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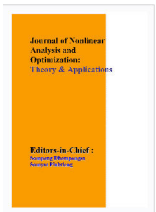

Strong convergence of a new two-step iterative scheme for two quasi-nonexpansive multi-valued maps in Banach spaces

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ABSTRACT: In this paper, a new two-step iterative scheme is introduced for two quasi-nonexpansive multi-valued maps in Banach spaces. Strong convergence theorem of the purposed iterative scheme is established for quasi-nonexpansive multi-valued maps in Banach spaces. The result obtained in this paper improve and extend the corresponding one announced by Shahzad and Zegeye [N. Shahzad, H. Zegeye, On Mann and Ishikawa iteration schemes for multi-valued maps in Banach spaces, Nonlinear Analysis 71 (2009) 838-844.].

KEYWORDS: Quasi-nonexpansive multi-valued map; Nonexpansive multi-valued map; Common fixed point; Strong convergence; Banach space.

1. Introduction

Let D be a nonempty convex subset of a Banach spaces E. The set D is called *proximinal* if for each $x \in E$, there exists an element $y \in D$ such that $\|x - y\| = d(x, D)$, where $d(x, D) = \inf\{\|x - z\| : z \in D\}$. Let $CB(D), K(D)$ and $P(D)$ denote the families of nonempty closed bounded subsets, nonempty compact subsets, and nonempty proximinal bounded subsets of D, respectively. The *Hausdorff metric* on $CB(D)$ is defined by

$$H(A, B) = \max\left\{\sup_{x\in A} d(x, B),\ \sup_{y\in B} d(y, A)\right\}$$

for $A, B \in CB(D)$. A single-valued map $T : D \to D$ is called *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in D$. A multi-valued map $T : D \to CB(D)$ is said to be *nonexpansive* if $H(Tx, Ty) \leq \|x - y\|$ for all $x, y \in D$. An element $p \in D$ is called a fixed point of $T : D \to D$ (respectively, $T : D \to CB(D)$) if $p = Tp$ (respectively, $p \in Tp$). The set of fixed points of T is denoted by $F(T)$. The mapping $T : D \to CB(D)$ is called *quasi-nonexpansive*[13] if $F(T) \neq \emptyset$ and $H(Tx, Tp) \leq \|x - p\|$ for all $x \in D$ and all $p \in F(T)$. It is clear that every nonexpansive multi-valued map T with $F(T) \neq \emptyset$ is quasi-nonexpansive. But there exist quasi-nonexpansive mappings that are not nonexpansive, see [12].

The mapping $T : D \to CB(D)$ is called *hemicompact* if, for any sequence $\{x_n\}$ in D such that $d(x_n, Tx_n) \to 0$ as $n \to \infty$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \to p \in D$. We note that if D is compact, then every multi-valued mapping $T : D \to CB(D)$ is *hemicompact*.

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A mapping $T : D \to CB(D)$ is said to satisfy *Condition (I)* if there is a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for $r \in (0, \infty)$ such that

$$d(x, Tx) \geq f(d(x, F(T)))$$

for all $x \in D$.

A family $\{T_i : D \to CB(D), i = 1, 2, ..., N\}$ is said to satisfy *Condition (II)* if there is a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for $r \in (0, \infty)$ such that

$$d(x, T_i x) \geq f(d(x, \bigcap_{i=1}^{N} F(T_i)))$$

for all $i = 1, 2, ..., N$ and $x \in D$.

In 1953, Mann [6] introduced the following iterative procedure to approximate a fixed point of a nonexpansive mapping T in a Hilbert space H:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T x_n, \quad \forall n \in \mathbb{N}, \tag{1}$$

where the initial point x_0 is taken in C arbitrarily and $\{\alpha_n\}$ is a sequence in [0,1].

However, we note that Mann's iteration process (1) has only weak convergence, in general; for instance, see [1, 3, 9].

In 2005, Sastry and Babu [10] proved that the Mann and Ishikawa iteration schemes for multi-valued map T with a fixed point p converge to a fixed point q of T under certain conditions. They also claimed that the fixed point q may be different from p. More precisely, they proved the following result for nonexpansive multi-valued map with compact domain.

In 2007, Panyanak [8] extended the above result of Sastry and Babu [10] to uniformly convex Banach spaces but the domain of T remains compact.

Later, Song and Wang [14] noted that there was a gap in the proofs of Theorem 3.1(see [8]) and Theorem 5 (see [12]). They further solved/revised the gap and also gave the affirmative answer to Panyanak [8] question using the following Ishikawa iteration scheme. In the main results, domain of T is still compact, which is a strong condition (see [14], Theorem 1) and T satisfies condition(I) (see [14], Theorem 1).

In 2009, Shahzad and Zegeye [10] extended and improved the results of Panyanak [8], Sastry and Babu [12] and Song and Wang [14] to quasi-nonexpansive multi-valued maps. They also relaxed compactness of the domain of T. The results provided an affirmative answer to Panyanak [8] question in a more general setting. They introduced a new iteration as follows: Let D be a nonempty convex subset of a Banach space E and $\alpha_n, \alpha'_n \in [0, 1]$. Let $T : D \to P(D)$ and $P_T x = \{y \in Tx : \|x - y\| = d(x, Tx)\}$. The sequence of Ishikawa iterates is defined by $x_0 \in D$,

$$\begin{aligned} y_n &= \alpha'_n z'_n + (1 - \alpha'_n) x_n, \quad n \geq 0, \\ x_{n+1} &= \alpha_n z_n + (1 - \alpha_n) x_n, \quad n \geq 0, \end{aligned} \tag{2}$$

where $z'_n \in P_T x_n$ and $z_n \in P_T y_n$.

Since 2003, the iterative schemes with errors for a single-valued map in Banach spaces have been studied by many authors, see [2, 4, 5, 7].

Question: How can we modify Mann and Ishikawa iterative schemes with errors to obtain convergence theorems for finding a common fixed point of two multi-valued nonexpansive maps ?

Motivated by Shahzad and Zegeye [12], we purpose a new two-step iterative scheme for two multi-valued quasi-nonexpansive maps in Banach spaces and prove strong convergence theorems of the purposed iteration.

2. **Main Results**

We use the following iteration scheme:
Let D be a nonempty convex subset of a Banach space E, $\alpha_n, \beta_n, \alpha'_n, \beta'_n \in [0, 1]$ and $\{u_n\}, \{v_n\}$ are bounded sequences in D.

Let T_1, T_2 be two quasi-nonexpansive multi-valued maps from D into $P(D)$ and $P_{T_i}x = \{y \in T_i x : \|x-y\| = d(x, T_i x)\}$, $i = 1, 2$. Let $\{x_n\}$ be the sequence defined by $x_0 \in D$,

$$y_n = \alpha'_n z'_n + \beta'_n x_n + (1-\alpha'_n-\beta'_n)u_n, \quad n \geq 0,$$
$$x_{n+1} = \alpha_n z_n + \beta_n x_n + (1-\alpha_n-\beta_n)v_n, \quad n \geq 0, \tag{3}$$

where $z'_n \in P_{T_1}x_n$ and $z_n \in P_{T_2}y_n$.

We shall make use of the following results.

Lemma 2.1. [15] *Let $\{s_n\}, \{t_n\}$ be two nonnegative sequences satisfying*

$$s_{n+1} \leq s_n + t_n, \quad \forall n \geq 1.$$

If $\sum_{n=1}^{\infty} t_n < \infty$ then $\lim_{n\to\infty} s_n$ exists.

Lemma 2.2. [11] *Suppose that E is a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$ for all positive integers n. Also suppose that $\{x_n\}$ and $\{y_n\}$ are two sequences of E such that $\limsup_{n\to\infty}\|x_n\| \leq r$, $\limsup_{n\to\infty}\|y_n\| \leq r$ and $\lim_{n\to\infty}\|t_n x_n + (1-t_n)y_n\| = r$ hold for some $r \geq 0$. Then $\lim_{n\to\infty}\|x_n - y_n\| = 0$.*

Theorem 2.3. *Let E be a uniformly convex Banach space, D a nonempty, closed and convex subset of E, and T_1, T_2 be two multi-valued maps from D into $P(D)$ with $F(T_1) \cap F(T_2) \neq \emptyset$ such that P_{T_1}, P_{T_2} are nonexpansive. Assume that*
(i) *$\{T_1, T_2\}$ satisfies condition (II);*
(ii) *$\sum_{n=1}^{\infty}(1-\alpha_n-\beta_n) < \infty$ and $\sum_{n=1}^{\infty}(1-\alpha'_n-\beta'_n) < \infty$;*
(iii) *$0 < \ell \leq \alpha_n, \alpha'_n \leq k < 1$.*
Then the sequence $\{x_n\}$ generated by (3) converges strongly to some elements in $F(T_1) \cap F(T_2)$.

Proof. We split the proof into three steps.

Step 1. Show that $\lim_{n\to\infty}\|x_n - p\|$ exists for all $p \in F(T_1) \cap F(T_2)$.

Let $p \in F(T_1) \cap F(T_2)$. Then $P_{T_1}p = \{p\}$ and $P_{T_2}p = \{p\}$. Since u_n, v_n are bounded, therefore exists $M > 0$ such that $\max\{\sup_{n\in\mathbb{N}}\|u_n - p\|, \sup_{n\in\mathbb{N}}\|v_n - p\|\} \leq M$. Then

$$\begin{aligned}
\|y_n - p\| &\leq \alpha'_n\|z'_n - p\| + \beta'_n\|x_n - p\| + (1-\alpha'_n-\beta'_n)\|u_n - p\| \\
&\leq \alpha'_n d(z'_n, P_{T_1}p) + \beta'_n\|x_n - p\| + (1-\alpha'_n-\beta'_n)M \\
&\leq \alpha'_n H(P_{T_1}x_n, P_{T_1}p) + \beta'_n\|x_n - p\| + (1-\alpha'_n-\beta'_n)M \\
&\leq (\alpha'_n + \beta'_n)\|x_n - p\| + (1-\alpha'_n-\beta'_n)M \\
&\leq \|x_n - p\| + (1-\alpha'_n-\beta'_n)M. \qquad (4)
\end{aligned}$$

It follows that

$$\begin{aligned}
\|x_{n+1} - p\| &\leq \alpha_n\|z_n - p\| + \beta_n\|x_n - p\| + (1-\alpha_n-\beta_n)\|v_n - p\| \\
&= \alpha_n d(z_n, P_{T_2}p) + \beta_n\|x_n - p\| + (1-\alpha_n-\beta_n)M \\
&\leq \alpha_n H(P_{T_2}y_n, P_{T_2}p) + \beta_n\|x_n - p\| + (1-\alpha_n-\beta_n)M \\
&\leq \alpha_n\|y_n - p\| + \beta_n\|x_n - p\| + (1-\alpha_n-\beta_n)M \\
&\leq \alpha_n\big(\|x_n - p\| + (1-\alpha'_n-\beta'_n)M\big) + \beta_n\|x_n - p\| \\
&\quad + (1-\alpha_n-\beta_n)M \\
&= (\alpha_n + \beta_n)\|x_n - p\| + \big(\alpha_n(1-\alpha'_n-\beta'_n) + (1-\alpha_n-\beta_n)\big)M \\
&\leq \|x_n - p\| + \big(\alpha_n(1-\alpha'_n-\beta'_n) + (1-\alpha_n-\beta_n)\big)M \\
&= \|x_n - p\| + \varepsilon_n, \qquad (5)
\end{aligned}$$

where $\varepsilon_n = \big(\alpha_n(1-\alpha'_n-\beta'_n) + (1-\alpha_n-\beta_n)\big)M$. By (ii), we have $\varepsilon_n \to 0$ as $n \to \infty$. Thus by Lemma 2.1, we have $\lim_{n\to\infty}\|x_n - p\|$ exists for all $p \in F(T_1) \cap F(T_2)$.

Step 2. Show that $\lim_{n\to\infty}\|z_n - x_n\| = 0 = \lim_{n\to\infty}\|z'_n - x_n\|$.

Let $p \in F(T_1) \cap F(T_2)$. By Step 1, By Step 1, there is a real number $c > 0$ such that $\lim_{n\to\infty}\|x_n - p\| = c$. Let $S = \max\{\sup_{n\in\mathbb{N}}\|v_n - y_n\|, \sup_{n\in\mathbb{N}}\|u_n - x_n\|\}$. From 4, we get

$$\limsup_{n\to\infty}\|y_n - p\| \leq c. \tag{6}$$

Next, we consider

$$\begin{aligned}\|z_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\| &\le \|z_n - p\| + (1-\alpha_n-\beta_n)\|v_n - x_n\| \\ &\le d(z_n, P_{T_2}p) + (1-\alpha_n-\beta_n)S \\ &\le H(P_{T_2}y_n, P_{T_2}p) + (1-\alpha_n-\beta_n)S \\ &\le \|y_n - p\| + (1-\alpha_n-\beta_n)S\end{aligned}$$

It follows that

$$\limsup_{n\to\infty}\|z_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\| \le c.$$

Also

$$\begin{aligned}\|x_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\| &\le \|x_n - p\| + (1-\alpha_n-\beta_n)\|v_n - x_n\| \\ &\le \|x_n - p\| + (1-\alpha_n-\beta_n)S\end{aligned}$$

which implies that

$$\limsup_{n\to\infty}\|x_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\| \le c.$$

Since

$$\begin{aligned}\lim_{n\to\infty} \| &\quad \alpha_n\big(z_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\big) \\ + &\quad (1-\alpha_n)\big(x_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\big)\| = \lim_{n\to\infty}\|x_{n+1} - p\| = c,\end{aligned}$$

by Lemma 2.2, we obtain that

$$\lim_{n\to\infty}\|z_n - x_n\| = 0. \tag{7}$$

By the nonexpansiveness of P_{T_2}, we have

$$\begin{aligned}\|x_n - p\| &\le \|x_n - z_n\| + \|z_n - p\| \\ &= \|x_n - z_n\| + d(z_n, P_{T_2}p) \\ &\le \|x_n - z_n\| + H(P_{T_2}y_n, P_{T_2}p) \\ &\le \|x_n - z_n\| + \|y_n - p\|\end{aligned}$$

which implies

$$c \le \liminf_{n\to\infty}\|y_n - p\| \le \limsup_{n\to\infty}\|y_n - p\| \le c.$$

Hence $\lim_{n\to\infty}\|y_n - p\| = c$. Since

$$\begin{aligned}y_n - p &= \alpha'_n\big(z'_n - p + (1-\alpha'_n-\beta'_n)(u_n - x_n)\big) \\ &\quad + (1-\alpha'_n)\big(x_n - p + (1-\alpha'_n-\beta'_n)(u_n - x_n)\big),\end{aligned}$$

we have

$$\begin{aligned}\lim_{n\to\infty} \| &\quad \alpha'_n\big(z'_n - p + (1-\alpha'_n-\beta'_n)(u_n - x_n)\big) \\ + &\quad (1-\alpha'_n)\big(x_n - p + (1-\alpha'_n-\beta'_n)(u_n - x_n)\big)\| = c.\end{aligned}$$

Moreover, we get

$$\begin{aligned}\|z'_n - p + (1-\alpha'_n-\beta'_n)(u_n - x_n)\| &\le \|z'_n - p\| + (1-\alpha'_n-\beta'_n)\|u_n - x_n\| \\ &\le d(z'_n, P_{T_1}p) + (1-\alpha'_n-\beta'_n)S \\ &\le H(P_{T_1}x_n, P_{T_1}p) + (1-\alpha'_n-\beta'_n)S \\ &\le \|x_n - p\| + (1-\alpha'_n-\beta'_n)S.\end{aligned}$$

This yields that

$$\limsup_{n\to\infty}\|z'_n - p + (1-\alpha'_n-\beta'_n)(u_n - x_n)\| \le c.$$

Also

$$\begin{aligned}\|x_n - p + (1-\alpha'_n - \beta'_n)(u_n - x_n)\| &\leq \|x_n - p\| + (1-\alpha'_n - \beta'_n)\|u_n - x_n\| \\ &\leq \|x_n - p\| + (1-\alpha'_n - \beta'_n)S.\end{aligned}$$

This implies that

$$\limsup_{n\to\infty} \|x_n - p + (1-\alpha'_n - \beta'_n)(u_n - x_n)\| \leq c.$$

Again by Lemma 2.2, we have

$$\lim_{n\to\infty} \|z'_n - x_n\| = 0. \tag{8}$$

Step 3. Show that $\{x_n\}$ converges strongly to q for some $q \in F(T_1) \cap F(T_2)$

From Step 2, we know that $\lim_{n\to\infty} \|z_n - x_n\| = 0 = \lim_{n\to\infty} \|z'_n - x_n\|$. Also $d(x_n, T_1x_n) \leq d(x_n, P_{T_1}x_n) \leq \|z'_n - x_n\| \to 0$ as $n \to \infty$. Since $\{x_n\}, \{u_n\}$ are bounded, so is $\{u_n - z'_n\}$. Now, let $K = \sup_{n\in\mathbb{N}} \|u_n - z'_n\|$. By assumption and (8), we get

$$\begin{aligned}\|y_n - z'_n\| &\leq \|\alpha'_n z'_n + \beta'_n x_n + (1-\alpha'_n - \beta'_n)u_n - z'_n\| \\ &\leq \beta'_n\|x_n - z'_n\| + (1-\alpha'_n - \beta'_n)\|u_n - z'_n\| \\ &\leq \beta'_n\|x_n - z'_n\| + (1-\alpha'_n - \beta'_n)K \\ &\to 0 \qquad (9)\end{aligned}$$

as $n \to \infty$. It follows from (8) and (9) that

$$\begin{aligned}\|y_n - x_n\| &\leq \|y_n - z'_n\| + \|z'_n - x_n\| \\ &\to 0 \qquad (10)\end{aligned}$$

as $n \to \infty$. It follows from (7) and (10) that

$$\begin{aligned}d(x_n, T_2x_n) &\leq d(x_n, P_{T_2}x_n) \\ &\leq d(x_n, P_{T_2}y_n) + H(P_{T_2}y_n, P_{T_2}x_n) \\ &\leq \|x_n - z_n\| + \|y_n - x_n\| \\ &\to 0.\end{aligned}$$

Since that $\{T_1, T_2\}$ satisfies the condition (II), we have $d(x_n, F(T_1) \cap F(T_2)) \to 0$. Thus there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and a sequence $\{p_k\} \subset F(T_1) \cap F(T_2)$ such that

$$\|x_{n_k} - p_k\| < \frac{1}{2^k} \tag{11}$$

for all k. From (5), we obtain

$$\begin{aligned}\|x_{n_{k+1}} - p\| &\leq \|x_{n_{k+1}-1} - p\| + \varepsilon_{n_{k+1}-1} \\ &\leq \|x_{n_{k+1}-2} - p\| + \varepsilon_{n_{k+1}-2} + \varepsilon_{n_{k+1}-1} \\ &\vdots \\ &\leq \|x_{n_k} - p\| + \sum_{i=0}^{n_{k+1}-n_k-1} \varepsilon_{n_k+i}\end{aligned}$$

for all $p \in F(T_1) \cap F(T_2)$. This implies that

$$\|x_{n_{k+1}} - p_k\| \leq \|x_{n_k} - p_k\| + \sum_{i=0}^{n_{k+1}-n_k-1} \varepsilon_{n_k+i} < \frac{1}{2^k} + \sum_{i=0}^{n_{k+1}-n_k-1} \varepsilon_{n_k+i}.$$

Next, we shall show that $\{p_k\}$ is Cauchy sequence in D. Notice that

$$\begin{aligned}\|p_{k+1}-p_k\| &\le \|p_{k+1}-x_{n_{k+1}}\|+\|x_{n_{k+1}}-p_k\|\\ &< \frac{1}{2^{k+1}}+\frac{1}{2^k}+\sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i}\\ &< \frac{1}{2^{k-1}}+\sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i}.\end{aligned}$$

This implies that $\{p_k\}$ is Cauchy sequence in D and thus converges to $q \in D$. Since

$$d(p_k, T_i q) \le d(p_k, P_{T_i} q) \le H(P_{T_i} q, P_{T_i} p_k) \le \|q - p_k\|$$

for all $i = 1, 2$ and $p_k \to q$ as $n \to \infty$, it follows that $d(q, T_i q) = 0$ for all $i = 1, 2$ and thus $q \in F(T_1) \cap F(T_2)$. It implies by (11) that $\{x_{n_k}\}$ converges strongly to q. Since $\lim_{n\to\infty} \|x_n - q\|$ exists, it follows that $\{x_n\}$ converges strongly to q. This completes the proof. □

For $T_1 = T_2 = T$ and $\alpha_n + \beta_n = 1 = \alpha'_n + \beta'_n$ in Theorem 2.3, we obtain the following result.

Theorem 2.4. (See [12], Theorem 2.7) *Let E be a uniformly convex Banach space, D a nonempty, closed and convex subset of E, and $T : D \to P(D)$ a multi-valued map with $F(T) \neq \emptyset$ such that P_T is nonexpansive. Let $\{x_n\}$ be the Ishikawa iterates defined by (B). Assume that T satisfies condition (I) and $\alpha_n, \alpha'_n \in [a, b] \subset (0, 1)$. Then $\{x_n\}$ converges strongly to a fixed point of T.*

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