selfmap on C such that $\mu_n\|T^n x - Ty\|^2 \leq \mu_n\|T^n x - y\|^2$ for all $y \in C$. Then T has a fixed point.*

2. MAIN RESULTS

Now, we are ready to state and prove our main results. Our first result is another version of Theorem 3.1 in [7].

Theorem 2.1. *Let C be a nonempty, closed and convex subset of a Hilbert space H and T a selfmap on C such that*

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + 2\gamma Re < x - Ty, y - Tx > +k\|x - Tx - (y - Ty)\|^2$$

for all $x, y \in C$, where $\gamma + 2k < 0$. Then, T has a fixed point in C if and only if $\{T^n z\}$ is a bounded sequence for some $z \in C$.

Proof. Let $z \in F(T)$. Then $\{T^n z\} = \{z\}$ and so $\{T^n z\}$ is bounded. Now, suppose that there exists $z \in C$ such that $\{T^n z\}$ is bounded. Then, for each $x, y \in C$ we have

$$\| Tx - Ty \|^2 \leq \| x - y \|^2 + \gamma(\| x - Ty \|^2 + \| y - Tx \|^2)$$

$$+k\| x-y \|^2 + k\| Tx-Ty \|^2 + 2kRe\langle x-y, Ty-Tx\rangle.$$

Hence,

$$\| Tx-Ty \|^2 \le \| x-y \|^2 + \gamma\| x-Ty \|^2 + \gamma\| y-Tx \|^2$$
$$+k\| x-y \|^2 + k\| Tx-Ty \|^2 + k\| x-y \|^2 + k\| Tx-Ty \|^2$$

for all $x,y \in C$. Let μ be the Banach limit. Since μ is a positive linear functional on l^∞, for each $y \in C$ and $n \ge 0$ we have

$$\mu_n\| T^{n+1}z - Ty \|^2 \le \mu_n\| T^n z - y \|^2 + \gamma\mu_n\| T^n z - Ty \|^2 + \gamma\mu_n\| y - T^{n+1}z \|^2$$
$$+k\mu_n\| T^n x - y \|^2 + k\mu_n\| T^{n+1}z - Ty \|^2 + k\mu_n\| T^n z - y \|^2 + k\mu_n\| T^{n+1}z - Ty \|^2.$$

Thus, by using the property of μ we obtain

$$\mu_n\| T^n z - Ty \|^2 \le \mu_n\| T^n z - y \|^2 + \gamma\mu_n\| T^n z - Ty \|^2 + \gamma\mu_n\| y - T^n z \|^2$$
$$+k\mu_n\| T^n z - y \|^2 + k\mu_n\| T^n z - Ty \|^2 + k\mu_n\| T^n z - y \|^2 + k\mu_n\| T^n z - Ty \|^2.$$

Hence, $(1-\gamma-2k)\mu_n \, \| T^n z - Ty \|^2 \le (1+\gamma+2k)\mu_n \, \| T^n z - y \|$ and so

$$\mu_n\| T^n z - Ty \|^2 \le \frac{1+\gamma+2k}{1-\gamma-2k}\mu_n \, \| T^n z - y \| \le \mu_n \, \| T^n z - y \| .$$

Now by using Lemma 1.1, T has a fixed point. □

The following result is another version of Lemma 5.1 in [7].

Theorem 2.2. *Let C be a nonempty subset of a Hilbert space H and T a selfmap on C such that*

$$\| Tx-Ty \|^2 \le \| x-y \|^2 + 2\gamma Re\langle x-Tx, y-Ty\rangle + k\| (I-T)x - (I-T)y \|^2$$

for all $x,y \in C$, where γ and k are real fixed numbers with $k<1$. If $\{x_n\}$ converges weakly to z and $\{x_n - Tx_n\}$ tends to 0, then $I-T$ is demiclosed and $z \in F(T)$.

Proof. Suppose that $\{x_n\}$ converges weakly to z and $\{x_n - Tx_n\}$ tends to 0. Then, for each n we have

$$\| Tx_n - Tz \|^2 \le \| x_n - z \|^2 + 2\gamma Re\langle x_n - Tx_n, z - Tz\rangle + k\| x_n - Tx_n + Tz - z \|^2$$

and so

$$\| Tx_n - x_n \|^2 + \| x_n - Tz \|^2 + 2Re\langle Tx_n - x_n, x_n - Tz\rangle = \| Tx_n - x_n + x_n - Tz \|^2$$
$$\le \| x_n - z \|^2 + 2\gamma Re\langle x_n - Tx_n, z - Tz\rangle + k\| x_n - Tx_n + Tz - z \|^2.$$

Thus, we obtain

$$\mu(\{\| Tx_n - x_n \|^2\}) + \mu(\{\| x_n - Tz \|^2\}) + 2\mu(\{Re\langle Tx_n - x_n, x_n - Tz\rangle\})$$
$$\le \mu(\{\| x_n - z \|^2\}) + 2\gamma\mu(\{Re\langle x_n - Tx_n, z - Tz\rangle\})$$
$$+k\mu(\{\| x_n - Tx_n + Tz - z \|^2\}).$$

Since μ is the Banach limit, $\{x_n\}$ converges weakly to z and $\{x_n - Tx_n\}$ tends to 0, we get

$$\mu_n\| x_n - Tz \|^2 \le \mu_n\| x_n - z \|^2 + k\mu_n\| Tz - z \|^2$$

holds for all n. But, for each n we have

$$\mu_n\| x_n - z \|^2 + \mu_n\| z - Tz \|^2 + 2\mu_n Re\langle x_n - z, z - Tz\rangle$$
$$= \mu_n\| x_n - z + z - Tz \|^2 \le \mu_n\| x_n - z \|^2 + k\mu_n\| Tz - z \|^2.$$

Since $\{x_n\}$ converges weakly to z, we obtain $(1-k)\mu_n\| Tz - z \|^2 \le 0$ for all n. Hence, $\| z - Tz \|^2 \le 0$ and so $Tz = z$. This implies that $I-T$ is demiclosed. □

The following result is a generalization of Lemma 2.7 in [2].

Theorem 2.3. *Let C be a nonempty, closed and convex subset of a Hilbert space H and $\{x_n\}$ a sequence in C. Suppose that $T : C \to H$ and $T' : C \to H$ are two mappings and $\{\xi_n\}$ and $\{\xi'_n\}$ are two sequences of real numbers. Define the sequence $\{z_n\}$ in C by $z_n = \frac{1}{n}\sum_{k=1}^{n} x_k$. Suppose that z is a weak cluster point of $\{z_n\}$,*

$$\xi_n + \xi'_n \leq \| x_n - z \|^2 - \| x_{n+1} - Tz \|^2 + \| x_n - z \|^2 - \| x_{n+1} - T'z \|^2$$

holds for all n, $\frac{1}{n}\sum_{k=1}^{n} \xi_k \longrightarrow 0$ and $\frac{1}{n}\sum_{k=1}^{n} \xi'_k \longrightarrow 0$. Then z is a common fixed point of T and T'.

Proof. First, note that

$$\xi_k + \xi'_k \leq \| x_k - z \|^2 - \| x_{k+1} - Tz \|^2 + \| x_k - z \|^2 - \| x_{k+1} - T'z \|^2$$

$$= \| x_k - Tz + Tz - z \|^2 - \| x_{k+1} - Tz \|^2 + \| x_k - T'z + T'z - z \|^2 - \| x_{k+1} - T'z \|^2$$

$$= \| x_k - Tz \|^2 - \| x_{k+1} - Tz \|^2 + 2Re\langle x_k - Tz, Tz - z\rangle + \| Tz - z \|^2$$

$$+ \| x_k - T'z \|^2 - \| x_{k+1} - T'z \|^2 + 2Re\langle x_k - T'z, T'z - z\rangle + \| T'z - z \|^2$$

holds for all k. By summing these inequalities from $k = 1$ to n and dividing by n, we obtain

$$\frac{1}{n}\sum_{k=1}^{n} \xi_k + \frac{1}{n}\sum_{k=1}^{n} \xi'_k \leq \frac{1}{n}(\| x_1 - Tz \|^2 - \| x_{n+1} - Tz \|^2)$$

$$+2Re\langle \frac{1}{n}\sum_{k=1}^{n} x_k - Tz, Tz - z\rangle + \| Tz - z \|^2 + \frac{1}{n}(\| x_1 - T'z \|^2 - \| x_{n+1} - T'z \|^2)$$

$$+2Re\langle \frac{1}{n}\sum_{k=1}^{n} x_k - T'z, T'z - z\rangle + \| T'z - z \|^2$$

$$\leq \frac{1}{n}\| x_1 - Tz \|^2 + 2Re\langle z_n - Tz, Tz - z\rangle + \| Tz - z \|^2$$

$$+ \frac{1}{n}\| x_1 - T'z \|^2 + 2Re\langle z_n - T'z, T'z - z\rangle + \| T'z - z \|^2$$

for all n. Since z is a weak cluster point of $\{z_n\}$, there is a subsequence $\{z_{ni}\}$of $\{z_n\}$ such that $z_{ni} \longrightarrow z$. By replacing n by n_i, we get

$$\frac{1}{n_i}\sum_{k=1}^{n_i} \xi_k + \frac{1}{n_i}\sum_{k=1}^{n_i} \xi'_k \leq \frac{1}{n_i}\| x_1 - Tz \|^2 + 2Re\langle z_{ni} - Tz, Tz - z\rangle$$

$$+ \| Tz - z \|^2 + \frac{1}{n_i}\| x_1 - T'z \|^2 + 2Re\langle z_{ni} - T'z, T'z - z\rangle + \| T'z - z \|^2.$$

since $\frac{1}{n_i}\sum_{k=1}^{n_i} \xi_k \longrightarrow 0$, $\frac{1}{n_i}\sum_{k=1}^{n_i} \xi'_k \longrightarrow 0$ and $z_{ni} \longrightarrow z$, we obtain

$$0 \leq 2Re\langle z - Tz, Tz - z\rangle + \| Tz - z \|^2 + 2Re\langle z - T'z, T'z - z\rangle + \| T'z - z \|^2$$

$$= -\| Tz - z \|^2 - \| T'z - z \|^2.$$

Hence, $Tz = z$ and $T'z = z$. □

In 2011, Kocourek, Takahashi and Yao provided the notion of (α,β)-generalized hybrid mappings. Let C be a nonempty subset of a Hilbert space H and $\alpha,\beta\in\mathbb{R}$. We say that $T:C\longrightarrow C$ is a (α,β)-generalized hybrid mapping whenever

$$\alpha\| Tx-Ty\|^2+(1-\alpha)\| x-Ty\|^2\leq\beta\| Tx-y\|^2+(1-\beta)\| x-y\|^2$$

for all $x,y\in C$ ([7] and [14]). Note that, each (α,β)-generalized hybrid mapping is a nonexpansive mapping for $\alpha=1$ and $\beta=0$, a nonspreading mapping for $\alpha=2$ and $\beta=1$ and a hybrid mapping for $\alpha=\frac{3}{2}$ and $\beta=\frac{1}{2}$. Also, each (α,β)-generalized hybrid mapping is a quasi-nonexpansive mapping ([7]).

The following example shows that the conditions of Theorem 2.2 hold while a similar result is not true for (α,β)-generalized hybrid mappings.

Example 2.4. Consider $C=\{(1,0,0),(0,1,0),(0,0,0)\}$ in Euclidean metric space $\mathbb{R}^3$ and define the selfmap T on C by $T(1,0,0)=(1,0,0)$, $T(0,0,0)=(0,1,0)$ and $T(0,1,0)=(0,0,0)$. Then, T satisfies the conditions of Theorem 2.2 while T is not a (α,β)-generalized hybrid mapping, because by setting $x=(1,0,0)$ and $y=(0,0,0)$ we get a contradiction.

The following example shows that there is a $(2,1)$-generalized hybrid mapping which is not a nonexpansive mapping. One can find its main idea in [10].

Example 2.5. Let H be a Hilbert space. Consider the sets $E=\{x\in H:\| x\|\leq 1\}$, $D=\{x\in H:\| x\|\leq 2\}$ and $C=\{x\in H:\| x\|\leq 3\}$. Define the selfmap S on C by

$$Sx=\begin{cases}0 & x\in D\\ P_E(x) & x\in C\backslash D\end{cases}$$

where P_E is the metric projection on E. It is easy to see that S is not a nonexpansive mapping while it is a $(2,1)$-generalized hybrid mapping.

The proof of the following result is straightforward (note that, $(\frac{\alpha-1}{\alpha})<1$).

Proposition 2.6. *Let C be a nonempty subset of a Hilbert space H, $\alpha>0$ and T a selfmap on C. Then, T is a $(\alpha,\alpha-1)$-generalized hybrid mapping if and only if T is a $\frac{\alpha-1}{\alpha}$-nonexpansive mapping if and only if T is a $(2-\alpha)$-hybrid mapping.*

The following example shows that there are discontinuous (α,β)-generalized hybrid mappings. Main idea of this example provided by Aoyama and Kohsaka in [3].

Example 2.7. Let E be a Banach space and $S,T:E\longrightarrow E$ two firmly nonexpansive mappings such that $S(E)$ and $T(E)$ are contained by rB_E for some $r>0$. Let α and δ be real numbers such that $1<\alpha\leq 2$ and $\delta\geq(1+\frac{2}{\sqrt{\frac{\alpha-1}{\alpha}}})r$. Define the map $U:E\longrightarrow E$ by

$$Ux=\begin{cases}Sx & x\in\delta B_E\\ Tx & otherwise\end{cases}$$

Then, U is a discontinuous $(\alpha,\alpha-1)$-generalized hybrid mapping.

Also, Kocourek, Takahashi and Yao provided the notion of (α,β,γ)-super hybrid mappings. Let C be a nonempty, closed and convex subset of a Hilbert space H and $\alpha,\beta,\gamma\in\mathbb{R}$ with $\gamma\geq 0$. We say that $T:C\longrightarrow C$ is a (α,β,γ)-super hybrid mapping whenever

$$\alpha\| Tx-Ty\|^2+(1-\alpha+\gamma)\| x-Ty\|^2\leq(\beta+(\beta-\alpha)\gamma)\| Tx-y\|^2$$
$$+(1-\beta-(\beta-\alpha-1)\gamma)\| x-y\|^2+(\alpha-\beta)\gamma\| x-Tx\|^2+\gamma\| y-Ty\|^2$$

for all $x, y \in C$ ([7]). Note that, each $(\alpha, \beta, 0)$-super hybrid mapping is a (α, β)-generalized hybrid mapping. By using this idea, we are going to generalize the notion of α-nonexpansivity in the following form. Let E be a Banach space, C a nonempty subset of E and α and β two real numbers with $\beta > \frac{-1}{2}$. A mapping $T : C \longrightarrow E$ is said to be (α, β)-nonexpansive if

$$(1-\alpha) \parallel Tx - Ty \parallel^2 + \alpha \parallel T^2x - T^2y \parallel^2 \leq (\frac{1}{2} - (\alpha+\beta)) \parallel Tx - y \parallel^2$$

$$+ (\frac{1}{2} - (\alpha+\beta)) \parallel x - Ty \parallel^2 + \alpha \parallel T^2x - Ty \parallel^2 + \alpha \parallel Tx - T^2y \parallel^2 + 2\beta \parallel x - y \parallel^2$$

for all $x, y \in C$. If $\alpha = 0$ and $\alpha' = \frac{1}{2} - \beta$, then the notion of (α, β)-nonexpansivity reduces to the notion of α'-nonexpansivity. It is easy to see that each (α, β)-nonexpansive mapping is a quasi-nonexpansive mapping. Finally, note that by using a similar proof in Theorem 2.1, we can prove the following result.

Theorem 2.8. *Let C be a nonempty, closed and convex subset of a Hilbert space H, α and β two real numbers with $\beta > \frac{-1}{2}$ and $\alpha \leq 0$ and T a (α, β)-nonexpansive selfmap on C. Then, T has a fixed point in C if and only if $\{T^n z\}$ is a bounded sequence for some $z \in C$.*

The proof of the following result is straightforward (note that, $\frac{\frac{1}{2}-\beta}{1-\alpha} < 1$).

Proposition 2.9. *Let C be a nonempty subset of a normed space E, α and β two real numbers with $\alpha < 1$ and $\alpha - \beta < \frac{-1}{2}$ and T a (α, β)-nonexpansive selfmap on C such that T^2 is the identity map. Then T is a $\frac{\frac{1}{2}-\beta}{1-\alpha}$-nonexpansive mapping.*

Theorem 2.10. *Let C be a nonempty, closed and convex subset of a strictly convex Banach space E and T a (α, β)-nonexpansive selfmap on C. Then $F(T)$ is a closed and convex subset of E.*

Proof. If $F(T)$ is empty, then it is clear that $F(T)$ is closed and convex. Let $F(T) \neq \emptyset$. Since T is quasi-nonexpansive, by using a result of Itoh and Takahashi ([5]), we get that $F(T)$ is a closed and convex subset of E. □

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